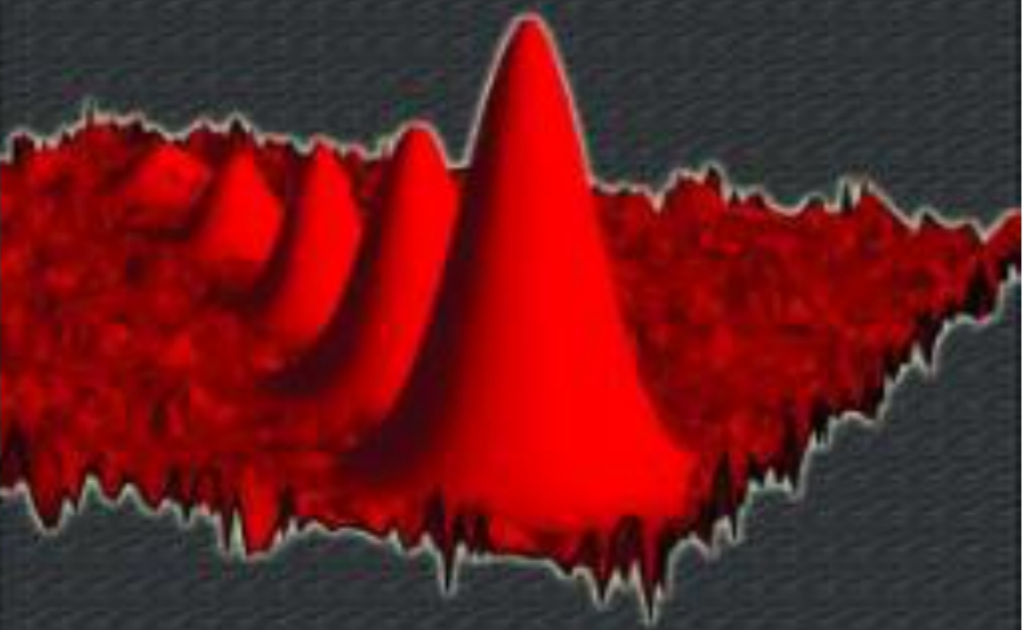
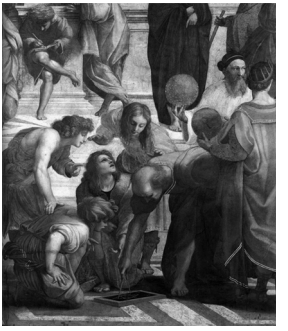
A New Vision on PHYSIS

Eurhythmy, Emergence and Nonlinearity



Edited by

J. R. Croca and J. E. F. Araújo



Center for Philosophy of Science
University of Lisbon

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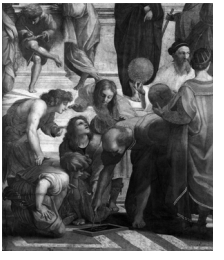

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J. R. Croca and J. E. F. Araújo

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FOREWORD

The present book aims to be a Manifest for a new vision, a new way to look at the natural phenomena. That is why a global approach with innovative radical conceptual tools and a new formal apparatus is proposed to overcome the old and traditional simplistic Cartesian linear model, which has shown evident signs of exhaustion and wornness.

The traditional Cartesian linear model, initiated mainly with Galileo Galilee, where the *whole* is equal to the sum of the composing parts and the action is proportional to the reaction, was indeed a great human achievement. It is on the basis of the emergence of the scientific revolution and the consequent enormous technological development. The modifications brought by this scientific and technological revolution drastically changed the everyday life of people, thus leading to the industrial revolution and subsequent political convulsions such as the French revolution and the Marxist socialist revolution.

Nonetheless and nowadays, about four hundred years after Galilee, this simplistic linear model has shown its limits of applicability, clearly indicating the necessity of a change. Still, this necessary change is not a simple mitigated adjustment that could eventually be accomplished with a few minor modifications which would keep the hard core of the Cartesian method untouched. It is, on the contrary, a basic radical change. It is necessary to assume from the very beginning that the natural phenomena - the reciprocal interactions among the diverse physical systems - are of an extreme complexity and, consequently, need an adequate description in a global nonlinear kind of approach.

In this new way to look at the natural phenomena it is assumed that, in general, the *whole* is different from the sum of the constituent parts and that a minute action may, under certain conditions, give origin to a huge reaction (or vice-versa). These conclusions do not come as a surprise since, in these cases, the emergent entities behave in a completely different way so that it cannot be inferred from the properties of the constituent parts. The mathematical formulation of the new Physics is done through the organizing principle of Eurhythmy.

The present book comes as a result from the long, continued and enlightening discussions among the members of the Research Group on

Foundation of Quantum Physics of the Center for Philosophy of Sciences of the University of Lisbon (Project FCT: Philosophy of Quantum Physics PTDC/FIL/66598/2006), a group devoted to the study of Natural Philosophy. Each researcher of the team has contributed, according to his particular view and formation, with its own part to the body of the present book.

The first contribution by J.R. Croca, *Hyperphysis, The Unification of Physics*, lays down the general conceptual and formal framework for the new Physics.

The second part by M. Gatta, *Preliminary Results from Computer Simulations of Elementary Particle Interactions as an Application of the Principle of Eurhythmy*, presents a beautiful numerical simulation of the stochastic interaction for the stability of the complex particle. Furthermore numerically simulates with great skill coalescence and anti-coalescence theorems.

The third part by A. Rica da Silva, *Elementary Nonlinear Mechanics of Localized Fields*, deals with the mathematical analytical resolution of the fundamental nonlinear master equation for the most typical cases namely, the harmonic oscillator, hydrogen atom, Kepler problem and the one-dimensional potential.

The fourth part by A. Rica da Silva, *Symmetry Generated Solutions of a Nonlinear Schrödinger Equation*, studies related nonlinear Schrödinger Equations and the symmetry generated solutions of the pair of equations that classically correspond to the Hamilton-Jacobi equation for an action and the continuity equation for an extended density.

The fifth by J.E.F. Araújo, *Investigating the Infinity Slope in a Nonlinear Approach*, presents a mathematical solution to the nonlinear master equation for a particular idealized case.

The sixth part by J.R. Croca and M.M. Silva, *Some Fundamentals of Particle Physics in the Light of Hyperphysis*, presents an interesting application of the new Physics for the deep and clear explanation of some problems that plague particle physics.

The seventh part by J.E.F. Araújo, *A Physical Theory on the Complex and the Nonlinear. A Cartographic Perspective*, proposes a brief analysis of the Global new Physics, nonlinear and causal. In this sense he puts forward a

provisional cartography of that theory and, at the same time, he highlights some critical essential topics of the Orthodox Quantum Theory.

The eight part by J. Cordovil, *Some Principles of Philosophy of Physical Nature,* presents in a sketchy like-wise philosophical experiment a small number of propositions that are, according to him, *a priori* to the new Physics that is the subject of the present book. To be more precise, this contribution deals with propositions concerning the nature of physical objects.

The ninth part by R.N. Moreira, *The Crisis in Theoretical Physics Science, Philosophy and Metaphysics***,** presents a general panoramic on the crisis in natural sciences calling the attention to some philosophical consequences of the new Way of looking at the complex phenomena namely in the so-called soft sciences.

The tenth part by G. Magalhães, *On Eurhythmy as a Principle for Growing Order and Complexity in the Natural World*, is the introductory part of a more extensive work on the organizing principle of eurhythmy applied to the diverse sciences and to the problem of the emergent order in nature.

The eleventh part by G.C. Santos, *Between two Worlds. Nonlinearity and a New Mechanistic Approach*, presents a remarkable historical and philosophical view of the birth and evolution of the emergence concept and discusses the origins of the linear and nonlinear ways of understanding the natural phenomena.

The twelfth by P. Alves, *Theses Towards a New Natural Philosophy*, presents in a most daring way a proposal, an attitude for doing philosophy. Due to the increasing development of the natural sciences, he proposes a line of philosophical research which could indeed prove capable of improving our knowledge of reality – that is: a research not for the sake of interpretation of the consecrated authors, but for the sake of truth.

The last part by J.R. Croca and M.M. Silva, *The Concept of Time*, presents the challenging idea that Time, understood in the sense of becoming, that is, of change, is a much more basic concept than space.

The editors.

HYPERPHYSIS
THE UNIFICATION OF PHYSICS

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Summary: Hyperphysis is a global physics that represents a step forward in the sense that it consents the unification of its branches. This new global approach is developed in its fundamentals in the present work. Hyperphysis, assumes, as a starting point, that natural physical phenomena are indeed very complex and consequently to be best described need a nonlinear approach. This method, based on the organizing principle of eurhythmy, not only allows the unification of physics but also leads to a deeper and clearer understanding of the physical reality. In light of this nonlinear approach for studying the complex phenomena and with the help of the basic organizing principle of eurhythmy classical, relativist and quantum physics are susceptible of a unique unitary and causal description. Furthermore it allows a deeper understanding of what is commonly called gravitation, gravitic interaction, the concept of mass and what is to be understood by gravitic particle and what is behind the electromagnetic phenomena. In addition this nonlinear approach furnishes the basis for understanding what lies behind the invariance of the velocity of the light in the most common circumstances.

Keywords: Hyperphysis, unity of physics, nonlinear physics, linear physics, principle of eurhythmy, subquantum medium, emergence, concept of complex particle, extreme principles, nonlinear quantum physics, photonic motion, gravitic interaction, universal attraction law, gravitic particle, concept of mass, electromagnetic interaction, nonlinear electromagnetic phenomena.

1. OVERTURE

In the present work I intend to lay down the fundamentals of a new way, a new method to look at the natural phenomena. After some hesitations this method, this body of knowledge, is named from now on by Hyperphysis[1]. Prior to the discussion and presentation of this global complex integrating new physics it is convenient to look at what has been till now essentially the physical knowledge.

The scientific revolution of the XVIIth Century initiated mainly with Galileo came out, in its essentials, as a consequence of assuming, as basic starting point, the simplistic linear approach for describing the natural phenomena. Galileo[2] reached the notable accomplishment of explaining the motion of the bodies as a composition of two motions: The vertical and the horizontal motions. Being that the final motion resulted to be the simple addition of the two individual motions. In this way the principle of superposition was implanted in physics. On the other hand also in the study of the relative motion, initiated with Giordano Bruno[3], this principle of superposition, that later came to be called the principle of relativity of Galileo, also showed to be very adequate in the description of the then studied phenomena, that is, at the classic, or macroscopic scale of description of physis.

Descartes[4] went further by proceeding with the systematization and generalization of this principle of superposition, the principle of addition of velocities of Galileo, as a simple consequence of the linear approach where, as known, it is assumed that the whole is equal to the sum of the parts. In this process of description of the phenomena, to study a part or to study the whole is precisely the same thing. Implicitly, it is assumed that the different parts that constitute the whole, when in interaction, do not modify themselves reciprocally. On the other hand, there exists a direct linear relationship between the action and reaction. The greater the action, the greater the reaction or vice-versa. Newton[5] goes even further when in his Principia he postulates the principle of superposition as the conceptual instrumental basis for his mechanics. Another implicit direct consequence of these assumptions is the abstract notion of referential. Also this concept of non physical abstract referential infinite in time and space was originated mainly by Descartes. From then on this simplistic linear approach stands rooted not only in physics but also practically in every domain of human activity as the very cornerstone, as the only and the true process for understanding all phenomena. Naturally, we must be aware that due to its very nature the complex nonlinear phenomena were always present even at the classical scale of description of physical reality. Still, even when it could not be neglected it was introduced surreptitiously, as nuisance, as a kind of

resistance, a disturbance that is, a noise shadowing the so called purity of the "true laws of Nature".

Yet, this was indeed a very great human achievement, not only because of the great conceptual and practical simplicity of the method but also due to its enormous capacity of explaining and predicting the then observed phenomena and consequently of acting in the World. Nevertheless, we must bear in mind that although its great effectiveness, it was a no more than a mere human construction so, inevitably, sooner or later it would show its limits. Still, mainly owing to its great concrete success it was considered by the great majority of thinkers as the last, as the complete and the final work of mankind[6]. Naturally, this comfortable attitude, of believing that we have and furthermore own the Truth, more characteristic of a religious belief is against the true spirit of physics that in its essence is a continuous search for the Truth.

Still, this state of things of the omnipresence of the principle of superposition, of the Cartesian method for dealing with the problems, remains then in the main current of physics, known as classic physics, without great problems until the beginning of the XXth Century. Naturally, as expected, there were opponents, from which we must point out the great figure of Leibniz[7], followed by Huygens[8], Bernoulli[9] and others. The work of this sector of thinkers gave, later on, origin to the field theories[10]. However all these developments were, for better or worse, integrated in the mechanics paradigm where ruled omnipresent and omnipotent the simplistic linear Cartesian principle of superposition.

With the advent of the XXth Century, due to the development of experimental better tools, in the field of determinations of very short times, with the help of interferometers, allowing the minute determinations of the change at great velocities and in the field of microphysics, things started to get a bit confused. Nevertheless, since people were so habituated, or better to say completely addicted, to the Cartesian linear method that had been proven extremely useful for about three hundred years they, naturally as it should be expected, looked for the explanation of the new experimental phenomena in the traditional Cartesian conceptual framework.

The inability of the traditional Cartesian method showed mainly on two fronts: The first one, in the domain of great velocities, velocities near to the speed of the light. In this field, that is, at this short temporal scale of observation of the natural phenomena, the habitual law of the addition of velocities, the principle of superposition, showed its limits by being incapable of explaining the result of certain experiences, namely the interferometric experiments of Michelson and Morley[11]. What happens is that at these high velocities, velocities near the saturation, the principle of superposition, a simple consequence of the simplistic Cartesian linear

method, is no more adequate to describe the real physical situation. As understood, we are dealing essentially with a nonlinear phenomenon where the reciprocal interaction between the system and the medium no longer can be neglected and therefore must be taken in account. In this case it is not a surprise to find out that the law of addition of velocities based in the notion of an abstract physically nonexistent, infinite in time and space, referential does not hold anymore. In fact what happens is that the acron or luminous corpuscle integrated in the field generated by its theta wave, although having a very great velocity, (its instantaneous natural velocity), due to the reciprocal interaction with the subquantum chaotic medium undergoes a kind of retardation in its velocity, until it reaches the average velocity we then observe. This effect is sometimes known as saturation. According to the nature and physical conditions of the physical medium, such as for example the temperature, this average speed may eventually change. Nevertheless, and in habitual conditions this average velocity of saturation is independent of the velocity of the emitting source. Naturally, before reaching the saturation this initial velocity can be different, however it quickly reaches the saturation velocity, being for all practical purposes constant (c). Only in very special experimental conditions, as for instance in the tunnel effect, such average velocity of saturation can eventually be exceeded.

However, before these experimental facts what was done? Instead of considering that one was dealing typically with nonlinear very complex phenomena and that in these conditions the habitual simplistic Cartesian law of linear superposition did not work, since the velocities under consideration were necessarily in the saturation region and consequently, no matter how hard we try, in the usual conditions in which the experiences were carried, the speed of the light did not increase nor decrease. Therefore, it was simply postulated[12] directly or indirectly the invariance of the velocity of the light. That is, the nonlinear intrinsic nature of the phenomena is hidden by means of a postulate, thus forcing extremely complex phenomena into the simplistic linear description, with the disastrous consequences that inevitably come out from it.

The second front is related with the microphysics that is, at the quantum scale of observation of the natural phenomena. In this physical domain the problem of the dualism wave-corpuscle imposed itself showing that the quantum beings somehow behave in a very complex and unexpected way. Once again, we were facing a typically nonlinear phenomenon. So, for its full description, it was necessary to abandon the habitual simplistic Cartesian linear approach. Unfortunately, Niels Bohr[13], instead of facing this situation, proceeded to hide, to disguise this inherent nonlinearity of the phenomena at the quantum scale in his well-known principle of complementarity leading directly to the postulate of the reduction or collapse

of the wave function. He then built a supposedly linear theory, that is to say, linear in the whole domain except in the act of the measurement leading to the collapse of the wave function. From this basic inconsistency, from this refusal in accepting the nonlinear complex nature of the physical phenomena, the well-known quantum paradoxes and related problems came out as a natural consequence.

Naturally, it is very comprehensible that the thinkers of the early XXth century full immersed in the Cartesian simplistic linear method could not do otherwise so, inevitably, they tried, at all costs, to apply it to the study of the new natural phenomena.

Now, that we are more distant in time, and therefore have a clearer vision of what really was the physics of the XXth century, (a coarse forceful attempt of linearization of essentially nonlinear phenomena), we got the conscience that it is necessary to develop a Better way to look into physics. That it is possible to unify physics. This unified global physics, the Hyperphysis, will assume, naturally, as basic starting point, that the phenomena we intend to describe, both at the quantum scale and in the domain of relative great velocities, velocities, near saturation, due to its complex permanent reciprocal interaction, needs a nonlinear global integrated approach rooted on the organizational principle of the eurhythmy.

Deeply inside us we feel that the aim of Physics has always been the Unity. This ideal means that physicists, the natural philosophers, have constantly searched for a very basic set of principles that is, assumptions, from which it would, at last in principle, be possible to derive all particular laws for describing the physical reality at different scales of observation. Now, mainly thanks to the principle of eurhythmy together with a few other basic assumptions integrated coherently in a new ontology, this aspiration seems near to be fulfilled. The principle of eurhythmy[14] comes from the Greek word ***euritmia***, which is the composition of the root *eu* plus *rhythmy*. With *eu* standing for the right, the good, the adequate, and *rhythmy*, for the way, the path, the harmonic motion. The composed word meaning: the adequate path, the good path, the good way, the right way, the golden path, and so on. At this point it is convenient to notice that here the words good, right, and so on, are devoid of any ethical meaning. They only try to convey the general tendency of the complex entities to persist existing as such.

In the present work I intend to show that the principle of eurhythmy is, indeed, the very organizing fundamental principle for the understanding of the complex physical world[15]. It is implicit, of course, that in this more general approach to understand physis, in this New Physics, the Hyperphysis, at the different scales of observation, it is necessary to assume, as basic starting point that the natural phenomena are indeed very complex,

and in general the reciprocal interactions among the diverse participants cannot be neglected. That is the reason for the need of a nonlinear approach. Only in very simple cases, those in which the reciprocal interaction among the participant entities can be overlooked, the simplistic Cartesian linear approach can be of utility.

Also the notion of the physically inexistent abstract referential, infinite in space and time, need to be treated with great care. What we have, in this more general physics, is a physical medium, the chaotic subquantum medium, and the complex particles which are no more than relatively stable organizations of the very same medium. These organized structures are in permanent reciprocal interaction among themselves. In these conditions in general it is not convenient to utilize even approximately the notion of a referential infinite in time and space, this because, after all, we are always dealing with bounded, even if more or less extended, real physical phenomena. What we really always have are local yet more or less extended complex interactions. Naturally, in certain cases where the simplistic linear approach holds the notion of referential can be of use, namely at the statistical approximation and when the dimension of the extended field is relatively very large. On the other hand one needs to be aware that the infinite division is not possible in this New Physics. Whenever the dividing, the Cartesian splitting process reaches a certain point where there is a change in the scale of observation. In such conditions we are before a new realm with different entities, having consequently different properties. The parts that make the whole, that is the emergent beings behave and have naturally quite different properties. These new emergent physical beings make a group, corresponding to a scale of observation. So, if we proceed with the splitting of the elements of this group we arrive at a final limit, which is the initial basic emergent being. Now, if we proceed with the splitting, assuming it possible, the basic being de-emerge, that is gives origin to something related with the initial components. These new beings have properties of their own and quite different from the ones of the de-emergent being. The different scales of observation are bounded by the emergence.

Quantum physics, relativity physics, classical physics including electromagnetism can all be understood in terms of this global nonlinear New Physics, the Hyperphysis that, as expected, contains, naturally, the linear domains of physics as a particular case.

Furthermore, the notions of mass, of charge, of force, of gravitic and electromagnetic interactions and others can now be understood in clear and intuitive way in the framework of the general approach, of the hyperphysis. It will also be shown that what is usually called electron and gravitic particle, just like any other quantum particle, are indeed very complex entities sharing at the same time the attributes of localization and of

extension. The visitation hypothesis plus the principle of eurhythmy are enough to help us to understand the stability of the complex particle. From these basic processes, what is commonly called attraction, in the general case of gravitic particles; repulsion or attraction for electric particles can easily be derived and furthermore comprehended in a very simple and intuitive way as mere particular cases of the coalescence theorem. The universal attraction law, a basic postulate in classical physics, and the habitual concept of mass are, in this approach, seen as simple useful derived concepts. The invariance of the velocity of the light in the common circumstances is also understood as a natural consequence of this nonlinear approach rooted in the principle of eurhythmy. From the deep understanding of these basic principles it should be possible, in principle, to devise practical means to control gravitation, harness the production of energy from the subquantum medium, and of allowing the way for developing physical devices not bounded by the usual velocity limits, thus able of attaining superluminal velocities.

In order to carry out this giant task we must, as mentioned, from the very beginning, start changing our way of looking at the world.

Till know, as we have previously seen, mainly following Descartes, it was assumed, as a working process, to unravel the complex physical problems - the simplistic linear approach. That is, if we have a complex problem to solve we start splitting the initial problem into two sub problems and try to solve them independently. If they continue to prove difficult to solve, then these sub problems are split into other simpler sub problems. This splitting process continues till each one of the final sub problems can be solved. Therefore, the solution of the initial problem is the sum of the all particular solutions. This whole process is described by a linear differential equation and the final solution is the sum of all particular solutions.

In plain words this means that if we act on a system, the answer will be a proportional response from the system. In practice this means, that if the action practiced on the system is small the reaction is also small. When the action is huge its reaction is also huge.

All present theories with good physical solid concrete applications are essentially linear theories. Still we know that, in general, the complex problems posed by everyday life are not subject to a simplistic linear description. Everybody knows that, in certain circumstances, a minute action can give origin to a huge reaction. The principal virtue of the linear approach lies mainly in its great operational simplicity. About four hundred years of utilization of this method leave, even if we do not like it, deep marks, therefore it is not a surprise to realize that we are so addicted to this simplistic linear Cartesian method, so that implicitly we try to apply it to the solution of every problem. In reality everything happens as if we were blind to other processes of problem solving.

Now, in order to progress in the understanding of the natural phenomena, that is, of the Physis it is necessary to assume from the very beginning the nonlinear complex way of thinking. Naturally, we need to be aware that at the average statistical level some intrinsically nonlinear complex problems can be approached by the simpler linear process.

This New Physics assumes that the systems are in permanent reciprocal interaction modifying therefore the interacting medium and being modified at the same time. On the other hand one needs to be aware that we are dealing with a local physics. In this sense it is not possible to derive universal physical laws, that is, universal rules and universal constants. The best we are allowed to do is to propose local, both in time and space, expressions that could approximately describe the complex interacting phenomena. Still if the interacting conditions or the level of description, change, these expressions and naturally the constants linking the variables, traducing the phenomena, may also change. Hopefully we may, and always with great care, in certain broad general conditions enunciate general principles such as the principle of eurhythmy. Still we need always to be very careful. We must bear in mind that, as the human knowledge increases the domain of this new complex and local physics also enlarges. Theories, being a human construction, resulting mainly from the information we are able to gather, and depending significantly on our mental tools, do not stand for final definite and eternal laws. The best we may hope is to have an increasingly better description of the Reality.

2. BASIC ASSUMPTIONS

In order to develop this local new physics it is necessary to clearly state the basic assumptions upon which the Hyperphysis building is constructed. At this point, it is convenient to state, as clear as possible, its true ontology and the very basic starting points or assumptions. We call them assumptions not postulates because we are well aware they are no more than That: simple assumptions! Nevertheless the virtue of these assumptions lies in the fact that they will allow us to develop a new more general unified physics, the Hyperphysis, with the capacity of not only permitting us to understand the old linear physics but also to disclose a whole new realm of experimental and technological possibilities...

1. First assumption

There is an objective Reality. This reality is observer-independent, yet, it is understood that the observer interacts with the very same reality being able to change it and of course of being changed in a greater or lesser degree.

2. Second assumption

There is a basic physical natural chaotic medium named the subquantum medium. All physical processes occur in this natural chaotic medium.

3. Third assumption

What are called physical entities that is, the particles, fields and so on, are more or less stable local organizations of the basic chaotic subquantum medium.

4. Fourth assumption

In general the complex particles, stable organizations of the subquantum medium, are composed of an extended region, the so called theta wave, and inside it there is a kind of a very small localized structure, the acron.

5. Fifth assumption

The principle of eurhythmy. This organizing principle states that the acron inside the theta wave field follows a stochastic path that in average leads it to the regions were the intensity of the theta wave field is greater.

The first assumption of essentially philosophical nature stands for the realistic belief that there is something out of which our ideas and science derive. Science is no more than an attempt to describe the way we interact and understand the same very reality. The need to explicitly state this very fundamental assumption comes from the fact that we aim at making as clear as possible, since the very beginning, the critical starting points, upon which hyperphysics is built, contrary to what happened in orthodox quantum mechanics, where the basic indeterminism, the idealistic attitude was deeply hidden in the diverse explicit or implicit assumptions, and never clearly stated. In order to show out this inner idealistic basic implicit assumption it was necessary an enormous work that lead to the so called quantum

paradoxes were the indeterminist assumptions of the theory could no longer be hidden.

The second basic fundamental statement assumes that there is a natural physical medium of chaotic nature, the subquantum medium. By chaotic medium we understand an indefinite medium where, in general, we cannot make causal relationships. Still, in this subquantum medium there can be more or less stable localized organized regions. These local stable organized regions are precisely where we can establish causal relationships. What we call physics is no more than the description, at the different scales of observation, of the reciprocal interactions among these local more or less stable organizations of the subquantum medium. So, in this sense, time and space are not anymore primary concepts, but only emergent helpful concepts helping us to establish a causal relationship among the diverse interacting organized regions of the subquantum medium.

The concept of subquantum medium was introduced early in the first quarter of the XXth by the great French physicist Louis de Broglie[1] to describe quantum phenomena, namely the duality wave-corpuscle. Here this natural basic concept is generalized from the strict domain of quantum physics to include all physics.

The third assumption characterizes the nature of the physical entities, commonly known as particles, fields and so that here are seen as very complex entities. In this approach to describe reality it is assumed that the natural phenomena, at the different scales of observation, is no more than a reflex of the evolution and interaction of these local stable organizations of the subquantum medium. In this sense physics looks for describing the behavior of these organized structures and their reciprocal mutual interactions.

The fourth assumption concretely specifies the nature of the complex physical particle. Also in this case the initial proposal for the nature of the complex particle was made by de Broglie[1] in quantum domain in order to explain the duality wave-corpuscle. Now this concept of de Broglie complex particle was generalized[2] from pure quantum physics to include all physics. In this assumption it is stated that the complex particles are compose of an extended part, the wave, and inside there is a well localized and in general indivisible, yet complex structure relatively very small compared to its wave. The wave is named theta wave and the small localized structure the acron. Mathematically we could write

$$\phi = \phi(\theta, \xi) \tag{2.1}$$

or, assuming the simplest linear approach, as de Broglie did,

$$\phi = \theta + \xi, \tag{2.2}$$

where ξ stands for the acron and θ, naturally for the theta wave.

In previous works, following de Broglie, this very small high energetic region of the complex particle was called, singularity or even corpuscle. Still due to the confusion with the concept of mathematical singularity and from the fact that this region of the particle has an inner very complex structure it is now named by the Greek word acron[3]. This word comes from the Greek άκρον meaning the higher pike like in acropolis, the higher city.

The following drawing, Fig.2.1, tries, roughly, to picture the real part of the complex particle.

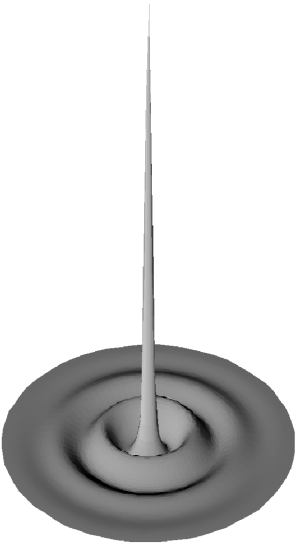

Fig.2.1 – Graphic sketch of a complex particle.

The fifth assumption corresponds to the laying down of the principle of eurhythmy as the cornerstone, as the very organizing principle for helping in the description of the natural phenomena. The principle of eurhythmy concretely states that the acron possesses a kind of extended sensorium, its theta wave, with which it feels the surrounding medium. The acron being immersed in its theta wave moves in a stochastic way preferentially to the regions where the intensity of the theta wave field is greater. This is a natural consequence of that fact that if the acron moves to other regions of lesser intensity is mean life is unsurprisingly shorter. This principle has been

generalized by some authors[4] to include other sciences like for instance biology and others.

Furthermore, in order to better clarify the starting basic assumptions, I would like to stress that in this work the concept of chaos refers to an epistemological chaos and not an ontological chaos.

On the other hand it is convenient to keep in mind that due to the complex nonlinear nature of the phenomena from one scale of observation to the next it happens that the new emergent entities, even if they are a composition of parts their properties cannot, in general, be derived from the properties of the building parts. This because the composite parts interact and therefore modify themselves reciprocally in a greater or lesser degree, so that the resulting emergent entity has properties of its own.

3. SOME SIMPLE APPLICATIONS OF THE PRINCIPLE OF EURHYTHMY

3.1. INTRODUCTION

In order to introduce the reader into the deep meaning of the principle of eurhythmy, to its explaining capacity, prior to the full development of it, we shall start by presenting a simple yet very interesting example of the applicability of this basic organizing principle.

One of the first principles for understanding the physical world was discovered by Heron of Alexandria back in the first century A.D. In order to explain the reflection of light and derive the respective law Heron established the principle of the minimum path. This principles states that light coming from a point like source S and going to the observer O, after being reflected by the mirror, from the multiple possible paths follows the shorter one. From this principle Heron derived[1], by geometrical considerations, the reflection law stating that the angle of reflection equals the incident angle, $\theta_i = \theta_r$.

About a millennium and half later, Pierre de Fermat established the principle of minimum time. With the help of this principle Fermat was able to derive Snell´s empirical law for the refraction of light. This law describes how light propagates in different optical medium. The principle of minimum time states that from all possible paths, from S to O, see Fig. 3.1, the light follows the path such that the time taken in the whole course is a minimum.

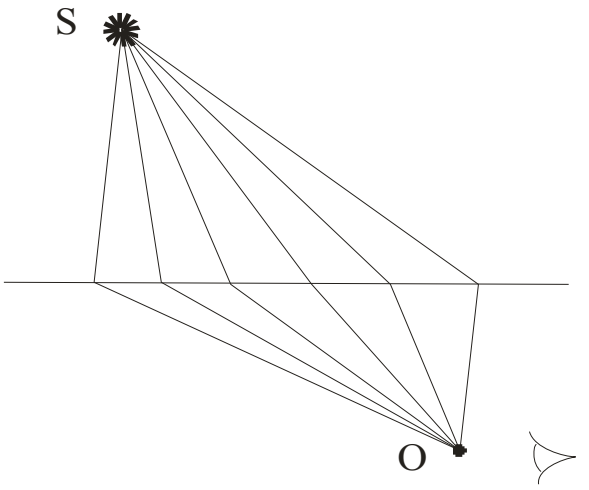

Fig.3.1 – The principle of minimum time

That is, the light coming from a point like source, the point S, in order to reach the observer at point O, after travelling through two different optical media, follows a path such that the time taken in the whole course is a minimum.

That is, the light behaves in such a way that it "chooses" from among all possible paths the one that takes less time. From this principle Fermat derived Snell's law for the transmission of the light in different optical media.

3.2. Derivation of Snell's Refraction Law from Fermat's Principle of Minimum Time

This derivation can be found in almost all good textbooks on optics[1]. Nevertheless for the sake of future reference we shall present here a simplified version. In order to do that, consider the following sketch where the light is emitted from point S and later is detected at point P, after travelling through two optical media.

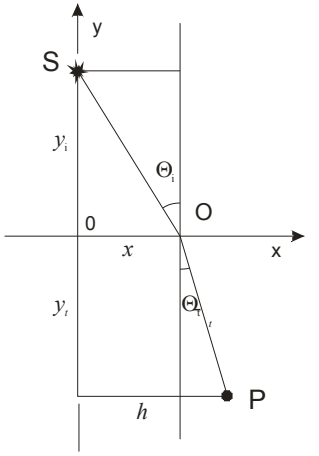

Fig.3.2 – Sketch for derivation of Snell's law
from Fermat principle of minimum time

The time t taken by the light in the course from point S to point P is equal to the time in the incidence region t_i plus the time in the transmission medium t_t,

$$t = t_i + t_t \,. \tag{3.2.1}$$

These individual times being given by the travelled path divided by the velocity, which must be expressed in terms of the variable x,

$$\begin{cases} t_i = \dfrac{\overline{SO}}{v_i} = \dfrac{1}{v_i}(x^2 + y_i^2)^{1/2} \\ t_t = \dfrac{\overline{OP}}{v_t} = \dfrac{1}{v_t}((h-x)^2 + y_t^2)^{1/2} \end{cases} \tag{3.2.2}$$

Since the total possible times is now a function of the variable x, $t = t(x)$, in order to find the minimum of all those possible times we use the habitual calculus technique

$$\frac{d\,t(x)}{d\,x} = 0\,, \tag{3.2.3}$$

which gives the condition

$$\frac{1}{v_i}\frac{x}{(x^2-y_i^2)^{1/2}}=\frac{1}{v_t}\frac{(h-x)}{((h-x)^2-y_t^2)^{1/2}}$$

or

$$\frac{1}{v_i}\frac{x}{\overline{SO}}=\frac{1}{v_t}\frac{(h-x)}{\overline{OP}}. \tag{3.2.4}$$

From the sketch Fig.3.2 we see that

$$\sin\theta_i=\frac{x}{\overline{SO}} \quad \text{and} \quad \sin\theta_t=\frac{(h-x)}{\overline{OP}}, \tag{3.2.5}$$

which by substitution in (4.2.4) gives

$$\frac{1}{v_i}\sin\theta_i=\frac{1}{v_t}\sin\theta_t, \tag{3.2.6'}$$

or, multiplying by the constant c, it gives finally Snell´s refraction law

$$n_i\sin\theta_i=n_t\sin\theta_t, \tag{3.2.6}$$

where $n_i=c/v_i$ and $n_t=c/v_t$ are the indexes of refraction in the incident and transmission media.

The basic idea behind this principle of minimum time, as we have seen, is that the light that goes from point S to reach point O, after travelling through different optical media, where it has different velocities, follows, from all possible paths, the one that takes less time. In short, this natural entity we call light "chooses" the best, the most adequate path, the one that mostly preserves it has an independent being, that is, the photon, the light follows the principle of eurhythmy.

Naturally, Heron's principle of the shortest path is a particular case of Fermat's principle of minimum time. This is easily seen because in the reflection phenomena the light travels always in the same optical medium and with the same velocity. In such conditions, the shortest path is the one that takes less time.

The principle of minimum time of Fermat was soon followed by Maupertuis's principle of least action for classical mechanics. These principles were and still are commonly named extreme principles. It was

shown[2] that classical mechanics can formally be derived from these principles of extreme.

Since these principles are mere particular formulations of the same basic principle, it is only natural to assume that classical physics is clearly integrated in this new general nonlinear and causal physics.

The principle of eurhythmy states that in nature the complex systems in order to keep existing as such must behave, that is, follow a path that, in average, is the best. If the natural complex systems do not, in average, follow the best possible path they will not survive for long. In such conditions all systems in order to be observable, the ones that exist for a certain time, even if very small, need to follow the principle of eurhythmy.

4. THE COMPLEX PARTICLE

4.1. INTRODUCTION

In this chapter we present a deeper view into the concept of the complex particle. This history began in the first quarter of the twentieth century when physicists were trying to understand the quantum world. In order to explain in a rational causal way the wave-corpuscle duality, characteristic of the quantum entities, the great French physicist Louis de Broglie[1] stated, that a quantum particle is indeed a very complex system sharing at the same time wave and corpuscle properties. Quantum particles, implicitly assumed previously to be only structureless point like entities, are, in reality, composed of an extended yet finite part, the wave, plus a highly complex localized structure, immersed in the sensing extended theta wave field. Later[2] this early model was developed and the complex particle, quantum or otherwise, is now mathematically described by a wavelet[3] solution to a nonlinear master equation that describes both extend and localized properties.

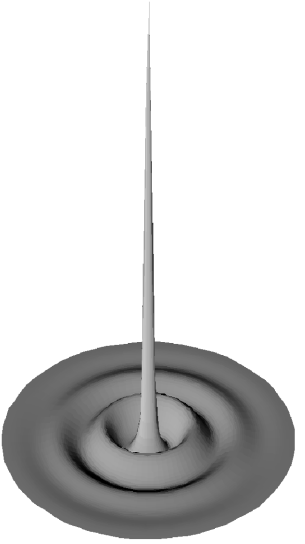

Fig.4.1 – Rough representation of the complex particle

Recently[2] this concept of complex quantum particle was extended from the quantum physics to all physics.

In this new global unified physics, the real physical particles, quantum particles, gravitic particles, electronic particles, and so, at each scale of description of the physical reality, are understood as highly complex structured entities, wave-like organized perturbations of a basic chaotic subquantum medium. The particle is then a composition of a very small region with a relatively very high energetic content the acron, plus an extended part the wave, the theta wave.

The extended theta wave behaves like a kind of an extended sensorium for the acron that moves randomly inside it according to the principle of eurhythmy. In a way such that the acron moves preferentially to the regions where the intensity of the theta wave has greater intensity, that is, according to the principle of eurhythmy.

The theta wave is in general the composition of the mother theta wave plus many other theta waves forming a more or less extended pack of wavelets. Naturally, if it happens that many acrons are located at a small section the overall theta wave may increase significantly in this region. We know that the overlapping of the two theta waves can not, in general, be described by the sum of the two independent waves, since we are dealing with an essentially nonlinear process where two or more physical systems react and modify themselves in function of their mutual interaction. Yet, when dealing with a very large number of subquantum entities, and making

the average statistical approximation, in most common cases the additive linear rule holds approximately.

The average size of the basic mother wavelet is naturally related with the wavelength by the most basic relation

$$\sigma = M\,\lambda, \tag{4.1.1}$$

where *M* is a constant.

On the other hand as we shall see the concept of energy for the complex quantum particle may be formally expressed by Planck formula

$$E_{\phi} = \hbar\,\omega. \tag{4.1.2}$$

or, more precisely it needs to be written into two formulae: One for the acron

$$E_{\xi} = \hbar_{\xi}\omega \tag{4.1.3}$$

were $\hbar_{\xi}$ is roughly about the value of the common the Planck constant, the other of the same type for the basic wavelet

$$E_{\theta} = \hbar_{\theta}\omega. \tag{4.1.4}$$

Now, with $\hbar_{\theta}$ standing for the very small constant, the subquantum medium constant, in a certain way similar to the Planck constant, relating the temporal frequency of the basic mother theta wave with its own minute energy. In this conditions expression (4.1.2) needs to be written

$$E_{\phi} = (\hbar_{\xi} + \hbar_{\theta})\,\omega. \tag{4.1.5}$$

or

$$E_{\phi} \cong \hbar_{\xi}\,\omega. \tag{4.1.6}$$

Since, for all practical purposes, the energy of the quantum particle is the one of the acron. The energy of the mother theta wave is so small that the common detectors are unable to see it. Indeed, a rough estimation[4] for the ratio between the energy of the photonic acron and that of its associated theta wave gives

$$\frac{E_{\xi}}{E_{\theta}} \approx 10^{+54}. \tag{4.1.7}$$

As we shall see latter, Planck's and de Broglie's expressions still hold formally at other different scales of observation namely at the classical scale, and furthermore they shall be derived as a kind of corollary from the very nature of the complex particle.

So far we have been using the term of energy in its habitual sense still, as we shall see, the concept of energy and energy conservation in Hyperphysis are to be understood only as very useful derived concepts.

Now a question arises! How are the theta wave and the acron connected?

The answer to the last question is, as expected, given by the organizing principle of eurhythmy. The concept of particle in this new unitary physics involves implicitly a chaotic interaction between the acron and the theta wave field described precisely by the principle of eurhythmy.

A first simplified version of this principle was proposed by de Broglie under the name of the guiding principle. This guiding, devised to explain the outcome of the single quantum particle double slit experiment, states that the probability of localizing the acron is proportional to the intensity of the theta wave. In this sense it represents a kind of stationary, static average version of the principle of eurhythmy. Implicitly implying that the acron is guided, through a nonlinear process, preferentially to the regions where the theta wave has higher intensity. Still this name does not give full account of the complex physical process involved. What really happens is that the acron, or the corpuscle, necessarily immersed in the theta wave field when moving, randomly from one point to another, follows, in average, the best possible path. The best possible path is the one that in average leads the acron to the regions where the theta wave has higher intensity. So, in reality it is not a simple action of guiding. The extended theta wave field naturally guides the acron, but this guiding action is of a very special kind. The theta wave guides the acron to the best possible path, the average path which leads the acron to the regions where the intensity of the theta wave is greater. The acron avoids the regions where the intensity of the theta field is null because in these regions its very physical existence is in danger, in the sense that in such zones the acron needs to regenerate the surrounding field at the expenses of its own energy. Therefore the motion of the acron in the theta wave field follows always the best possible path, the most adequate path, that is, follows the principle of eurhythmy. The acrons that do not follow the average best path shortly cease to exist as independent entities. In this sense in order to be relatively permanent, that is to have a mean life of some significance, at the scale of observation, the acron ought to follow the most adequate path, in short to follow the principle of eurhythmy.

It is assumed, of course, that any acron, as long as it keeps its existence, has an inner motion of its own and furthermore keeps always

moving with its natural velocity v_N. The acron moves incessantly in a chaotic way, in the theta wave field, with an instantaneous huge velocity, called the natural velocity. Nevertheless, due to the chaotic nature of the subquantum field the average velocity $\bar{v}$, which is the observed velocity, can go from zero to the natural velocity, $0 \le \bar{v} \le v_N$.

In these circumstances the principle of eurhythmy unifies both the principle of minimum time of Fermat, the principles of extreme of classical physics and the guiding principle of de Broglie. Furthermore, this basic organizing principle of Nature gives to all of them their true natural deep meaning.

The next section is devoted to the study of the interaction between the acron and theta wave, both very highly organized perturbations of the basic chaotic medium, the subquantum field.

4.2. VISITATION HYPOTHESIS

The existence of the acron or corpuscle is here simply assumed. Its origin in this approximation is not discussed. More complete developments need to take the emergence problem into account. The acron being a very well localized region, a kind of autonomous generator, localized in the subquantum medium, while moving in a chaotic way, generates in its neighbourhood a surrounding wave of feeble amplitude. This minute field is called the fundamental theta wave or mother theta wave. The size of this basic mother wavelet with its wavelength is given by formula (4.1.1). In this approach it is also assumed that the acron has a very large mean life $T_\xi \to \infty$. In other more rigorous approaches this restriction needs to be removed. The mean life T_θ of the theta wave is relatively small.

When the acron moves to another region of the subquantum medium, it generates there another theta wave. The initial remaining theta wave being a perturbation of the medium having only a small amount of energy starts losing energy to the chaotic subquantum medium. As a consequence, if the theta wave is not visited by the generating acron it starts disappearing till it vanishes completely. Another visitation from the acron increases the field of the theta wave.

Naturally, it is assumed that the acron loses very minute amounts of energy in each transition. Nevertheless this loss is so small that, in this approach, it is assumed, as we have seen, that the acron behaves like an infinite energy reservoir. Still, when the acron moves to more chaotic regions of the subquantum medium this loss of energy is relatively much

larger. This happens, because if the acron "chooses" always these chaotic unorganized destructive regions its mean life is unsurprisingly much shorter.

In order to proceed with the calculations the subquantum medium is assumed to be divided into small cubic cells and the acron in each transition can only go to the adjacent cells. So, at time zero we have the initial theta wave $\theta^0 = \theta(r_0)$ surrounding the acron. In the next transition, the acron may remain in the same cell or move to the adjacent cells giving origin to another theta wave $\theta_1 = \theta(r_1)$, with $r_1 = \text{or} \neq r_0$. In the mean time the initial theta wave $\theta^0 = \theta(r_0)$ undergoes a decay process. In the following transition the acron moves from position r_1 to position r_2, with $r_2 = \text{or} \neq r_1$, while the two previous theta waves lose intensity. This process keeps going on till equilibrium is reached. The final theta wave results from the composition of all these particular waves.

In order to be able to proceed with the calculations in the following it is assumed the quasi static approach, that is, we only take in account the final end sub states. So, the processes in-between transitions are not here taken in account. Furthermore, it is assumed that we are dealing with a very large number of waves which have approximately a random phase relationships among them. In this case the interferometric terms cancel out in average.

4.2.1. STABILITY OF A PARTICLE: DISCRETE APPROACH

In this one-dimensional quasi-static approach for the stochastic process, it is assumed that at the initial time the acron is localized in cell 0,

$$\theta^0(x) = \theta_0(x) \tag{4.2.1}$$

where the upper index corresponds to the time, or transition number and the lower index to the cell position.

According to what was said in section **4.1**, in the first transition, the acron may remain in the same cell, $0 \to 0$ with the probability $p^1_{0\to 0} \equiv p^1_{00}$, move backwards $0 \to -1$, with the probability $p^1_{0\to -1} \equiv p^1_{0-1}$ or to the front $0 \to 1$ with the probability $p^1_{0\to 1} \equiv p^1_{0+1}$. The upper index in the probability corresponds to the transition number, still since there is no danger of confusion, because the lower indexes contain the history of the process, the

upper index shall be dropped. The form of the transition probabilities are given by the principle of eurhythmy

$$
\begin{aligned}
p_{0-1} &= \frac{|\theta^0(-1)|^2}{|\theta^0(-1)|^2 + |\theta^0(0)|^2 + |\theta^0(1)|^2} \\
p_{00} &= \frac{|\theta^0(0)|^2}{|\theta^0(-1)|^2 + |\theta^0(0)|^2 + |\theta^0(1)|^2} \\
p_{0+1} &= \frac{|\theta^0(1)|^2}{|\theta^0(-1)|^2 + |\theta^0(0)|^2 + |\theta^0(1)|^2}
\end{aligned}
\tag{4.2.2}
$$

In such conditions at the time $t = 1$ we have three possible outcomes

$$0 \to -1, \quad 0 \to 0, \quad 0 \to 1 .$$

To each of these possibilities corresponds a wave that is, in this first and statistical approach assumed to be approximately described by the sum of the initial wave corrected by the decaying factor with the theta wave resulting from the transition:

$$
\begin{aligned}
\theta_{0-1}(x) &= \theta_0(x)\, e^{-\lambda \cdot 1} + \theta_{-1}(x) \\
\theta_{00}(x) &= \theta_0(x)\, e^{-\lambda \cdot 1} + \theta_0(x) \\
\theta_{0+1}(x) &= \theta_0(x)\, e^{-\lambda \cdot 1} + \theta_{+1}(x)
\end{aligned}
\tag{4.2.3}
$$

That is, even if we know that we are dealing with a nonlinear process it is assumed, in this approximation, that the composition rule for the final wave resulting from the composition of the overlapping waves can, for a very large number of waves, be approached by the linear rule.

Assuming that all these possibilities occur at the same time the average total theta wave at time $t = 1$ is given by

$$\theta^1(x) = p_{0-1}\, \theta_{0-1}(x) + p_{00}\, \theta_{00}(x) + p_{0+1}\, \theta_{0+1}(x) . \tag{4.2.4}$$

Supposing, in a first approximation, that the theta wave, generated in the subquantum medium by the acron, has a gaussian form it is possible to write, for the wave at the initial time

$$\theta^0(x) = \theta_0(x) = e^{-\frac{x^2}{2\sigma^2}}, \tag{4.2.5}$$

So, at the time $t = 1$, the transition probabilities assume the concrete form

$$\begin{aligned} p_{0-1} &= \frac{e^{-\frac{(-1)^2}{\sigma_2}}}{e^{-\frac{(-1)^2}{\sigma_2}} + e^{-\frac{(0)^2}{\sigma_2}} + e^{-\frac{(1)^2}{\sigma_2}}} \\ p_{00} &= \frac{e^{-\frac{(0)^2}{\sigma_2}}}{e^{-\frac{(-1)^2}{\sigma_2}} + e^{-\frac{(0)^2}{\sigma_2}} + e^{-\frac{(1)^2}{\sigma_2}}} \\ p_{0+1} &= \frac{e^{-\frac{(1)^2}{\sigma_2}}}{e^{-\frac{(-1)^2}{\sigma_2}} + e^{-\frac{(0)^2}{\sigma_2}} + e^{-\frac{(1)^2}{\sigma_2}}} \end{aligned} \tag{4.2.6}$$

the waves corresponding to the possible stochastic paths

$$\begin{aligned} \theta_{0-1}(x) &= e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda\cdot 1} + e^{-\frac{(x+1)^2}{2\sigma^2}} \\ \theta_{00}(x) &= e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda\cdot 1} + e^{-\frac{(x-0)^2}{2\sigma^2}} \\ \theta_{0+1}(x) &= e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda\cdot 1} + e^{-\frac{(x-1)^2}{2\sigma^2}} \end{aligned} \tag{4.2.7}$$

and consequently the average theta wave has the form

$$\theta^1(x) = \frac{1}{e^{-\frac{(-1)^2}{\sigma^2}} + e^{-\frac{(0)^2}{\sigma^2}} + e^{-\frac{(1)^2}{\sigma^2}}} \times$$
$$\times \left[e^{-\frac{(-1)^2}{\sigma^2}} \left[e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda \cdot 1} + e^{-\frac{(x+1)^2}{2\sigma^2}} \right] + \right.$$
$$+ e^{-\frac{(0)^2}{\sigma^2}} \left[e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda \cdot 1} + e^{-\frac{(x+1)^2}{2\sigma^2}} \right] +$$
$$\left. + e^{-\frac{(1)^2}{\sigma^2}} \left[e^{-\frac{(x-0)^2}{2\sigma^2}} e^{-\lambda \cdot 1} + e^{-\frac{(x-1)^2}{2\sigma^2}} \right] \right] \tag{4.2.8}$$

For the second transition, that is, at time $t = 2$ we will have $N = 3^2 = 9$ possible outcomes:

$$\begin{array}{lll} 0 \to -1 \to -2, & 0 \to -1 \to -1, & 0 \to -1 \to 0 \\ 0 \to \quad 0 \to -1, & 0 \to \quad 0 \to 0, & 0 \to \quad 0 \to +1 \\ 0 \to +1 \to 0, & 0 \to +1 \to +1, & 0 \to +1 \to +2 \end{array} \tag{4.2.9}$$

Accordingly the corresponding waves are

$$\begin{array}{lll} \theta_{0-1-2}(x), & \theta_{0-1-1}(x), & \theta_{0-10}(x) \\ \theta_{00-1}(x), & \theta_{000}(x), & \theta_{00+1}(x) \\ \theta_{0+10}(x), & \theta_{0+1+1}(x), & \theta_{0+1+2}(x) \end{array} \tag{4.2.10}$$

to which are associated the probabilities

$$\begin{array}{lll} p_{0-1-2}, & p_{0-1-1}, & p_{0-10} \\ p_{00-1}, & p_{000}, & p_{00+1} \\ p_{0+10}, & p_{0+1+1}, & p_{0+1+2} \end{array} \tag{4.2.11}$$

The probabilities are equal to the product of the probabilities of transiting from the initial state to the first and to transit from this position to

the next, for instance: the probability p_{0-1} from initial state 0 to the cell -1 plus the probability of transiting from cell -1 to cell -2, p_{-1-2} . That is:

$$p_{0-1-2} = p_{0-1}p_{-1-2}, \quad p_{0-1-1} = p_{0-1}p_{-1-1}, \cdots \tag{4.2.12}$$

with the first transition probabilities given by previous expressions (4.2.2). The second transition probabilities are given by

$$\begin{aligned} p_{-1-2} &= \frac{|\theta^1(-2)|^2}{|\theta^1(-2)|^2 + |\theta^1(-1)|^2 + |\theta^1(0)|^2} \\ p_{-1-1} &= \frac{|\theta^1(-1)|^2}{|\theta^1(-2)|^2 + |\theta^1(-1)|^2 + |\theta^1(0)|^2} \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \tag{4.2.13}$$

Thus, as can be seen, the transition probabilities contain the history of the entire process.

In these conditions, the final theta wave at time $t = 2$ is therefore

$$\begin{aligned} \theta^2(x) = {} & p_{0-1-2}\theta_{0-1-2}(x) + p_{0-1-1}\theta_{0-1-1}(x) + p_{0-10}\theta_{0-10}(x) + \\ & p_{00-1}\theta_{00-1}(x) + p_{000}\theta_{000}(x) + p_{001}\theta_{001}(x) + \\ & p_{0+10}\theta_{0+10}(x) + p_{0+1+1}\theta_{0+1+1}(x) + p_{0+1+2}\theta_{0+1+2}(x) \end{aligned} \tag{4.2.14}$$

with

$$\begin{aligned} \theta_{0-1-2}(x) &= e^{-\lambda\cdot 2}\theta_0(x) + e^{-\lambda\cdot 1}\theta_{-1}(x) + e^{-\lambda\cdot 0}\theta_{-2}(x) \\ \theta_{0-1-1}(x) &= e^{-\lambda\cdot 2}\theta_0(x) + e^{-\lambda\cdot 1}\theta_{-1}(x) + e^{-\lambda\cdot 0}\theta_{-1}(x) \\ \theta_{0-10}(x) &= e^{-\lambda\cdot 2}\theta_0(x) + e^{-\lambda\cdot 1}\theta_{-1}(x) + e^{-\lambda\cdot 0}\theta_0(x) \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \theta_{0+1+2}(x) &= e^{-\lambda\cdot 2}\theta_0(x) + e^{-\lambda\cdot 1}\theta_{+1}(x) + e^{-\lambda\cdot 0}\theta_{+2}(x) \end{aligned} \tag{4.2.15}$$

At time $t = 3$ we have $N = 3^3 = 27$ possible paths

$$
\begin{array}{lll}
0\to-1\to-2\to-3 & 0\to0\to-1\to-2 & 0\to+1\to+2\to+3\\
0\to-1\to-2\to-2 & 0\to0\to-1\to-1 & 0\to+1\to+2\to+2\\
0\to-1\to-2\to-1 & 0\to0\to-1\to0 & 0\to+1\to+2\to+1\\
0\to-1\to-1\to-2 & 0\to0\to0\to-1 & 0\to+1\to+1\to+2\\
0\to-1\to-1\to-1 & 0\to0\to0\to0 & 0\to+1\to+1\to+1\\
0\to-1\to-1\to0 & 0\to0\to0\to+1 & 0\to+1\to+1\to0\\
0\to-1\to0\to-1 & 0\to0\to+1\to0 & 0\to+1\to+2\to+1\\
0\to-1\to0\to0 & 0\to0\to+1\to+1 & 0\to+1\to+2\to+2\\
0\to-1\to0\to+1 & 0\to0\to+1\to+2 & 0\to+1\to+2\to+3
\end{array}
\qquad (4.2.16)
$$

In these conditions the average final state at the third transition is:

$$
\theta^3(x)=p_{0-1-2-3}\theta_{0-1-2-3}(x)+p_{0-1-2-2}\theta_{0-1-2-2}(x)+\cdots
\cdots+p_{0+1+2+3}\theta_{0+1+2+3}(x) \qquad (4.2.17)
$$

with the probabilities given by expressions analogues to the (3.2) and (3.13) and the same for the waves corresponding to the third possible transition paths.

For N transition we have 3^N possible waves corresponding to the respective paths in these conditions the final average theta wave is given by

$$
\theta^N(x)=p_{0\cdots-N}\theta_{0\cdots-N}(x)+\cdots+p_{0\cdots+N}\theta_{0\cdots+N}(x)\,. \qquad (4.2.18)
$$

The form of the final theta wave can then be approached by numeric calculation. These calculations have being already done, see M. Gatta[5].

4.2.2. STABILITY OF A PARTICLE: MORE GENERAL APPROACH

In order to treat the process in a more general way we consider the initial representation in terms of transitions of length r_k. For this representation the transition stochastic possible paths are:

$$
\begin{aligned}
&r_0^1 \to r_1^1 \to r_2^1 \to \cdots \to r_N^1 \\
&r_0^2 \to r_1^2 \to r_2^2 \to \cdots \to r_N^2 \\
&r_0^3 \to r_1^3 \to r_2^3 \to \cdots \to r_N^3 \\
&\text{.....} \\
&r_0^j \to r_1^j \to r_2^j \to \cdots \to r_N^j \\
&\text{.....} \\
&r_0^{3^N} \to r_1^{3^N} \to r_2^{3^N} \to \cdots \to r_N^{3^N}
\end{aligned}
\tag{4.2.19}
$$

where the upper index represents a possible stochastic pass and the lower index the transition. Naturally, we can have $r_i^j = \text{or} \neq r_{i+1}^j$, meaning that the acron may remain in the same cell or transit for the adjacent ones.

The corresponding wave associated with each path is:

$$\theta^j(x) = e^{-\lambda N}\theta_{r_0^j}(x) + e^{-\lambda(N-1)}\theta_{r_1^j}(x) + e^{-\lambda(N-2)}\theta_{r_2^j}(x) + \cdots + e^{-\lambda\cdot 0}\theta_{r_N^j}(x),$$

or

$$\theta^j(x) = \sum_{k=0}^{N} e^{-\lambda(N-k)}\theta_{r_k^j}(x), \tag{4.2.20}$$

and, consequently, the total theta wave assuming that all possible paths 3^N occur

$$\theta(x) = \sum_{j=0}^{3^N} p^i\theta^j(x), \tag{4.2.21}$$

where the transition probabilities are given by

$$p^j = p_{r_0^j \to r_1^j \cdots r_{N-1}^j \to r_N^j} = p_{r_0^j \to r_1^j} \cdot p_{r_1^j \to r_2^j} \cdot p_{r_2^j \to r_3^j} \cdots p_{r_{N-1}^j \to r_N^j} \tag{4.2.22}$$

$$p^j = \prod_{m=1}^{N} p_{r_{m-1}^j \to r_m^j} \tag{4.2.23}$$

.

In these conditions, and for this representation, the total theta wave assumes the form

$$\theta\ (x) = \sum_{j=0}^{3^N} \left[\left(\prod_{m=1}^{N} p_{r_{m-1}^j \to r_m^j} \right) \sum_{k=0}^{N} e^{-\lambda(N-k)} \theta_{r_k^j}(x) \right]. \qquad (4.2.24)$$

4.2.2.1. STABILITY OF THE COMPLEX PARTICLE: A CRUDE APPROXIMATION

Our problem now is to calculate the form of the transition probabilities p^j corresponding to each path. In principle the last term of transition probabilities p^j, $p_{r_{N-1}^j \to r_N^j}$, contains all the information on all the history of the path. The direct calculation of this term is extremely difficult, if not even impossible. Still if we look for an expedite reasonable solution, we can see that each path starts from the same initial point, cell, and after a very large number N of transitions, finishes at an end cell. Even if the time taken in each stochastic path is the same the final length they reach is completely different as can be inferred from (4.16). In such conditions a first approximation could be gathered from the known fact that the probability of attaining the length $0 \to +1 \to +2 \to +3 \to \cdots \to +N$, after N transitions, is much smaller than, for instance, of returning to the same cell $0 \to \cdots \to 0$, after the same N transitions, since this last state can be reached in many different ways. So, even if all paths take the same time, in a first approach, it is reasonable to assume that the probability of a stochastic path depends on its final length r and furthermore has a gaussian form

$$p^j \approx p_r \approx \alpha_1 e^{-\frac{r^2}{2\sigma_r^2}}, \quad r = 0 = \pm 1 \pm 2 \cdots \pm N, \qquad (4.2.25)$$

where α_1 is a normalization constant.

The other crude simplifying approach is that, for $N >>> 1$, even if the variables r_k^j are independent they nevertheless follow approximately the same distribution, that is

$$\theta_{r_k^j}(x) \approx \theta_{r_k}(x) \approx \theta_r(x). \qquad (4.2.26)$$

This approximation means that the state depends only on the final end position. Therefore, since the initial position is always 0 we have

$$\theta_{0\cdots r}(x) \approx \theta_r(x) \approx \alpha_2 e^{-\frac{(x-r)^2}{2\sigma^2}}, \tag{4.2.27}$$

where it was also assumed that the final form of the wave can approximately be described by a gaussian. In these conditions formula (4.2.24) is approximated to

$$\theta^N(x) \approx \sum_{m=0}^{N}\sum_{r}^{N} e^{-\lambda(N-m)} p_r \theta_r(x),$$

or

$$\theta^N(x) \approx \sum_{m=0}^{N} e^{-\lambda(N-m)} \sum_{r}^{N} p_r \theta_r(x). \tag{4.2.28}$$

For N very large and making the continuous approximation we have

$$\theta(x) \approx \int_0^N e^{-\lambda(N-m)} dm \int_{-\infty}^{+\infty} \alpha_1 e^{-\frac{r^2}{2\sigma_r^2}} \alpha_2 e^{-\frac{(x-r)^2}{2\sigma^2}} dr, \tag{4.2.29}$$

which by integration gives

$$\theta(x) \approx \frac{A}{\lambda}\sqrt{\frac{2\pi}{1/\sigma_r^2 + 1/\sigma^2}}\, e^{-\frac{x^2}{2(\sigma_r^2+\sigma^2)}}. \tag{4.2.30}$$

As expected, it was shown, even if under some rather crude approximations, that the final distribution is also approximately gaussian. That is, as time goes on the initial gaussian distribution for the initial theta wave field keeps basically the same form only a little broader.
These results have been confirmed by numerical calculation done by M. Gatta[5] in his chapter of this book.

5. MOTION OF THE SINGLE ACRON

5.1. INTRODUCTION

In the previous chapter we have seen that the chaotic nature of the subquantum medium plus organizing the principle of eurhythmy guarantee the relative stability of the complex particle. Now we are going to study the average motion of a single acron immersed generally in a composite theta wave field.

In order to do so we consider a complex particle described by the mother theta wave plus the acron assuming that under the simplest approximation may be written[1]

$$\phi = \theta + \xi = \left[\theta_0 e^{-\frac{(x-v_g t)^2}{2\sigma_\theta^2}} + A_\xi e^{-\frac{(x-\bar{v}_\xi t-\varepsilon)^2}{2\sigma_\xi^2}}\right] e^{ik(x-v_f t)}, \qquad (5.1.1)$$

where

$$|A_\xi| \ggg |\theta_0| \text{ and } \sigma_\xi \lll \sigma_\theta, \qquad (5.1.2)$$

with v_g being the group velocity or the velocity of the theta wave, $\bar{v}_\xi$ is the average velocity of the acron and v_f is the phase velocity all relative to the surrounding medium, in general a very large extended theta wave field. The frequency of the mother theta wave and the acron are naturally the same.

As we shall have opportunity to see, this no dispersive function, describing the complex particle is a natural solution to the nonlinear master equation.

In this situation we have approximately

$$v_g = \bar{v}_\xi. \qquad (5.1.3)$$

Since $\sigma_\theta \ggg \sigma_\xi$ and $\sigma_\theta = M\lambda_\theta$, this implies that the spatial dimension of the acron is very small relative to the wavelength of the mother theta wave

$$\sigma_\xi \ll \lambda_\theta. \qquad (5.1.4)$$

In other words, in most cases everything happens as if the acron had no relative physical size, behaving therefore as a point-like entity. Still we must be aware that the acron is indeed a very complex, a highly structured

entity, therefore naturally with a real physical size. So, for all practical purposes and in most cases, the acron behaves as if devoid of wavelength. Still we must recall, as previously seen, that an isolated acron generates its own theta wave, the mother theta wave.

Nevertheless if this acron and its associated mother theta wave are immersed in a very extended large theta wave of greater intensity relative to the intensity of the mother theta, that is,

$$\theta_T = \phi + \theta_L$$

which for one instant of time gives

$$\theta_T = \left[\theta_0 e^{-\frac{x^2}{2\sigma_\theta^2}} + A_\xi e^{-\frac{(x-\varepsilon)^2}{2\sigma_\xi^2}}\right] e^{i(k_1 x - \delta_1)} + \theta_{0L} e^{\frac{x^2}{2\sigma_L^2} + i(k_2 x - \delta_2)}, \quad (5.1.5)$$

and the respective intensity

$$I_T = |\phi|^2 + |\theta_L|^2 + 2\left[\theta_0 e^{-\frac{x^2}{2\sigma_\theta^2}} + A_\xi e^{-\frac{(x-\varepsilon)^2}{2\sigma_\xi^2}}\right] \times$$
$$\times\, \theta_{oL} e^{-\frac{x^2}{2\sigma_L^2}} cos[(k_1 - k_2)x - (\delta_1 - \delta_2)] \quad (5.1.6)$$

taking in consideration the relative values of the amplitudes (5.1.2) and for values $x \neq \varepsilon$ we have

$$I_T \cong |\phi|^2 + |\theta_L|^2 +$$
$$+2\theta_0\theta_{0L} e^{-\frac{x^2}{2\sigma_\theta^2}}\, e^{-\frac{x^2}{2\sigma_L^2}} cos[(k_1 - k_2)x - \delta_{12}], \quad (5.1.7)$$

or, developing

$$I_T \cong \theta_0^2 e^{-\frac{x^2}{\sigma_\theta^2}} + \theta_{0L}^2 e^{-\frac{x^2}{\sigma_L^2}} +$$
$$+2\theta_0\theta_{0L} e^{-\frac{x^2}{2\sigma_\theta^2}}\, e^{-\frac{x^2}{2\sigma_L^2}} cos[(k_1 - k_2)x - \delta_{12}], \quad (5.1.8)$$

taking in consideration that, in general

$$\lambda_L \ggg \lambda_\theta, \quad (5.1.8)$$

we have

$$k_1 = \frac{2\pi}{\lambda_\theta} \ggg k_2 = \frac{2\pi}{\lambda_T}, \tag{5.1.8'}$$

leading to

$$I_T \cong \theta_0^2 e^{-\frac{x^2}{\sigma_\theta^2}} + \theta_{0L}^2 e^{-\frac{x^2}{\sigma_L^2}} +$$

$$+2\theta_0\theta_{0L} e^{-\frac{x^2}{2\sigma_\theta^2}} e^{-\frac{x^2}{2\sigma_L^2}} cos(k_1 x - \delta_{12}), \tag{5.1.9}$$

This expression means that for the case where the wavelength of the extended theta wave field is much greater than the one of the mother theta wave (5.1.8) the frequency felt by the acron is its own, that is, the one of the mother theta wave.

Now supposing that the extended theta wave has an intensity much greater than the one of the mother theta wave

$$\theta_{0L} \ggg \theta_0, \tag{5.1.10}$$

it is reasonable to write the expression for the theta wave field seen by the acron

$$I_T \cong \theta_{0L}^2 e^{-\frac{x^2}{\sigma_L^2}}. \tag{5.1.11}$$

This expression indicates that in a large theta wave field which intensity is much greater than the one of the mother theta wave, the intensity field felt by the acron is the one of the large theta wave.

In the opposite case that is, when the intensity of the extended field is much less that the one of the mother theta wave, the acron ignores the extended field.

In the next section we are going to study the motion of the acron in the case where the intensity of the extended wave is much greater than the one of the entering theta wave.

5.2. General Motion

In this section we clarify and demonstrate in a systematic way some statements that were made previously which are fundamental for the understanding of the collective processes.

As understood, these collective processes are the ones in which the average intensity of the subquantum theta wave field is a composition of many single theta waves.

Furthermore it is assumed that this field may have immersed in it many acrons. Since a very large number of acrons share the same total theta wave field it is supposed that, in this approach, they behave independently, so that, taking in account the previous considerations, the additive rule holds, approximately, at the statistical average level.

5.2.1. Basic Starting Points

In order to proceed systematically with the study of the collective processes, the ones that involve many theta waves and many acrons, it is convenient to summarize explicitly the starting points.

As known the principle of Eurhythmy states that the complex acron immersed in its theta wave field, naturally "chooses" in average the best possible path. Meaning the path leading it to the regions where the intensity of the theta wave field has higher intensity.

If the acron follows other possible paths it will have a very short life. To keep on with the derivation it is convenient to precise the concrete meaning of the principle of eurhythmy for this particular case. In these conditions, in order to do that it is helpful to summarize the basic starting assumptions:

Generic:

G1 - The space available to the acron is only and only the theta wave field. No acrons are found where the theta wave field is null.

G2 - In this approach there is energy conservation of the acron. More complex cases where energy dissipation is important are not considered. That is, the mean life of the acron, in this approach, is considered, for all practical purposes, infinite.

G3 - The natural velocity of the acron v_N is enormous, $v_N >> c$. Still the observed velocity, the average, velocity can be very small or even null.

G4 - The acron moves, according to the principle of eurhythmy, preferentially to the zones where the organized theta wave field has higher intensity.

G5 - It is assumed, in this approximation, that the average theta wave field is much bigger than the field of a single particle. So, for all practical purposes, in this approach, the average theta wave field does not depend on the position of the acron.

Formal:

F1 - The space available to the acron is divided into cubic cells of linear size ℓ_0.

F2 - In each transition, which takes a very short time t_0, the acron may remain in the same cell or move to the adjacent cells.

F3 - The explicit form of the probabilities of transition is, for all practical purposes, the mathematical expression of the principle of eurhythmy.

In this context the natural velocity has the form

$$v_N = v_0 = \frac{\ell_0}{t_0} >>> c \,. \tag{5.2.1}$$

where c is the habitual constant standing for velocity of the light in a medium with index of refraction one.

In the three dimensional approach we can assume for the generic cell and the respective transition probabilities for the acron to move may be represented as in Fig.5.1.

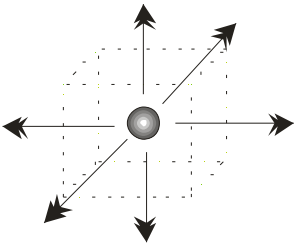

Fig.5.1 – Generic three dimensional cell and the transition probabilities represented by arrows

For this generic cell the probability conservation equation is

$$p_{back} + p_{forward} + p_{right} + p_{left} + p_{up} + p_{down} = 1\,, \tag{5.2.2}$$

where the p_i are the eurhythmic probabilities of the acron to move in the different directions. It was assumed that, in three dimensional approach, the acron always moves away from the initial cell. If the stochastic field seen by the acron is homogeneous then

$$p_i = p = \text{const}\,, \tag{5.2.3}$$

it implies that

$$p = \frac{1}{6}.$$

5.2.2. ONE-DIMENSIONAL APPROACH

In order to simplify the calculations and without loss of generality it is convenient instead of the more general three dimensional stochastic process to use the one-dimensional approach.

For this case, the three dimensional probability components are represented by the probability for the acron remaining in the same cell. In such circumstances the transition probabilities which are assumed constant in time can be represented, as can be seen in Fig.5.2

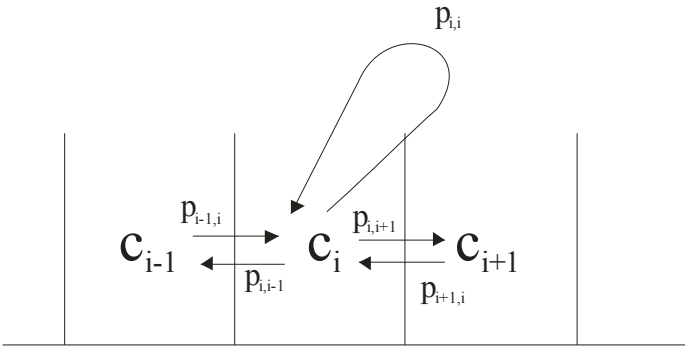

Fig.5.2 – One dimensional generic cell
and respective transition probabilities

with

$$q_i = p_{i,i-1}; \quad \delta_i = p_{i,i}; \qquad p_i = p_{i,i+1} \tag{5.2.4}$$

where $q_i = p_{i,i-1}$ represents the probability for the acron to move from cell i to cell i-1, and successively. The probability conservation equation leads us to write

$$q_i + \delta_i + p_i = 1. \tag{5.2.5}$$

These transition probabilities can be expressed in terms of the intensity of the theta wave field according to the principle of eurhythmy

$$\begin{cases} q_i = p_{i.i-1} = \dfrac{I_{i-1}}{I_{i-1} + I_i + I_{i+1}} \\ \delta_i = p_{i.i} = \dfrac{I_i}{I_{i-1} + I_i + I_{i+1}} \\ p_i = p_{i.i-1} = \dfrac{I_{i+1}}{I_{i-1} + I_i + I_{i+1}} \end{cases} \tag{5.2.6}$$

A stochastic evolution equation can be written recalling that the number of acrons in the generic cell C_i in the instant t +1 is equal to the number of acrons that remain plus the ones that enter from the right and the left

$$n_{i,t+1} = p_{i-1}n_{i-1,t} + \delta_i n_{i,t} + q_{i+1}n_{i+1,t}\,, \tag{5.2.7'}$$

or

$$n_{i,t+1} = (1 - q_i - p_i)n_{i,t} + p_{i-1}n_{i-1,t} + q_{i+1}n_{i+1,t} \tag{5.2.7''}$$

$$n_{i,t+1} = n_{i,t} - q_i n_{i,t} - p_i n_{i,t} + p_{i-1}n_{i-1,t} + q_{i+1}n_{i+1,t}\,. \tag{5.2.7'''}$$

Naming

$$\begin{cases} t_i = p_i n_i \\ r_i = q_i n_i \end{cases} \tag{5.2.8}$$

the stochastic equation assumes the form

$$n_{i,t+1} = n_{i,t} - r_{i,t} - t_{i,t} + t_{i-1,t} + r_{i+1,t}\,. \tag{5.2.9}$$

This expression may also be understood as describing the result of a very large number of similar independent experiments with a single acron.

The discrete equation can be approached to the continuous form and in this case *n(x,t)* stands for the probability density distribution

$$\begin{aligned} n(x,t+\tau_0) \;&= n(x,t) - r(x,t) - t(x,t) + \\ &\quad + t(x-\ell_0,t) + r(x+\ell_0,t). \end{aligned} \tag{5.2.10}$$

Now *n(x,t)* stands not the number of acrons at the position $x = i\,\ell_0$ at certain instant of time *t* but the density probability, that is, a function proportional to the number of acrons, such that $n_{i,t} \propto \ell_0\; n\;(x)$.

This formula can be Taylor-expanded, and using the customary cutoff criteria taking in account that the linear size ℓ_0 of the cell is very small and furthermore the velocity of transition is very high, see (5.2.2), which implies that

$$\ell_0 >>> t_0 \equiv \tau_0\,, \tag{5.2.11}$$

it is reasonable to cut the expansion from τ_0^2 and ℓ_0^3 on, giving

$$n+\tau_0\, n_t = n - r - t + t - \ell_0\, t_x + \frac{1}{2}\ell_0^2\, t_{xx} + r + \ell_0\, r_x + \frac{1}{2}\ell_0^2\, r_{xx}$$

$$\tau_0\, n_t = -\ell_0\, t_x + \frac{1}{2}\ell_0^2\, t_{xx} + \ell_0\, r_x + \frac{1}{2}\ell_0^2\, r_{xx}, \tag{5.2.12}$$

where

$$t_x = \frac{\partial t}{\partial x}, \quad t_{xx} = \frac{\partial^2 t}{\partial x^2}, \quad \cdots \tag{5.2.13}$$

but

$$\begin{cases} t = p\, n \\ t_x = p_x n + p\, n_x \\ t_{xx} = p_{xx} n + 2 p_x n_x + p\, n_{xx} \end{cases} \qquad \begin{cases} r = q\, n \\ r_x = q_x n + q\, n_x \\ r_{xx} = q_{xx} n + 2 q_x n_x + q\, n_{xx} \end{cases} \tag{5.2.14}$$

then by substitution in (5.2.12) we got

$$\begin{aligned} \tau_0\, n_t = &-\ell_0\, p_x\, n - \ell_0\, p\, n_x + \frac{1}{2}\ell_0^2\, p_{xx}\, n + \ell_0^2\, p_x\, n_x + \frac{1}{2}\ell_0^2\, p\, n_{xx} \\ &+ \ell_0\, q_x\, n + \ell_0\, q\, n_x + \frac{1}{2}\ell_0^2\, q_{xx}\, n + \ell_0^2\, q_x\, n_x + \frac{1}{2}\ell_0^2\, q n_{xx} \end{aligned} \tag{5.2.15'}$$

or,

$$\begin{aligned} \tau_0\, n_t = &-\ell_0 (p_x - q_x) n - \ell_0 (p - q) n_x + \\ &+ \frac{1}{2}\ell_0^2 (p_{xx} + q_{xx}) n + \ell_0^2 (p_x + q_x) n_x + \frac{1}{2}\ell_0^2 (p + q) n_{xx} \end{aligned} \tag{5.2.15}$$

Naming, somehow in analogy with the Brownian motion, the difference between the two probabilities as the reduced drift

$$\mu(x) = p(x) - q(x), \tag{5.2.16}$$

and the sum

$$d(x) = p(x) + q(x) \tag{5.2.17}$$

by the basic diffusion coefficient we have for the derivatives

$$\begin{cases} p - q = \mu \\ p_x - q_x = \mu_x \end{cases} \quad \begin{cases} p + q = d \\ p_x + q_x = d_x \\ p_{xx} + q_{xx} = d_{xx} \end{cases} \tag{5.2.18}$$

we have

$$\frac{\tau_0}{\ell_0} n_t = (\frac{1}{2} \ell_0 \, d_{xx} - \mu_x) \, n + (\ell_0 \, d_x - \mu) \, n_x + \frac{1}{2} \ell_0 \, d \, n_{xx}, \tag{5.2.19}$$

making

$$A(x) = \frac{1}{2} \ell_0 d, \quad B(x) = \ell_0 d_x - \mu, \quad C(x) = \frac{1}{2} \ell_0 d_{xx} - \mu_x, \tag{5.2.20}$$

and by substitution in (5.2.17) it gives the generic form for the fundamental stochastic evolution equation

$$A(x) \frac{\partial^2 n}{\partial x^2} + B(x) \frac{\partial n}{\partial x} + C(x) \, n = \frac{1}{v_N} \frac{\partial n}{\partial t} . \tag{5.2.21}$$

The explicit form for the transition probabilities can be obtained from equations (5.2.6) which may be written

$$\begin{cases} q(x) = \dfrac{I(x - \ell_0)}{I(x - \ell_0) + I(x) + I(x + \ell_0)} \\ \delta(x) = \dfrac{I(x)}{I(x - \ell_0) + I(x) + I(x + \ell_0)} \\ p(x) = \dfrac{I(x + \ell_0)}{I(x - \ell_0) + I(x) + I(x + \ell_0)} \end{cases} \tag{5.2.22}$$

when using the integration rectangle formula.

Expanding in Taylor power series and making the cutoff at the habitual position we have

$$\begin{aligned} I(x \pm \ell_0) &\cong I \pm \ell_0 I_x + \frac{1}{2}\ell_0^2 I_{xx} \\ I(x-\ell_0) + I(x) + I(x+\ell_0) &= I - \ell_0 I_x + \frac{1}{2}\ell_0^2 I_{xx} + I + I + \ell_0 I_x + \frac{1}{2}\ell_0^2 I_{xx} \\ &= 3I + \ell_0^2 I_{xx} \end{aligned} \tag{5.2.23}$$

that by substitution in (5.2.20) gives

$$\begin{cases} q(x) = \dfrac{I - \ell_0 I_x + \frac{1}{2}\ell_0^2 I_{xx}}{3I + \ell_0^2 I_{xx}} \\ \delta(x) = \dfrac{I}{3I + \ell_0^2 I_{xx}} \\ p(x) = \dfrac{I + \ell_0 I_x + \frac{1}{2}\ell_0^2 I_{xx}}{3I + \ell_0^2 I_{xx}} \end{cases} \tag{5.2.24}$$

In these circumstances the drift and the diffusion coefficients assume the form

$$\begin{cases} \mu(x) = \frac{2\ell_0 I_x}{3I + \ell_0^2 I_{xx}} \\ d(x) = 1 - \frac{I}{3I + \ell_0^2 I_{xx}} \end{cases} \tag{5.2.25}$$

This formula can be further simplified for the cases where the following approximation is valid

$$3I + \ell_0^2 I_{xx} \cong 3I. \tag{5.2.26}$$

In such approximation we have for the drift and diffusion coefficients

$$\begin{cases} \mu(x) = \frac{2}{3}\ell_0 \frac{I_x}{I} \\ d(x) = \frac{2}{3} \end{cases} . \tag{5.2.27}$$

In the following subsections we are going to study some particular cases for the intensity of the theta wave field.

5.2.2.1. VARIATION INTENSITY FIELD OF THE GENERIC FORM $I = ax^{\alpha}$

For theta wave field intensity of the generic form

$$I(x) = ax^{\alpha} \tag{5.2.28}$$

we have for reduced drift and diffusion parameters

$$\begin{cases} \mu = \frac{2}{3}\ell_0 \frac{\alpha}{x} \\ d = \frac{2}{3} \end{cases} \quad \begin{cases} \mu_x = \frac{2}{3}\ell_0 \frac{-\alpha}{x^2} \\ d_x = d_{xx} = 0 \end{cases} \tag{5.2.29}$$

5.2.2.1.1. CONSTANT INTENSITY FIELD

Now, assuming the simplest possible case, that is, when it can be approached to a constant variation for the intensity of the field, we have

$$p_i = q_i = p, \text{ we have } \alpha = 0 \tag{5.2.30}$$

then the stochastic terms transform into

$$\begin{cases} \mu = 0 \\ d = \frac{2}{3} \end{cases} \tag{5.2.31}$$

which give for the coefficients of the master stochastic equation

$$\begin{cases} A = \ell_0 /3 \\ B = 0 \\ C = 0 \end{cases} \tag{5.2.32}$$

By inserting in the master stochastic equation we got the heat equation

$$\frac{\partial^2 n}{\partial x^2} = \frac{3}{\ell_0 v_N}\frac{\partial n}{\partial t}. \tag{5.2.33}$$

which can be written in the canonical form of the diffusion equation

$$D\,\frac{\partial^2 n}{\partial x^2} = \frac{\partial n}{\partial t}, \tag{5.2.34}$$

with

$$D = \frac{1}{3}\ell_0 v_N \tag{5.2.35}$$

being the common diffusion coefficient.
The typical solution for this equation, for the initial condition $n(x,0) = \delta(x)$, is

$$n(x,t) = \frac{A\prime}{2\sqrt{\pi D t}}\, e^{-\frac{x^2}{4Dt}}, \tag{5.2.36}$$

with *A'* constant and $\delta(x)$ the Dirac delta function.
As can be seen from the plot of the equation (5.2.36),

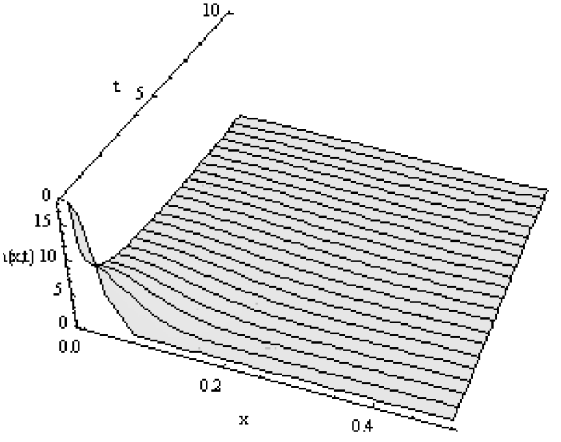

Fig.5.3 – Plot of the probability density
for the acron in a constant intensity field

for a constant density probability distribution, no preferential drift exists. This result is not surprising since, we know that when the intensity of the theta wave field is constant, according to the principle of eurhythmy, the tendency for the acron to move in any direction is constant. No privileged direction for the motion exists, so the average velocity is null.

For the case in which $n \neq n(t)$, meaning that we are in a kind of statistical equilibrium where the number of acrons does not, in average, change with time the equation (5.2.33) transforms into $D\frac{\partial^2 n}{\partial x^2} = 0$, having as a particular solution $n = \text{const}$. In this simple way we arrive at the Broglie guiding principle for quantum physics. This principle states that the probability of finding an acron in a quantum measurement process is proportional to the intensity of the theta wave field, $p \propto |\theta|^2 = I$. In this case, of a constant theta wave field, this equilibrium average probability is, as expected, constant, $p \propto |\theta|^2 = I = \text{const}$.

5.2.2.1.2. LINEAR INTENSITY FIELD

The second simplest case is to assume that the intensity of the theta wave field follows a linear variation, that is, when we have $\alpha = 1$

$$I(x) = \alpha x \tag{5.2.37}$$

And by substitution in (5.2.27) we have

$$\begin{cases} \mu(x) = \dfrac{2}{3}\dfrac{\ell_0}{x} \\ d = \dfrac{2}{3} \end{cases} \qquad \begin{cases} \mu_x = -\dfrac{2}{3}\dfrac{\ell_0}{x^2} \\ d_x = d_{xx} = 0 \end{cases} \tag{5.2.38}$$

which by substitution in equation (5.2.21) gives for the coefficients

$$A(x) = \frac{1}{3}\ell_0, \quad B(x) = -\frac{2}{3}\frac{\ell_0}{x}, \quad C(x) = \frac{2}{3}\frac{\ell_0}{x^2}, \tag{5.2.39}$$

and by substitution in equation (5.2.21) we finally obtain for the master stochastic equation

$$\frac{\partial^2 n}{\partial x^2}-\frac{2}{x}\frac{\partial n}{\partial x}+\frac{2}{x^2}n=\frac{3}{\ell_0 v_N}\frac{\partial n}{\partial t}. \tag{5.2.40}$$

In order to integrate this equation it is convenient to make the transformation

$$n=x\phi. \tag{5.2.41}$$

In this case the derivatives give

$$\begin{cases}\dfrac{\partial n}{\partial x}=\dfrac{\partial}{\partial x}(x\phi)=\phi+x\dfrac{\partial \phi}{\partial x}\\ \dfrac{\partial^2 n}{\partial x^2}=2\dfrac{\partial \phi}{\partial x}+x\dfrac{\partial^2 \phi}{\partial x^2}\\ \dfrac{\partial \phi}{\partial x}=\dfrac{\partial}{\partial x}(\dfrac{1}{x}n)=-\dfrac{1}{x^2}n+\dfrac{1}{x}\dfrac{\partial n}{\partial x}=-\dfrac{1}{2}(-\dfrac{2}{x}\dfrac{\partial n}{\partial x}+\dfrac{2}{x^2}n)\end{cases} \tag{5.2.42}$$

and inserting in (5.2.40) we get

$$\frac{\partial^2 \phi}{\partial x^2}=\frac{3}{\ell_0 v_N}\frac{\partial \phi}{\partial t},\quad \frac{1}{3}\ell_0 v_N\frac{\partial^2 \phi}{\partial x^2}=\frac{\partial \phi}{\partial t},$$

$$D\frac{\partial^2 \phi}{\partial x^2}=\frac{\partial \phi}{\partial t} \tag{5.2.43}$$

an equation equal to (5.2.31) whose solution is

$$\phi(x,t)=\frac{A\prime}{2\sqrt{\pi D t}}e^{-\frac{x^2}{4Dt}}, \tag{5.2.44}$$

and consequently (5.2.41) becomes

$$n(x,t)=\frac{A\prime}{2\sqrt{\pi D t}}\,x\,e^{-\frac{x^2}{4Dt}}. \tag{5.2.45}$$

which plot is shown in Fig.5.4

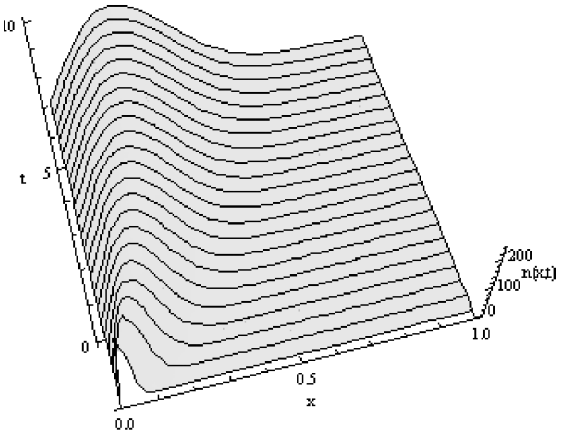

Fig.5.4 – Plot of the probability density
for the acron in a linear intensity field

From this solution it can be seen that the global average velocity of the acron, for a linear intensity field, decreases inversely with the distance. In fact taking the maximum of (5.2.45) and equaling to zero

$$n_x = 0 = \frac{A\prime}{2\sqrt{\pi Dt}}\left(e^{-\frac{x^2}{4Dt}} - \frac{x^2}{2Dt}e^{-\frac{x^2}{4Dt}}\right), \tag{5.2.46}$$

that is

$$x^2 - 2Dt = 0 \tag{5.2.47}$$

differentiating in respect to time

$$\frac{d}{dt}(x^2 - 2Dt = 0) \xrightarrow{yields} v = \frac{dx}{dt} = \frac{D}{x} \tag{5.2.48}$$

substituting the value of D (5.2.35) we get

$$v(x) = \frac{1}{3}\frac{\ell_0}{x}v_N. \tag{5.2.49}$$

This indicates that the average velocity for the acron is, as expected, proportional to the reduced drift

$$v(x) \propto \mu(x) = \frac{2}{3}\frac{\ell_0}{x} \tag{5.2.50}$$

From (5.2.48) one sees that only for relatively small values of the space variable the average velocity of the acron has some significance. Still, for relatively long distances from the origin it goes to zero meaning the eurhythmic tendency to advance in the direction of the increasing intensity field decreases inversely with the distance, till, in the limit, it goes to zero.

This apparently unexpected consequence is perfectly understood in light of the principle of eurhythmy. In fact recalling (5.2.6) we have for the reduced drift

$$p_i - q_i = \frac{I_{i+1}}{I_{i-1} + I_i + I_{i+1}} - \frac{I_{i-1}}{I_{i-1} + I_i + I_{i+1}} = \frac{I_{i+1} - I_{i-1}}{I_{i-1} + I_i + I_{i+1}}$$

giving for the continuous linear variation and for the rectangular integration rule the same drift

$$\mu(x) = p - q \ = \frac{\alpha(x + \ell_0) - \alpha(x - \ell_0)}{\alpha(x - \ell_0) + \alpha\, x + \alpha(x + \ell_0)} = \frac{2}{3}\frac{\ell_0}{x}$$

From the expression one sees that the difference is, in this case of the linear variation, always constant. Nevertheless the bulk intensity increases linearly. The reduced drift is, in this situation, no more than the weighting of the constant difference by increasing bulk. So, it means that the eurhythmy propensity for the acron to move forward decreases in each step till for large values of the distance it practically vanishes.

In the case of a decreasing linear intensity field

$$I(x) = \beta - \alpha x.$$

the reduced drift is

$$\mu(x) = -\frac{\alpha}{\beta - \alpha x}.$$

The plot of the linear decreasing intensity field together with the absolute value of the reduced drift are shown in Fig.5.5

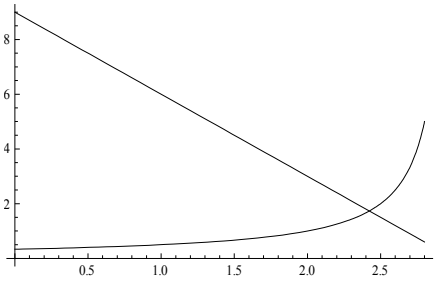

Fig.5.5 – Linear decreasing field
with the absolute value of the drift.

Again, writing equation (5.2.40) for the average statistical time independent case we have $D\frac{\partial^2\phi}{\partial x^2} = 0$, giving as a particular solution $p \propto |\theta|^2 = I = n = \alpha x$, we arrive at the Broglie equilibrium average expression for the principle of eurhythmy that is, his guiding principle.

5.2.2.1.3. INTENSITY FIELD DECAYING IN $1/R^2$

The other interesting case is the one with $\alpha = -2$. In such situation we have a theta wave field intensity decreasing in $1/x^2$, or, more generally, in $1/r^2$. In this particular case the intensity of the theta wave field is to be written

$$I(r) = \frac{a}{r^2}, \tag{5.2.51}$$

recalling again the previous formulas in which we changed the variable *x* by *r* without any loss since all derivations that lead to them do not change.

$$\begin{cases} \mu(r) = \frac{2}{3}\ell_0\frac{I_r}{I} \\ d(r) = \frac{2}{3} \end{cases}$$

we have

$$\frac{I_r}{r} = \frac{-2a\,r^{-3}}{ar^{-2}} = \frac{-2}{r}$$

and by substitution we have

$$\begin{cases} \mu(r) = -\dfrac{4}{3}\dfrac{\ell_0}{r} \\ d = \dfrac{2}{3} \end{cases} \qquad \begin{cases} \mu_r = \dfrac{4}{3}\dfrac{\ell_0}{r^2} \\ d_R = d_{rr} = 0 \end{cases} \tag{5.2.52}$$

an expression in a way similar to the linear case with a double value for the reduced drift and the signs changed. By substituting in the equation the habitual relations

$$A(r) = \frac{1}{2}\ell_0 d, \quad B(r) = \ell_0 d_r - \mu, \quad C(r) = \frac{1}{2}\ell_0 d_{rr} - \mu_r$$

we have for the coefficients

$$A = \frac{1}{3}\ell_0, \quad B(r) = \frac{4}{3}\frac{\ell_0}{r}, \quad C(r) = -\frac{4}{3}\frac{\ell_0}{r^2}, \tag{5.2.53}$$

and by introducing in the basic differential equation (5.2.21) we finally get for the master stochastic equation

$$\frac{\partial^2 n}{\partial r^2} + \frac{4}{r}\frac{\partial n}{\partial r} - \frac{4}{r^2}n = \frac{3}{\ell_0 v_N}\frac{\partial n}{\partial t}. \tag{5.2.54}$$

Let us now try the transformation used previously for the linear case

$$n = r\,\phi.$$

In this situation we have for the derivatives

$$\begin{cases} \dfrac{\partial n}{\partial r} = \dfrac{\partial}{\partial r}(r\phi) = \phi + r\dfrac{\partial \phi}{\partial r} \\ \dfrac{\partial^2 n}{\partial r^2} = 2\dfrac{\partial \phi}{\partial r} + x\dfrac{\partial^2 \phi}{\partial r^2} \\ \dfrac{\partial n}{\partial t} = \dfrac{\partial}{\partial t}(r\phi) = r\dfrac{\partial \phi}{\partial t} \end{cases}$$

And by substitution and simplification into (5.2.54) we get

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{6}{r}\frac{\partial \phi}{\partial r} = \frac{3}{\ell_0 v_N}\frac{\partial \phi}{\partial r}. \tag{5.2.55}$$

In this case, the transformation of variables did not lead to a direct soluble equation like in the case of the linear intensity variation. In order to do so it is convenient to use the method of separation of variables, with the loss of generality since it will make the time independent of the space. In any case making

$$\phi(r,t) = m(r)f(t). \tag{5.2.56}$$

we have by substitution

$$\frac{1}{m}\frac{\partial^2 m}{\partial r^2} + \frac{1}{m}\frac{6}{r}\frac{\partial m}{\partial r} = \frac{1}{f}\frac{3}{\ell_0 v_N}\frac{\partial f}{\partial t} = c^{te} = -\omega^2. \tag{5.2.57}$$

or

$$\frac{\partial^2 m}{\partial r^2} + \frac{6}{r}\frac{\partial m}{\partial x} + \omega^2 m = 0, \quad \frac{3}{\ell_0 v_N}\frac{\partial f}{\partial t} = -\omega^2 f. \tag{5.2.58}$$

The solution for the temporal part is

$$f(t) = e^{-\frac{\ell_0 v_N}{3}\omega^2 t}, \tag{5.2.59}$$

while the spatial part of the solution is a little bite complicated

$$m(r) = \sqrt{\frac{2}{\pi\omega}}\frac{1}{r^3}\begin{Bmatrix} C_1\left[-\frac{3\cos(\omega\, r)}{\omega\, r} - \sin(\omega\, r) + \frac{3\sin(\omega\, r)}{\omega^2 r^2}\right] + \\ C_2\left[\cos(\omega\, r) - \frac{3\cos(\omega\, r)}{\omega^2 r^2} - \frac{3\sin(\omega\, r)}{\omega\, r}\right] \end{Bmatrix} \quad (5.2.60)$$

The particular solution for the stationary case then reads

$$n(r,t) = \sqrt{\frac{2}{\pi\omega}}\frac{1}{r^2}\begin{Bmatrix} C_1\left[-\frac{3\cos(\omega\, r)}{\omega\, r} - \sin(\omega\, r) + \frac{3\sin(\omega\, r)}{\omega^2 r^2}\right] + \\ C_2\left[\cos(\omega\, r) - \frac{3\cos(\omega\, r)}{\omega^2 r^2} - \frac{3\sin(\omega\, r)}{\omega\, r}\right] \end{Bmatrix} e^{-\frac{\ell_0 v_N}{3}\omega^2 t} \quad (5.2.61)$$

where C_1 and C_2 are constants. This expression indicates that the probability density for the distribution of acrons, apart from the oscillatory variation, decreases with the inverse square of the distance.

Since it was not possible to solve the basic differential equation for time and space we could not arrive at a direct expression for the average velocity of the acron when immersed in a theta wave field with a variation in $1/r^2$. In such circumstances the average velocity is obtained from the expression of the reduced drift

$$v(x) \propto \mu(r) = \frac{4}{3}\ell_0\frac{1}{r}. \quad (5.2.62)$$

5.2.2.2. EXPONENTIALLY DECAYING INTENSITY FIELD

Assuming that the theta wave field has an exponential decreasing variation given by the expression

$$I(x) = A'e^{-\alpha x}, \quad (5.2.63)$$

taking the approximation given by (5.2.27) we have for the coefficients and derivatives

$$\begin{cases} \mu = \dfrac{2}{3}\ell_0\alpha \\ d = \dfrac{2}{3} \end{cases} \qquad \begin{cases} \mu_x = 0 \\ d_x = 0; \quad d_{xx} = 0 \end{cases} \tag{5.2.64}$$

by substitution in (5.2.20) it gives for the coefficients of the master stochastic equation

$$A = \frac{1}{3}\ell_0, \quad B = \frac{2}{3}\ell_0\alpha, \quad C = 0 \tag{5.2.65}$$

which can be written

$$\frac{1}{3}\ell_0 v_N \frac{\partial^2 n}{\partial x^2} + \frac{2}{3}\ell_0 v_N \alpha \frac{\partial n}{\partial x} = \frac{\partial n}{\partial t} \tag{5.2.66'}$$

or, written in the usual form

$$D\frac{\partial^2 n}{\partial x^2} + u\frac{\partial n}{\partial x} = \frac{\partial n}{\partial t} \tag{5.2.66}$$

we get the known diffusion equation where the diffusion and drift coefficients have the form

$$D = \frac{1}{3}\ell_0 v_N, \qquad u = \frac{2}{3}\ell_0 v_N \alpha = \mu v_N \tag{5.2.67}$$

Again a particular solution to this equation, for the same initial condition, is the well known formula

$$n(x,t) = \frac{A'}{\sqrt{4\pi D t}} e^{-\frac{(x+ut)^2}{4Dt}}, \tag{5.2.68}$$

whose plot is shown in Fig.5.6

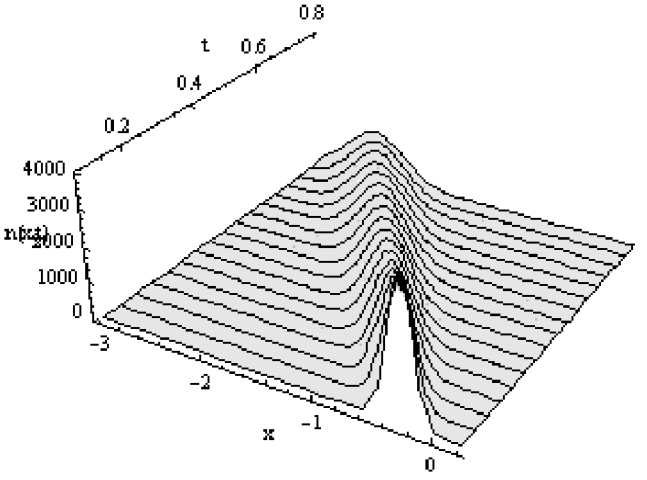

Fig.5.6 – Plot of the probability density for the acron
in an exponentially decreasing intensity field

From this solution one sees that the global average velocity of the acron, for an exponential decreasing intensity field, is constant. In fact taking the maximum of (5.2.68) and equaling to zero

$$n_x = 0 = \frac{A\prime}{2\sqrt{\pi Dt}} \frac{-2(x+ut)}{2Dt} e^{-\frac{(x+ut)^2}{4Dt}}, \tag{5.2.69}$$

implying that

$$x = -ut \tag{5.2.70}$$

consequently the average velocity of the acron, for this case, is given by

$$v = -u = -\frac{2}{3}\alpha \ell_0 v_N. \tag{5.2.71}$$

Since the reduced drift is constant the gradient of it is null implying that the acceleration is zero consequently the average velocity of the acron is constant in accordance with the last expression.

Furthermore, the average velocity of the acron is shown also to be proportional to the reduced drift.

$$v \propto \mu$$

As expected, the average sense of the motion of the acron is towards the higher theta wave field intensity according to the principle of eurhythmy. In this case, the average velocity is constant even if the field increases exponentially. This comes from the fact that the chaotic medium offers what may be called a kind of resistance to the motion of the acron, such that is completely balanced in the case of an exponentially increasing field leading to a constant average velocity.

Also, in this case, a time independent particular solution of equation (5.2.21) is, according to de Broglie guiding principle a decreasing exponential

$$p \propto |\theta|^2 = I \ = n(x) = Ae^{-\alpha x}$$

5.2.2.3 GAUSSIAN DECAYING INTENSITY FIELD

Assuming now that the intensity field has a Gaussian variation, such that it can be written

$$I(x) = A'e^{-\alpha x^2}, \tag{5.2.72}$$

recalling

$$\begin{cases} \mu(x) = \frac{2}{3}\ell_0 \frac{I_x}{I} \\ d(x) = \frac{2}{3} \end{cases}$$

we have

$$I_x = -2\alpha\, x\, I, \tag{5.2.73}$$

meaning that

$$\mu(x) = -\frac{4}{3}\ell_0 \alpha\, x \ , \tag{5.2.74}$$

And

$$\begin{cases} \mu = -\dfrac{4}{3}\ell_0 \alpha\, x \\ d = \dfrac{2}{3} \end{cases} \qquad \begin{cases} \mu_x = -\dfrac{4}{3}\ell_0 \alpha \\ d_x = 0; \quad d_{xx} = 0 \end{cases} \tag{5.2.75}$$

Now recalling (5.2.20),

$$A(x) = \frac{1}{2}\ell_0 d, \quad B(x) = \ell_0 d_x - \mu, \quad C(x) = \frac{1}{2}\ell_0 d_{xx} - \mu_x$$

and by substitution we have for the coefficients of the master stochastic equation for the case of a gaussian decreasing intensity theta wave field

$$A = \frac{1}{3}\ell_0, \quad B = \frac{4}{3}\ell_0 \alpha\, x, \quad C = \frac{4}{3}\ell_0 \alpha. \tag{5.2.76}$$

In such conditions the master stochastic equation reads

$$\frac{1}{3}\ell_0 v_N \frac{\partial^2 n}{\partial x^2} + \frac{4}{3}\ell_0 v_N \alpha\, x \frac{\partial n}{\partial x} + \frac{4}{3}\ell_0 v_N \alpha\, n = \frac{\partial n}{\partial t} \tag{5.2.77'}$$

or

$$D\frac{\partial^2 n}{\partial x^2} + u\, x \frac{\partial n}{\partial x} + u\, n = \frac{\partial n}{\partial t} \tag{5.2.77}$$

with

$$D = \frac{1}{3}\ell_0 v_N, \qquad u = \frac{4}{3}\ell_0 v_N \alpha \quad . \tag{5.2.78}$$

Since we are dealing with a diffusion type equation with a non constant drift, in order to solve the equation it is convenient to make the separation of variables. Naturally, by doing so we lose the correlation between space and time and consequently are unable to calculate the average

velocity of the acron directly and have to obtain it by the expression of the reduced drift. In such conditions, by making

$$n(x,t) = m(x)f(t), \tag{5.2.79}$$

we have by substitution

$$\frac{D}{m}\frac{\partial^2 m}{\partial x^2} + \frac{u}{m}x\frac{\partial m}{\partial x} + u = \frac{1}{f}\frac{\partial f}{\partial t} = c^{te} = -\omega\ , \tag{5.2.80}$$

or

$$D\frac{\partial^2 m}{\partial x^2} + u\,x\frac{\partial m}{\partial x} + (u+\omega)m = 0, \quad \frac{\partial f}{\partial t} = -\omega f\ . \tag{5.2.81}$$

Now dividing by *D* the spatial equation can be written

$$\frac{\partial^2 m}{\partial x^2} + 4\alpha\, x\frac{\partial m}{\partial x} + \frac{u+\omega}{D}m = 0, \tag{5.2.82}$$

a particular solution of this equation for the particular case of

$$\omega = 4\alpha D = \tfrac{4}{3}\ell_0 v_N \alpha = 2u \tag{5.2.83}$$

is

$$m(x) = C\ 2\sqrt{2\alpha}\ x\ e^{-2\alpha x^2}\ . \tag{5.2.84}$$

and a solution to the second equation is

$$f(t) = e^{-2ut}\ , \tag{5.2.85}$$

so the particular solution in the two variables for the stationary case can be written

$$n(x,t) = C\ 2\sqrt{2\alpha}\ x\ e^{-2\alpha x^2}e^{-2ut}, \tag{5.2.86'}$$

or

$$n(x,t) = C\ 2\sqrt{2\alpha}\ x\ e^{-2(\alpha x^2+2ut)}, \tag{5.2.86}$$

whose plot is seen in Fig.5.7.

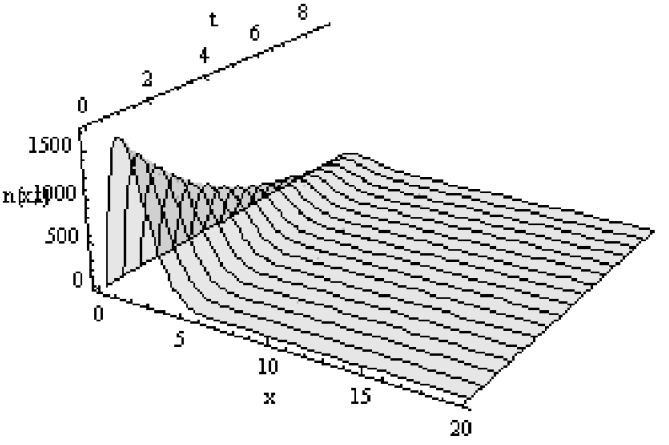

Fig.5.7 – Plot of the probability density for the acron in a gaussian decreasing intensity field

Now if we look for the statistical equilibrium, giving the probability density of acrons not depending on time, $n \neq n(t)$, equation (5.2.77) assumes the form

$$D\frac{\partial^2 n}{\partial x^2} + u\,x\frac{\partial n}{\partial x} + u\,n = 0\,, \tag{5.2.87}$$

and dividing by D it assumes the for

$$\frac{\partial^2 n}{\partial x^2} + 4\alpha\,x\frac{\partial n}{\partial x} + 4\alpha\,n = 0 \tag{5.2.88}$$

having as particular solution, as expected, a Gaussian

$$n(x) = Ae^{-2\alpha x^2}. \tag{5.2.89}$$

Again, the approximations made previously are robust enough to allow us to get de Broglie's guiding expression.

5.3. THE NATURE OF THE CONCEPT OF CLASSICAL FIELD

At this point it is convenient to call the attention to a very important point that till now has been misunderstood due to the subtleness of it. It concerns the status of the classical concept of field, be it gravitic or electromagnetic, and of the consequent related concept of potential. As everybody knows the gravitic field corresponds to the description of the forces, or accelerations that a gravitic particle placed in the field experiences. That is, corresponds to the description for the propensity of a particle to move in the real gravitic medium (the gravitic theta wave field) and consequently do not correspond to the real existing physical medium. So, it is very easy to realize that the gravitic field, electromagnetic field and so, are not a real physical media in the sense that they correspond to a description of real existent physical entity. Indeed the gravitic field, or any other of them, describes only a particular behavior of a particle, in this case the propensity to move. In our case, the propensity for the acron to move, when immersed in the real existing theta wave field. By choosing another possible property for the same physical entity we should expect another type behavior of the field for the very same real physical medium where the particle is immersed.

The gravitic field corresponds to the description of the forces, or accelerations that a gravitic particle experiences when placed in the field. That is, stands for the propensity for the particle to move in the real gravitic field (the gravitic theta wave field) and consequently do not correspond to the real existing real field. The necessary conclusion to draw is that classical fields are devoid of any ontological status, they are only mere operational useful concepts.

We have seen that the reduced drift $\mu = p - q$ stands for the eurhythmic propensity for the acron to move, in a large theta wave field such that $|\theta_0|^2 \lll |\theta_M|^2$, so that the immersed acron, for all practical purposes, is only sensitive to the extended field. This propensity for the acron to move, which is associated with the average velocity, relative to the theta wave field, in the continuous representation may be approached by $\bar{v} \propto \mu \cong I_r/I$. This expression tells us that the propensity for the acron to move is given by the gradient of the intensity normalized by the actual value of the extended field intensity.

Furthermore, we have seen that in the case of a real theta wave field of intensity varying with $1/r^2$, the expression for the reduced drift is

$$\mu(r) = \frac{4}{3}\ell_0 \frac{1}{r}. \tag{5.3.1}$$

Since the reduced drift is proportional is proportional to the average velocity of the acron we have

$$\bar{v}(r) \propto \mu(r) = \frac{4}{3}\ell_0 \frac{1}{r} \propto \frac{1}{r}. \tag{5.3.2}$$

Now, let us see which is the velocity of a particle placed in a gravitic field with the same variation

$$F \propto a \propto \frac{1}{r^2}. \tag{5.3.3}$$

In this case we have

$$a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = v\frac{dv}{dr} = \frac{1}{2}\frac{d}{dr}(v^2) \propto r^{-2} = -\frac{d}{dr}(r^{-1}), \tag{5.3.4}$$

that is

$$v \propto \frac{1}{\sqrt{r}}. \tag{5.3.5}$$

This formula indicates that the velocity of the gravitic particle decreases, with the inverse square root of the distance.

Summarizing: For two fields with the same variation in $1/r^2$, one the theta wave field, a real physical medium, and the gravitc field an abstract medium without real physical existence we have

$$\text{Real physical medium - } \bar{v}(r) \propto \frac{1}{r}$$
$$\text{Abstract physically inexistent medium - } v \propto \frac{1}{\sqrt{r}}.$$

Now the question is: what is the real physical theta wave field that is behind the abstract operational physically inexistent gravitic field with same variation in $1/r^2$? That is, which is the real physical theta wave field that gives the same expression for the average velocity of the acron?

Since the average velocity of the acron is proportional to the reduced drift

$$\bar{v}(x) \propto \mu(r) = \frac{I_r}{I}. \tag{5.3.6}$$

we have only to solve the differential equation

$$\frac{I_r}{I} = \bar{v} \propto \frac{1}{\sqrt{r}} \tag{5.3.7}$$

that is

$$I_r + ar^{-1/2} I = 0, \tag{5.3.8}$$

having as solution

$$I(r) = c_1 e^{-2a\sqrt{x}}. \tag{5.3.9}$$

In these conditions a theta wave field with this kind of intensity variation gives precisely the operational properties of the $1/r^2$ the classical field.

In the case of a linear field $I = a\,r$ we got for the value of the drift $\mu = 1/r$ implying that the propensity for the acron to move, even if the value of the slope of the field is constant, decreases till for large values of the distance r it approaches zero. In this case very far from the origin the propensity for the acron to move is practically zero.

Quite different is the notion of classical potential U which is, as we have already seen, only an operational useful concept where is assumed that $F \propto a \propto \nabla U$. Still there are some very special particular cases, and under adequate boundary conditions, were the stochastic drift may be approached by the classical notion of potential. For instance in the case we are studying the average velocity of the acron in a very small regions of the field, the distance from the origin may be described as a composition of two terms one constant and very big R, for instance the Earth radius, the other relatively very small r. In such case we may write

$$\begin{gathered} \bar{v} = \frac{1}{R+r}, \;\; v = \frac{\sqrt{2}}{\sqrt{R+r}}, \\ R = const \gg r \;\; \rightarrow \bar{v} = \frac{1}{R} = c_1, \\ v = \frac{\sqrt{2}}{\sqrt{R}} = c_2, \Longrightarrow \bar{v} \propto v. \end{gathered} \tag{5.3.10}$$

meaning that in this particular case and for these boundary conditions, the eurhythmic average velocity is proportional to the velocity of the classic particle.

The conclusion to draw is that, we need to be aware that the concept, of gravitic field, with the force being proportional to the inverse of the

square distance, or in the case of the electric field, stand only for abstract operational representations, descriptions of some deeper physical processes. So, these so-called fields are not, in reality, true physical fields, but something with some average properties derived from the interactions of the true basic real fields. The notion of abstract operational force fields, gravitic, electric, and so on, corresponds to a kind of inference made with the aim of describing the average behavior for the reciprocal interaction of the real physical systems. Furthermore, even if the real extended theta wave field keeps the same type of variation, say $1/r^2$, the reciprocal interaction of the complex particle with it depends, as we have seen, on the relative strength of the waves. In the case when the theta wave accompanying the acron has a relatively greater intensity, $|\theta_0|^2 \ggg |\theta_M|^2$, there is no interaction at all. The acron ignores completely the large theta wave field. Only in the opposite case, $|\theta_0|^2 \lll |\theta_M|^2$, the one we were considering the acron pays no attention to its own theta wave and is fully sensitive to the large extended theta wave field. For the intermediary cases things are a little bit more complicated because the final average motion of the acron depends on the conjugated effects of the two theta wave fields.

5.3.1. THE CLASSICAL CONCEPT OF INITIAL VELOCITY

The initial velocity is the velocity with which the acron is, let us say, injected in a large theta wave field. The acron, coming from a mother theta wave, assuming, naturally, for simplicity reasons that it is reasonably described by a modulated gaussian, a Morlet wavelet, of the form

$$\theta(x) = \frac{A}{\sqrt{\sqrt{\pi}\sigma}} e^{-\frac{x^2}{2\sigma^2}+ikx} \tag{5.3.11}$$

taking in consideration only the spatial part, starts seeing the field of the large composite theta wave, since in average the condition $|\theta|^2 \lll |\theta_L|^2$ is practically fulfilled. The velocity with which it is launched in the large theta wave is the initial velocity. This velocity is the instantaneous velocity related to its average velocity in the mother theta wave.

The intensity of the basic mother theta wave is

$$I(x) = \theta\theta^* = \frac{A^2}{\sqrt{\pi}\sigma} e^{-\frac{x^2}{\sigma^2}}, \tag{5.3.12}$$

which integrated over all space gives a kind of area

$$\mathcal{A} = \int_{-\infty}^{+\infty} \frac{A^2}{\sqrt{\pi}\sigma} e^{-\frac{x^2}{\sigma^2}} dx = A^2, \tag{5.3.13}$$

which means that the area of the basic theta wave is constant for any value of the width as can be seem from the plot

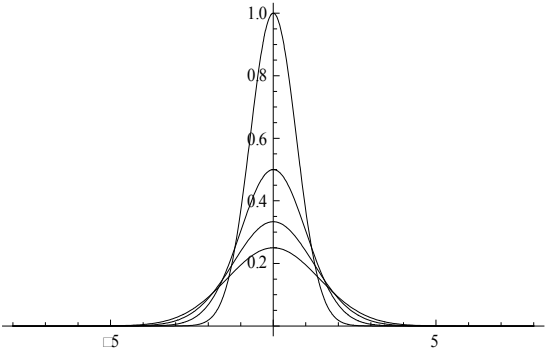

Fig.5.8 – Plot of the mother theta wave
for different values of the width

From the plot one sees that the shorter the width, the higher is the mother theta and consequently the gradient.

In such situation, recalling the previous study of the gaussian intensity we know that the reduced drift is (5.2.75)

$$\mu(x) = -\frac{4}{3}\ell_0 \frac{1}{\sigma^2}\, x \ . \tag{5.3.14}$$

This drift traduces the average velocity with which the acron moves in the theta wave field. So, the velocity with which the acron is injected in the extended theta wave field is given by the average of the drift,

$$v = \overline{\mu} = \frac{1}{\sigma}\int_0^{\sigma} \mu \, dx. \tag{5.3.15}$$

By substitution and integration we have

$$v = \overline{\mu} = \frac{1}{\sigma}\int_0^{\sigma} (-\frac{4}{3}\ell_0 \frac{1}{\sigma^2}\, x) dx = -\frac{2}{3}\ell_0 \frac{1}{\sigma}. \tag{5.3.16}$$

Recalling the fundamental formula relating the width of the basic theta wave with the wavelength

$$\sigma = M\lambda \tag{5.3.17}$$

we have by substitution

$$v = \overline{\mu} = -\frac{2}{3}\ell_0 \frac{1}{M\lambda} \tag{5.3.18}$$

that is

$$v \propto \frac{1}{\lambda}. \tag{5.3.19}$$

So, the initial velocity of an acron appearing in a relatively large intensity theta wave field is given approximately by the inverse of the wavelength.

In these conditions it is reasonable to express the average velocity of the acron as proportional to a kind of particular density type function of the theta wave. Since, let us say, the area of the mother theta wave is a constant, it is possible to define the generalized momentum, this momentum being a kind of density. So, dividing the area of the theta wave by the width we have

$$p = \frac{\mathcal{A}}{\sigma} = \frac{\mathcal{A}}{M}\frac{1}{\lambda} = \frac{3}{2}\frac{\mathcal{A}}{\ell_0}\, v = \eta\, v. \tag{5.3.20}$$

In these conditions it is possible to represent a kind of generalized mass by

$$\eta = \frac{3}{2}\frac{\mathcal{A}}{\ell_0}. \tag{5.3.21}$$

5.4. Motion in a Constant Intensity Field with Initial Velocity

In order to include this initial velocity in the description of the average motion of the acron in medium of constant intensity theta wave field such that $|\theta|^2 \ll |\theta_L|^2$ it is convenient again to make a sketchy representation of the space in order to write the explicit form for the probabilities of transition. In one dimension we have Fig.5.2

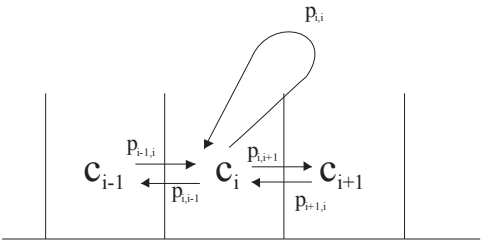

Fig.5.2 – Generic cell for the one-dimensional approach

Renaming, as previously, the transition probabilities

$$q_i = p_{i,i-1}; \quad \delta_i = p_{i,i}; \qquad p_i = p_{i,i+1} \tag{5.4.1}$$

with

$$q_i + \delta_i + p_i = 1 \tag{5.4.2}$$

Recalling that due to some initial preparation the acron has an initial velocity which is translated, as we have seen, in the generic form of the transition probabilities we have to write

$$\begin{aligned} p_i &= \widetilde{p}_i + \gamma, \quad \gamma = \text{constant} \\ q_i &= \widetilde{q}_i \\ \delta_i &= \widetilde{\delta}_i \end{aligned} \tag{5.4.3}$$

where $\widetilde{p}_i$ is the direct contribution of the principle of eurhythmy and γ resulting from the initial preparation of the acron represented usually by the initial velocity. In this situation the equation of conservation of probability (5.4.2) assumes the form

$$\widetilde{q}_i + \widetilde{p}_i + \widetilde{\delta}_i + \gamma = 1, \quad \widetilde{q}_i + \widetilde{p}_i + \widetilde{\delta}_i = 1 - \gamma\,. \tag{5.4.4}$$

These transition probabilities can be expressed in terms of the intensity of the theta wave field according to the principle of eurhythmy

$$\begin{cases} \widetilde{q}_i = \widetilde{p}_{i.i-1} = (1-\gamma)\dfrac{I_{i-1}}{I_{i-1}+I_i+I_{i+1}} \\ \widetilde{\delta}_i = \widetilde{p}_{i.i} = (1-\gamma)\dfrac{I_i}{I_{i-1}+I_i+I_{i+1}} \\ \widetilde{p}_i = \widetilde{p}_{i.i+1} = (1-\gamma)\dfrac{I_{i+1}}{I_{i-1}+I_i+I_{i+1}} \end{cases} \tag{5.4.5}$$

Since we are dealing with an homogeneous medium, $I_{i,j}$= constant, the eurhythmic probabilities of transition are, of course, also constant as can be inferred from (5.4.5)

$$\widetilde{p}_{i,i-1} = \widetilde{q}_i = \widetilde{\delta}_i = \frac{I_{i-1}}{I_{i-1}+I_i+I_{i+1}} = \text{constant}\ . \tag{5.4.6}$$

The equation of conservation of probability (5.4.4) can be written

$$q\ +\delta\ +p\ =1 \tag{5.4.7}$$

with

$$p-q=\gamma. \tag{5.4.8}$$

The stochastic evolution equation can then be written recalling that the number of acrons in the generic cell C_i in the instant t +1 is equal to the number of acrons that remain plus the ones that enter from the right and the left

$$n_{i,t+1} = p\, n_{i-1,t} + \delta\, n_{i,t} + q\, n_{i+1,t}\,, \tag{5.4.9}$$

or

$$n_{i,t+1} = (1-(q+p))n_{i,t} + p\, n_{i-1,t} + q\, n_{i+1,t} \tag{5.4.10'}$$

$$n_{i,t+1} = n_{i,t} - q\, n_{i,t} - p\, n_{i,t} + p\, n_{i-1,t} + q\, n_{i+1,t}\,. \tag{5.4.10''}$$

This discrete equation (5.4.10'') can, as usual, be approached by a continuous equation of the form

$$n(x,t+\tau_0) = p\,n(x-\ell_0,t) + \delta\,n(x,t) + q\,n(x+\ell_0,t) \qquad (5.4.11)$$

and now *n* stands for the density probability distribution. As habitual this expression can be expanded in Taylor series giving

$$\begin{aligned} & n(x,t) + \tau_0 \frac{\partial n(x,t)}{\partial t} + \frac{1}{2!}\tau_0^2 \frac{\partial^2 n(x,t)}{\partial t^2} + \ldots \\ & = pn(x,t) - p\ell_0 \frac{\partial n(x,t)}{\partial x} + \frac{1}{2!} p\ell_0^2 \frac{\partial^2 n(x,t)}{\partial x^2} - \frac{1}{3!} p\ell_0^3 \frac{\partial^3 n(x,t)}{\partial x^3} + \ldots \\ & + \delta\, n(x,y,t) \\ & + qn(x,t) + q\ell_0 \frac{\partial n(x,t)}{\partial x} + \frac{1}{2!} q\ell_0^2 \frac{\partial^2 n(x,t)}{\partial x^2} + \frac{1}{3!} q\ell_0^3 \frac{\partial^3 n(x,t)}{\partial x^3} + \ldots \end{aligned} \qquad (5.4.12)$$

since the linear size ℓ_0 of the cell is very small and furthermore the velocity of transition is very high it implies that

$$\ell_0 >> t_0 \equiv \tau_0\,, \qquad (5.4.13)$$

it is reasonable to cut off the expression (5.4.12)from τ_0^2 and ℓ_0^3 on, giving

$$n + \tau_0 \frac{\partial n}{\partial t} = (p+\delta+q)n - \ell_0(p-q)\frac{\partial n}{\partial x} + \frac{1}{2}\ell_0^2(p+q)\frac{\partial^2 n}{\partial x^2} \qquad (5.4.14)$$

recalling that

$$p+\delta+q = 1\,,$$

Then equation (5.4.14) can be written

$$\frac{\partial n}{\partial t} = -(p-q)\frac{\ell_0}{\tau_0}\frac{\partial n}{\partial x} + \frac{1}{2}(p+q)\frac{\ell_0^2}{\tau_0}\frac{\partial^2 n}{\partial x^2}\,, \qquad (5.4.15)$$

$$D\frac{\partial^2 n}{\partial x^2}-u\frac{\partial n}{\partial x}=\frac{\partial n}{\partial t}, \qquad (5.4.15')$$

which, as expected, is the common diffusion equation with constant drift

$$u=(p-q)\frac{\ell_0}{\tau_0}, \qquad (5.4.16)$$

and the diffusion coefficient

$$D=\frac{1}{2}(p+q)\frac{\ell_0^2}{\tau_0}. \qquad (5.4.17)$$

The drift given by equation (5.4.16) can also be written recalling that the reduced drift is given by

$$\mu = p - q = \gamma, \qquad (5.4.18)$$

and that $v_N = \ell_0 / \tau_0$

$$u = \mu\, v_N = \gamma\, v_N. \qquad (5.4.19)$$

The solution of the diffusion equation with constant drift (5.4.15´) is, as we have previously seen (5.2.68)

$$n(x,t) = \frac{A\prime}{\sqrt{4\pi D t}}\, e^{-\frac{(x-ut)^2}{4Dt}}, \qquad (5.4.20)$$

which means that, as expected, the average motion of the acron is constant and the average velocity is given in this case by the drift (5.4.19), that is, the natural velocity reduced by the γ term responsible for the initial velocity.

5.5. VELOCITY OF SATURATION

If the initial velocity is less than the saturation velocity in the medium then it will keep this initial velocity. This is somehow similar to the so called inertial law. Still we must bear in mind that this is only a rough description of what really happens. A more developed explanation needs to take in

account the eventual progressive decreasing of the average velocity due to the chaotic interaction with the subquantum field. If it happens that the initial velocity is greater than the velocity of saturation, it quickly reduces to the saturation velocity of the medium.

Each medium together with the specific acron behaves, as a result from theirs mutual reciprocal interaction, in such a way that the average velocity of the acron tends to reach a maximal possible value. No mater the preparation we may make onto the acron this average velocity can not increase.

When there is an initial preparation onto the acron that is, a kind of initial velocity is impinged onto the acron, the probability conservation equation (5.4.1) assumes the form

$$\begin{aligned} &p_{back} + (p_{forward} + \gamma) + p_{right} + p_{left} + p_{up} + p_{down} = 1 \\ &(p_{back} + p_{forward} + p_{right} + p_{left} + p_{up} + p_{down}) + \gamma = 1 \end{aligned} \qquad (5.5.1)$$

which, in a constant intensity field, gives

$$6p + \gamma = 1,$$

or

$$\gamma = 1 - 6p, \qquad (5.5.2)$$

where the γ term stands for this preparation action.

The plot, Fig.5.12, shows that the transition probability, in a constant intensity theta wave field, may go from zero to the maximal value 1/6, $0 \le p \le 1/6$, while the term responsible for the initial preparation, the initial velocity, being inversely proportional, goes from zero to one, $1 \ge \gamma \ge 0$.

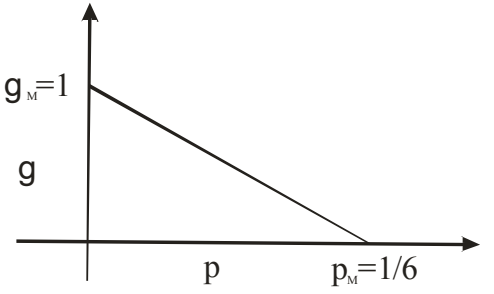

Fig.5.12 – Plot relating the initial velocity and the stochastic probabilities

When $\gamma = 0$ it means that no preparation was made onto the acron, that is the initial velocity is zero, therefore, $p = p_M = 1/6$, meaning that, in this case, only the stochastic field acts. In this situation the stochastic subquantum theta wave field has the maximal action on the corpuscle, so in average it does not move, $\bar{v} = 0$. In the extreme case when $\gamma = \gamma_M = 1$, it implies that $p = 0$, meaning that the medium is paved in such a way that no chaotic action, no obstacles, are put in the motion of the acron therefore is velocity approaches its natural velocity, $\bar{v} \to v_N$.

Still, in normal conditions, every subquantum theta wave medium has a chaotic nonlinear interaction with the acron leading it to reduce its natural velocity. This natural fact sets a boundary for the minimum possible value for the transition probability, $p_m \le p \le p_M = 1/6$, for any acron in the field. Naturally, there is an inverse reciprocal relation for the value of γ, that is, $\gamma_M \le \gamma \le 0$.

Since, in this case, we have $p' - q' = (p + \gamma) - p = \gamma = \mu = const$ it is possible to write $\bar{v} \propto \mu v_N$ since the reduced drift is constant then the acceleration its gradient is null. In this particular case of a constant reduced drift the average velocity is proportional to the reduced drift. This implies that for each medium there is a maximal possible velocity, depending on the medium and on the nonlinear interaction with the corpuscle, as shown in Fig.5.13

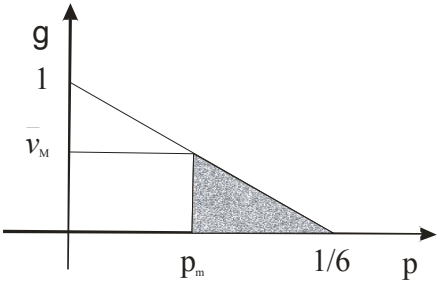

Fig.5.13 – Commonly accessible space for the possible average velocities in function of the chaotic interaction of the field characterized by the value of the transition probabilities.

So, for an acron immersed in a homogeneous constant subquantum theta wave field, due to the stochastic interaction, there is always a maximal possible average velocity $\bar{v}_M$. In the particular case of the photon, in the so

called vacuum, this velocity is (*c*). In normal circumstances when the initial velocity of the acron is less than the limit velocity, $0 \le \overline{v}_0 \le \overline{v}_M$, this velocity, $\overline{v} = \overline{v}_0$, is maintained in the homogenous medium.

If the acron entering the medium has a velocity greater than the maximal possible average velocity, $\overline{v}_0 > \overline{v}_M$, due to the chaotic reciprocal interaction corpuscle-field this velocity rapidly decreases till it reaches the maximum possible value, $\overline{v}_0 \to \overline{v}_M$.

Only in very special circumstances, like for instance in tunnelling conditions, this chaotic reciprocal interaction decreases till in the limit it can eventually reach the natural velocity $\overline{v}_0 \to \overline{v}_M = \mu_M v_N$. In this situation the forbidden region becomes accessible. Fig.5.14.

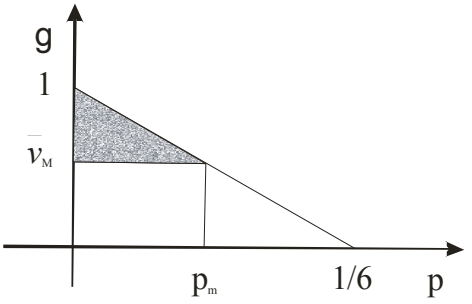

Fig.5.14 – Accessible space for the possible average velocities and transition probabilities for very special circumstances like tunnelling.

This constant term is a composition of two factors: One resulting from the initial velocity, the other, a subtracting term resulting from the chaotic nature of the medium representing the resistance opposed by it to the motion of the acron. It is because of this chaotic interacting process that there exists saturation leading to a maximal possible velocity allowed by each medium.

Consider now the case in which the acron moves in a certain theta wave field where it has an average maximal possible velocity and next enters a different theta wave field, for instance, the photonic acron coming from the air and entering an optical fiber or vice-versa.

In the first case the acron comes from a medium where its average maximal allowed velocity is greater, and enters a medium with a lesser maximal velocity, $\overline{v_1}=v_{1M} > \overline{v_2}=v_{2M}$. In such conditions the acron upon entering the second medium with a velocity larger than the maximal allowed in the medium starts, due to the increasing in the chaotic interactions, to

decrease the average velocity till it attains the maximal average velocity allowed in the medium as shown in Fig.5.15.

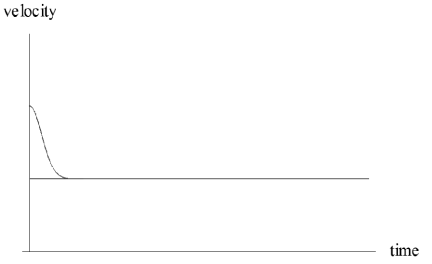

Fig.5.15 – The initial greater velocity soon attains
the maximal possible velocity

In the second case, the acron with an average velocity equal to the maximal possible allowed in a previous medium v_{1M} enters the other medium where the maximal possible velocity is greater. So, when the entering velocity is less than the maximal allowed by the chaotic nature of the second medium it happens that the subtracting probabilistic term starts decreasing so that the average velocity of the acron approaches the higher saturation velocity. Still this case offers no problem because even if the initial average velocity of the entering acron is less than the maximum allowed in the second medium $\overline{v_1} = v_{1M} < v_{2M}$ nonetheless its instantaneous velocity, its natural velocity, is much greater than the one allowed in such medium $v_N \gg v_{2M}$. So, after a short time again the average velocity approaches the maximum possible velocity allowed in the medium where now the acron moves, as shown in the picture, Fig.5.16.

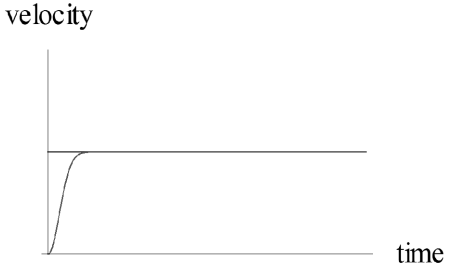

Fig.5.16 – The initial lesser velocity soon attain
the maximal possible velocity.

Everything seems to happen as if out of nothing the velocity of the acron starts increasing in the second medium. In reality due to the less

chaotic nature of the interactions in the second medium the average velocity of the acron is allowed to increase.

Now it is convenient to summarize some of the conclusions previously reached:

C1 – The average motion of the acron in the subquantum theta wave field depends on the principle of eurhythmy and on the previous preparation of the acron.

C2 – In a non-constant theta wave field such that $\mu \neq \text{const}$ the average velocity of the acron is not, in general, constant, $\overline{v} \neq \text{const}$.

C3 – In a homogeneous constant subquantum theta wave field such that $\mu = 0$ the average velocity of the acron is constant, $\overline{v} = \text{const}$.

C3.1 – The value of the constant average velocity depends on the previous preparation of the acron.

C3.2 – When no previous preparation is made on the acron, the motion of the acron is only the one due to the direct application of the principle of eurhythmy. Since the intensity of the subquantum theta wave field is equal in every direction it follows that also the probabilities are equal. So, in these conditions, the average velocity of the acron is always zero $\overline{v} = 0$.

C3.3 – Due to some previous preparation the acron entering the homogenous medium has an initial preferred motion, also commonly called initial velocity. In this case the average velocity of the acron in the constant theta wave field is in general different from zero, $\overline{v} \neq 0$.

C4 – When the acron is immersed in an increasing intensity subquantum theta wave field, due to the principle of eurhythmy, the average velocity may, in certain cases, increase. This increasing in the average velocity can not go forever. It has a limit given precisely by the natural velocity of the acron $\overline{v} \to v_N$. This value is attained when the probability of going in direction of motion approaches one. This means, that in these very special conditions, the path is paved in such a way that there are no obstacles for motion of the acron.

C4.1 – If after acquiring the average velocity $\overline{v}_0$ the acron enters a homogeneous medium, this velocity will be, in principle, its initial velocity in this second medium.

C5 – Each constant intensity, subquantum theta wave field, $\nabla|\theta|^2=0$ is characterized by a maximum possible average velocity $\overline{v}_M$. This velocity for each medium depends on the reciprocal chaotic interaction between the medium and the specific acron.

C5.1 – If the initial velocity of the acron, coming from the first medium, entering the second homogeneous medium is greater than the maximum possible velocity, $\overline{v}_0=\overline{v}>\overline{v}_M$, then the initial velocity, is quickly reduced till it equalizes it, $\overline{v}_0\rightarrow\overline{v}\cong\overline{v}_M$.

C5.2 – Even in mediums where $\nabla|\theta|^2\neq 0$ there is a maximal possible velocity, depending on the medium, such that $\overline{v}\rightarrow\overline{v}_M$, the average velocity may increase till the maximal possible velocity. Only in very special circumstances, see P4, this velocity can reach its natural velocity.

5.6. VELOCITY IN DIFFERENT THETA WAVE FIELDS

Now, in order to further progress we bear in mind that we need to consider two kinds of motions, or two velocities:

One is the velocity of the theta wave, v_θ
The other, the velocity of the acron v_ξ.

The velocity of the theta wave v_θ is relative to the organized local regions of the subquantum medium, while the velocity of the acron v_ξ is always relative to the theta wave field in which it is immersed.

When a complex particle, theta wave plus acron, enters another larger theta wave, as shown in Fig.5.17

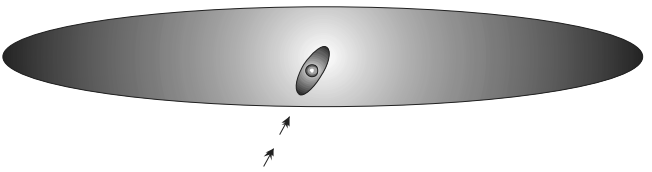

Fig.5.17 – A small theta wave with an acron enters a large theta wave

the acron will follow in the small theta wave or in the large theta wave field depending on the relative average strength of the respective fields:

a) The intensity of the small wave $|\theta_0|^2$ is relatively greater than the local intensity $|\theta_M|^2$ of the large theta wave,

$$|\theta_0|^2 / |\theta_L|^2 >> 1 . \tag{5.6.1}$$

In these conditions, the motion of the acron is primarily relative to its small theta wave. This happens because the theta wave field felt by the acron is its own

$$\theta_T \approx \theta_0 .$$

b) The intensity is smaller

$$|\theta_0|^2 / |\theta_L|^2 << 1 , \tag{5.6.2}$$

then everything happens as if the field seen by the acron is the one of the large theta wave. So, in these conditions, the theta wave field seen by the acron is basically the one of the large theta wave

$$\theta_T \approx \theta_L .$$

c) In between these two extreme cases the acron follows the conjugated action of the two theta wave fields

$$\theta_T = \theta_T(\theta, \theta_0). \tag{5.6.3}$$

When the condition $|\theta_0|^2 / |\theta_L|^2 >> 1$ is fulfilled, that is, the relative intensity of the wave with the acron is larger than that of the larger theta wave, the complex particle maintains its individuality. In such conditions the average motion of the acron is the one relative to the its theta wave θ_0.

In the opposite case, when $|\theta_0|^2 / |\theta_L|^2 << 1$, the initial particle loses its individuality and the velocity of the acron shall be, the average velocity relative to the large theta wave θ_L. Everything happens as if the field seen by the acron is the one of this theta wave, except for the initial tendency expressed in the form of the probability of transition, the so called initial velocity.

The motion of large bodies, which in last instance are a relatively stable composition of many acrons, eventually of different kinds, generating a surrounding theta wave field of lesser or greater strength, follows along the same principles, that is, depends on the relative strengths of the relative theta wave fields. In general, the macroscopic physical systems move with a velocity near its natural velocity. This natural velocity is, in last instance, a function of the average velocity of the ensemble of respective constituent acrons.

Naturally, we must be aware that the above description constitutes a first approach for representing the complex physical reality. A more complete model needs to take in account the details of the overlapping theta waves. These waves are oscillating so, not only their relative phase coherence must be considered but also, firstly the relative value of their respective wavelengths. For instance, if the incident theta wave is very small and has a very small wavelength relative to the extended theta wave field then in general the field seen by the acron is the one of its own theta wave.

5.7. TWO DIMENSIONAL MOTION IN A CONSTANT INTENSITY FIELD

The solution of this problem follows along the same lines of the previously studied cases. Only now we need to deal with two spatial dimensions. So, in order to study this case it is useful to make a sketchy representation of the space in order to write the explicit form for the probabilities of transition. In two dimensions we have Fig.5.18

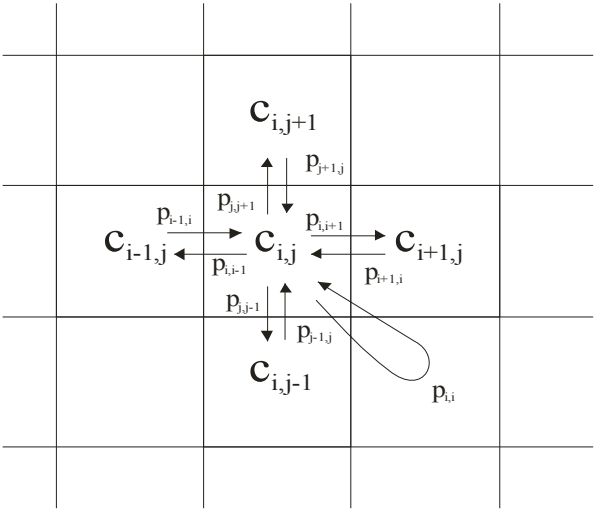

Fig.5.18 – Generic cell in a two-dimensional field with the transition probabilities

where $p_{i\to i+1,j}$ corresponds to the probability of transition from cell $C_{i,j}$ to cell $C_{i+1,j}$ and $p_{i\to i,j}$ is the probability of remaining in the same cell $C_{i,j}$. Since there is no danger of confusion, for notation simplicity reasons the index not corresponding to the transition is dropped. So, in the following we make $p_{i\to i+1,j} \equiv p_{i,i+1}$, $p_{i\to i,j} \equiv p_{i,i}$, and successively. For the other probabilities their meaning can be inferred from the sketch. Naturally, for the generic cell $C_{i,j}$ the sum of the exit probabilities plus the probability of remaining must be one, that is

$$p_{i,i-1} + p_{i,i+1} + p_{i,i} + p_{j,j-1} + p_{j,j+1} = 1\,. \tag{5.7.1}$$

Note, that for notation simplification, the index *j* was dropped in the horizontal representation and *i* in the vertical. To explicitate the analytic form of the probabilities of transition it is convenient to consider the following cases:

1 – The acron is in an initial condition such that it is without any preferential direction for the motion. In this case the form of the eurhythmic probabilities stems directly from the principle of eurhythmy, expressed by condition C4, giving

$$p_{i,i-1} = \frac{1}{1-\gamma}\frac{I_{i-1,j}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} \tag{5.7.2-1}$$

$$p_{i,i} = \frac{1}{1-\gamma}\frac{I_{i,j}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} \tag{5.7.2-2}$$

$$p_{i,i+1} = \frac{1}{1-\gamma}\frac{I_{i+1,j}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} \tag{5.7.2-3}$$

$$p_{j,j-1} = \frac{1}{1-\gamma}\frac{I_{i,j-1}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} \tag{5.7.2-4}$$

$$p_{j,j+1} = \frac{1}{1-\gamma}\frac{I_{i,j+1}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} \tag{5.7.2-5}$$

where γ is the generic term translating the initial velocity and $I_{i,j}$ is the intensity of the theta wave in the cell $C_{i,j}$ and respectively. In the case of a previous preparation, initial velocity, for the acron these expression need to be multiplied by the constant normalizing factor, just as was done in (5.4.5).

2 – The complex acron due to some initial preliminary process in the surrounding theta wave field behaves as if it has a preferred tendency for motion with a kind of initial velocity.

In this case the expression for the transition probabilities needs to have, as we have seen previously, two terms:

a) One due only to the principle of eurhythmy depending simply on the relative intensity of the organized theta wave field.

b) The other resulting from the direct modification of the field in the neighborhood of the acron, which usually is translated by the so called initial velocity. As usual, in this approach the absolute value of the initial velocity is assumed constant during all the process.

In such conditions for the generic form of the probabilities we got

$$p = \widetilde{p} + p_0, \quad p_0 = \text{constant} \tag{5.7.3}$$

where $\widetilde{p}$ is the direct contribution of the principle of eurhythmy and p_0 resulting from what is usually translated by the initial velocity. In this situation the equation of conservation of probability (5.7.1) assumes the form

$$\widetilde{p}_{i,i-1} + q_{oh} + \widetilde{p}_{i,i+1} + p_{oh} + \widetilde{p}_{i,i} + \widetilde{p}_{j,j-1} + q_{ov} + \widetilde{p}_{j,j+1} + p_{ov} = 1, \tag{5.7.4}$$

with q_{oh} corresponding to the term for the initial motion backwards, p_{oh} to the motion forwards, for the vertical initial component, q_{ov} downwards and p_{ov} upwards. Naturally

$$q_{oh} + p_{oh} + q_{ov} + p_{ov} = \text{const} = \gamma \tag{5.7.5}$$

with each one also constant.

The master stochastic equation describing the number of the corpuscles in the generic cell assumes the form

$$\begin{aligned} n_{i,j,t+1} = p_{i-1,i}\, n_{i-1,j,t} + p_{i+1,i}\, n_{i+1,j,t} + p_{i,i}\, n_{i,j,t} + \\ + p_{j-1,j}\, n_{i,j-1,t} + p_{j+1,j}\, n_{i,j+1,t} \end{aligned}, \tag{5.7.6}$$

with $n_{i,j,t}$ representing the number of corpuscles in the cell $C_{i,j}$ at time t.

According to the concrete physical situation the transition probabilities assume a fixed form allowing consequently to precise the structure of the master stochastic equation.

5.7.1. SNELL'S REFRACTION LAW AND FERMAT'S PRINCIPLE OF MINIMUM TIME

In the case of Snell's refraction law and Fermat's principle of minimum time we have two optical different mediums. In such a case we need to study the behaviour of the master stochastic equation in a homogeneous medium, that is, a medium where the average intensity of the theta wave field is approximately constant.

The eurhythmic probabilities of transition in a homogeneous medium, where $I_{i,j}$= constant, are, of course, also constant, as can be seen

$$\widetilde{p}_{i,i-1} = \frac{1}{1-\gamma}\frac{I_{i-1,j}}{I_{i-1,j} + I_{i+1,j} + I_{i,j} + I_{i,j-1} + I_{i,j+1}} = p = \text{constant} \cdot \qquad (5.7.7)$$

Assuming that the components due to the initial velocity are such that

$$q_{oh} = q_{ov} = 0\,, \qquad (5.7.8)$$

In such conditions the probabilities of transition can be written

$$p_{i-1,i} = p_h\,, \quad p_{i+1,i} = q_{h,}\ \ p_{i,i} = \delta\,, \quad p_{j-1,j} = p_v\,, \quad p_{j+1,j} = q_v\,, \qquad (5.7.9)$$

With

$$p_h = p + p_{oh}\,, \quad q_h = p, \quad \delta = p, \quad p_v = p + p_{ov}\,, \quad q_v = p\,, \qquad (5.7.10)$$

which by substitution in the stochastic master equation (5.7.5) give

$$n_{i,j,t+1} = p_h\, n_{i-1,j,t} + q_h\, n_{i+1,j,t} + \delta\, n_{i,j,t} + p_v\, n_{i,j-1,t} + q_v\, n_{i,j+1,t}\,. \qquad (5.7.11)$$

This discrete equation can, as habitual, be approached by a continuous equation of the form

$$\begin{aligned} &n(x,y,t+\tau_0) = \\ &\quad p_h\, n(x-\ell_0,y,t) + q_h\, n(x+\ell_0,y,t) + \delta\, n(x,y,t) + \\ &\qquad + p_v\, n(x,y-\ell_0,t) + q_v\, n(x,y+\ell_0,t) \end{aligned} \qquad (5.7.12)$$

that can be expanded in Taylor series giving

$$
\begin{aligned}
& n(x,y,t)+\tau_0\frac{\partial n(x,y,t)}{\partial t}+\frac{1}{2!}\tau_0^2\frac{\partial^2 n(x,y,t)}{\partial t^2}+\ldots \\
& = p_h n(x,y,t)-p_h\ell_0\frac{\partial n(x,y,t)}{\partial x}+\frac{1}{2!}p_h\ell_0^2\frac{\partial^2 n(x,y,t)}{\partial x^2}+ \\
& -\frac{1}{3!}p_h\ell_0^3\frac{\partial^3 n(x,y,t)}{\partial x^3}+\ldots+q_h n(x,y,t)+q_h\ell_0\frac{\partial n(x,y,t)}{\partial x}+ \\
& +\frac{1}{2!}q_h\ell_0^2\frac{\partial^2 n(x,y,t)}{\partial x^2}+\frac{1}{3!}q_h\ell_0^3\frac{\partial^3 n(x,y,t)}{\partial x^3}+\ldots+\delta n(x,y,t)+ \\
& +p_v n(x,y,t)-p_v\ell_0\frac{\partial n(x,y,t)}{\partial y}+\frac{1}{2!}p_v\ell_0^2\frac{\partial^2 n(x,y,t)}{\partial y^2}+ \\
& -\frac{1}{3!}p_v\ell_0^3\frac{\partial^3 n(x,y,t)}{\partial y^3}+\ldots+q_v n(x,y,t)+q_v\ell_0\frac{\partial n(x,y,t)}{\partial y}+ \\
& +\frac{1}{2!}q_v\ell_0^2\frac{\partial^2 n(x,y,t)}{\partial y^2}+\frac{1}{3!}q_v\ell_0^3\frac{\partial^3 n(x,y,t)}{\partial y^3}+\ldots
\end{aligned}
\tag{5.7.13}
$$

since the linear size ℓ_0 of the cell is very small and furthermore the velocity of transition is very high it implies that

$$\ell_0 >>> t_0 \equiv \tau_0, \tag{5.7.14}$$

again, it is reasonable to cut the expression (5.7.13) from τ_0^2 and ℓ_0^3 on, giving

$$
\begin{aligned}
n+\tau_0\frac{\partial n}{\partial t} &= (p_h+q_h+\delta+p_v+q_v)\,n \\
&-\ell_0(p_h-q_h)\frac{\partial n}{\partial x}+\frac{1}{2}\ell_0^2(p_h+q_h)\frac{\partial^2 n}{\partial x^2} \\
&-\ell_0(p_v-q_v)\frac{\partial n}{\partial y}+\frac{1}{2}\ell_0^2(p_v+q_v)\frac{\partial^2 n}{\partial y^2}
\end{aligned}
\tag{5.7.15}
$$

recalling that

$$p_h+q_h+\delta+p_v+q_v=1,$$

and making

$$\mu_h = p_h - q_h = p_{oh}, \quad \mu_v = p_v - q_v = p_{ov}, \tag{5.7.16}$$

$$d_h = p_h + q_h = 2p + p_{oh}, \quad d_v = p_v + q_v = 2p + p_{ov}$$

which are the reduced drift μ components and the diffusion coefficients *d*, along the horizontal and vertical axes. In this situation equation (5.7.15) can be written in the condensed form

$$d_h \frac{\partial^2 n}{\partial x^2} + d_v \frac{\partial^2 n}{\partial y^2} - \frac{2\mu_h}{\ell_0}\frac{\partial n}{\partial x} - \frac{2\mu_v}{\ell_0}\frac{\partial n}{\partial y} = \frac{2}{\ell_0 v_N}\frac{\partial n}{\partial t} \tag{5.7.17}$$

or,

$$D_h \frac{\partial^2 n}{\partial x^2} + D_v \frac{\partial^2 n}{\partial y^2} - u_h \frac{\partial n}{\partial x} - u_v \frac{\partial n}{\partial y} = \frac{\partial n}{\partial t} \tag{5.7.18}$$

With

$$D_h = \frac{1}{2} d_h \ell_0 v_N, \;\; D_v = \frac{1}{2} d_v \ell_0 v_N, \;\; u_h = \mu_h v_N, \;\; u_v = \mu_v v_N. \tag{5.7.19}$$

The differential equation (5.7.18) is the traditional diffusion equation with constant drift in two spatial dimensions.

A solution to this equation is

$$n(x,y,t) = \frac{A}{\sqrt{4\pi\sqrt{t}}} e^{-\frac{(x-u_h t)^2}{4D_h t} - \frac{(y-u_v t)^2}{4D_v t}}, \tag{5.7.20}$$

whose plot, for some particular values of the constants is shown in Fig.5.19

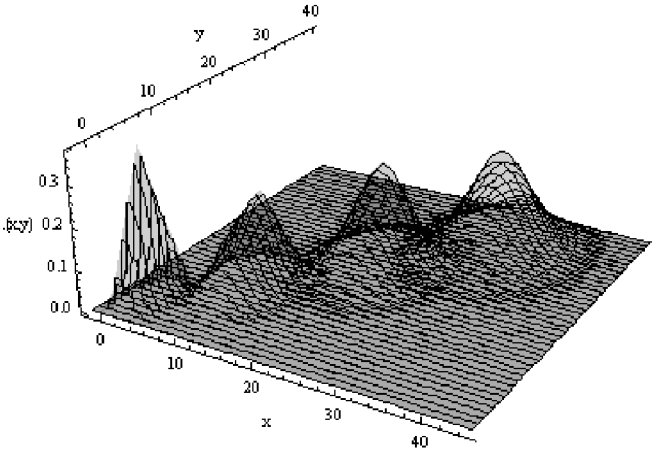

Fig.5.19 – Plot for different instants
of time for the probability distribution

From the equation (5.7.20) and the respective plot we gather that the drift indeed stands for the average motion of the acron. The drift components u_h and u_v are in practice the components of the velocity along the *xx* and *yy* axes.

$$\begin{cases} v_x = u_h = \mu_h v_N = (p_h - q_h)v_N \\ v_y = u_v = \mu_v v_N = (p_v - q_v)v_N \end{cases} \tag{5.7.21}$$

Needless to say that the drift $\mu \geq 0$ must be very small otherwise there would be a practically complete deterministic motion.

Previously we have seen that, under reasonable assumptions the average velocity may be assumed to be proportional to the inverse of the wavelength $v \propto 1/\lambda$, therefore since $k = 2\pi/\lambda$ one is allowed to write for the $\vec{k}$ wave vector

$$\vec{v} \propto \vec{\mu} \propto \vec{k} \tag{5.7.22}$$

Now that we have the necessary elements derived from the eurhythmic stochastic motion for the acron it is relatively easy to derive Snell's refraction law for the propagation of light that is, the luminous acrons, in two homogeneous mediums. The principle of eurhythmy tells us that the acron, in this case the photonic acron, follows in average the best possible path. From these considerations we derived expression (5.7.22) relating the average velocity, of the acron, with the drift and the wave vector

from which it is possible to derive again Snell's law as a direct consequence of the principle of eurhythmy. Therefore, stating that the photonic acron follows in average the best possible path, the principle of eurhythmy, is in a way similar to stating, for this particular case, Fermat's principle of minimum time, since it leads to the same result.

In such circumstances we can make a sketchy representation for the average motion of the acron in two homogeneous mediums

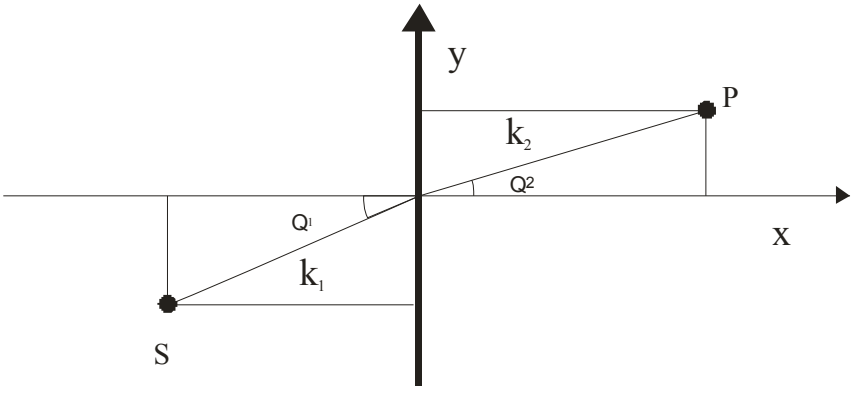

Fig.5.19 – Average motion of the acrons
in two optical mediums

where the drift may be written in the form

$$\vec{\mu} = \mu_x \vec{e}_x + \mu_y \vec{e}_y \,. \tag{5.7.23}$$

From Fig.5.19 it is seen that

$$\begin{cases} \mu_{1y} = \mu_1 \sin\theta_1 \\ \mu_{2y} = \mu_2 \sin\theta_2 \end{cases} \tag{5.7.24}$$

because along the *yy* direction there is no change in the medium we are allowed to make

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \,. \tag{5.7.25}$$

Since by (5.7.21)

$$\mu_x = \alpha\, k_x; \quad \mu_y = \alpha\, k_y, \tag{5.7.26}$$

where α is a proportionality constant we also have

$$k_1 \sin\theta_1 = k_2 \sin\theta_2, \tag{5.7.27-1}$$

recalling that $k = 2\pi / \lambda$, we have by substitution

$$\frac{2\pi}{\lambda_1}\sin\theta_1 = \frac{2\pi}{\lambda_2}\sin\theta_2, \tag{5.7.27-2}$$

or

$$\frac{1}{\nu\,\lambda_1}\sin\theta_1 = \frac{1}{\nu\,\lambda_2}\sin\theta_2, \tag{5.7.27-3}$$

because it is assumed that the energy of the acron that is, the frequency of light in the two medium is the same, $\nu_1 = \nu_2 = \nu$, since the light does not change color. Therefore, since $\upsilon = \lambda\nu$ we can write

$$\frac{1}{\upsilon_1}\sin\theta_1 = \frac{1}{\upsilon_2}\sin\theta_2, \tag{5.7.27-4}$$

which multiplied by *c* gives finally Snell's formula

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{5.7.27-5}$$

with $n = c / \upsilon$. This formula means, as we have seen, according to Fermat, that light, that is the acrons of light, follow a path such that the time taken in the whole course from S to P is minimum, or in short that the photonic acrons behaved according to the principle of eurhythmy.

6. MOTION OF MANY ACRONS

6.1. INTERACTION PROCESSES

So far we have only studied the interaction of single, or statistically multiple and assumed independent acrons, with the theta wave. Naturally we are aware that the behaviour of the acrons and respective theta wave are to be described by a nonlinear processes. Yet when we have a large number of acrons and under certain circumstances they can be studied as independent entities. In this chapter we are going to study the general collective interaction among the acrons behaving collectively and the respective theta wave fields.

Classical physics is mainly concerned with the gravitic and electromagnetic interactions. The gravitic interaction is described in terms of the so-called universal gravitational attraction law stating that the gravitic force is proportional to the inverse of the square of the distance separating the two bodies. The electromagnetic interaction is a little more complex since it depends, not only on the static Coulomb's law, similar to the universal attraction law, but also on the motion of the electrons. Anyway, these so- called laws of Nature are linear and furthermore have already shown its limits of application, with the advent of quantum and relativistic physics. Now, we are going to see how it is possible to derive these classic interactions from the new nonlinear approach based on the organizing principle of eurhythmy. In this way we can see how the New General approach, the Hyperphysis, really promotes, in a natural comprehensible way, the unification of Physics. For that we shall develop the general interaction process among particles and theta wave fields out of which, as a mere particular case, the classic interaction laws can be derived.

6.2. OVERLAPPING OF TWO THETA WAVES

The overlapping of two theta waves, each one assumed to be described by a gaussian Morlet wavelet, and being a relatively stable organized perturbation of the chaotic subquantum medium, can be written in one dimensional space

$$\theta_1(x,t) = e^{-\frac{x^2}{2\sigma^2}+i(kx-\omega t+\delta_1)}, \quad \theta_2(x,t) = e^{-\frac{(x-\ell)^2}{2\sigma^2}+i(kx-\omega t+\delta_2)} \tag{6.2.1}$$

The nonlinear overlapping of these two theta waves can approximately be described in the following cases:

6.2.1. TWO THETA WAVES AND ONE SINGLE ACRON

This case corresponds to the typical quantum interference of one single particle. In general, in such cases, the initial theta wave, carrying a single acron, by means of a beamsplitter, or any other similar device, is split in two secondary coherent theta waves that later are led to overlap. Experiments have shown that the composition rule for the overlapping of these theta waves can be assumed to be approximately the linear one, namely in the far field observation,

$$\theta = \theta(\theta_1, \theta_2) \approx \theta_1 + \theta_2 = \\ = A_1 e^{-\frac{x^2}{2\sigma^2} + i(k\,x - \omega t + \delta_1)} + A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2} + i(k\,x - \omega t + \delta_2)} \tag{6.2.2}$$

That is, the final theta wave can approximately be described, under the most common conditions, by the simple sum of the two theta waves. The acron is then localized inside this global theta wave field according to the principle of eurhythmy. That is, the acron is then to be localized preferentially in the regions where the intensity of the total theta wave field has greater intensity

$$I = |\,\theta\,|^2 = |\,\theta_1 + \theta_2\,|^2 = \\ = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} + \\ + 2A_1 A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-\ell)^2}{2\sigma^2}} \cos(\delta_1 - \delta_2) \tag{6.2.3}$$

For more than two theta waves the additive linear rule holds approximately as long as one is dealing with one single acron. If the theta waves are incoherent between themselves the averaging of the interference term goes to zero giving

$$I = |\,\theta\,|^2 = |\,\theta_1 + \theta_2\,|^2 = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} \tag{6.2.4}$$

The same rule is approximately true when we are dealing with a very large number of particles, that is, many theta waves and many acrons. In such conditions we have

$$\theta = \sum_{j=1}^{N} \theta_j, \quad \theta_j = A_j e^{-\frac{(x-\ell_j)^2}{2\sigma^2}+i(kx-\omega t+\delta_j)}$$

$$I = |\theta|^2 = |\theta_1|^2 + |\theta_2|^2 + \cdots + |\theta_N|^2 + \\ + 2|\theta_1||\theta_2|\cos\delta_{12} + 2|\theta_1||\theta_3|\cos\delta_{13} + \cdots \tag{6.2.5}$$

thus, the sum of the interference terms can be assumed as if it averages to zero and consequently we have simply

$$I = |\theta_1|^2 + |\theta_2|^2 + \cdots + |\theta_N|^2 . \tag{6.2.6}$$

Which amounts to say that the relative phase shift among the different theta waves varies randomly.

6.2.2. TWO THETA WAVES, EACH ONE WITH ITS OWN ACRON

In this case the additive linear rule does not hold since we are dealing with an essentially nonlinear process.

Each one of the two theta waves

$$\theta_1(x,t) = A_1 e^{-\frac{x^2}{2\sigma^2}+i(kx-\omega t+\delta_1)}, \quad \theta_2(x,t) = A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2}+i(kx-\omega t+\delta_2)}$$

when isolated that is when $\ell >> \sigma$ they do not interact with each other. On the other hand their relative phase difference, in line of principle, can be any. So, as the distance between the two waves decreases they start overlapping, and consequently interacting. If the linear additive rule was to be applied we would have a total intensity profile, resulting from the sum of the two overlapping waves, changing according to the value of the relative phase shift difference. In some cases it would increase leading to attraction, other times decreasing conducing to repulsion for the same type of particles. This conclusion resulting from the linear assumption is, as we are well ware of, against the observed behaviour of the particles that always, either attract or repel each other. So the linear approach, as we have said, can not, in general, be applied to describe this interacting process.

The interaction between two particles is essentially a nonlinear process. When the two theta waves approach, each one carrying its own acron, they start interacting in such a way, that their phases tend to equalize, $\delta_{12} \to 0$, in a process very much similar to the self-organizing interaction among oscillating systems[25]. In this case, after the equalizing process has taken place, the two theta waves can approximately be described by

$$\theta_1(x,t) = A_1 e^{-\frac{x^2}{2\sigma^2}+i(kx-\omega t+\delta)}, \quad \theta_2(x,t) = A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2}+i(kx-\omega t+\delta)}. \quad (6.2.7)$$

Once this kind of self-organizing interaction process has stabilized the linear additive rule can then be applied to the overlapping of the waves.

6.2.3. ATTRACTION

In this case we simply have

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} + 2A_1A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-\ell)^2}{2\sigma^2}} \cos\delta_{12},$$

with

$$\delta_{12} = \delta_1 - \delta_2 = 0,$$

and consequently

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} + 2A_1A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-\ell)^2}{2\sigma^2}},$$

or

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = \left[A_1 e^{-\frac{x^2}{2\sigma^2}} + A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2}} \right]^2 = (|\theta_1| + |\theta_2|)^2, \quad (6.2.8')$$

whose plot is seen in Fig.6.1

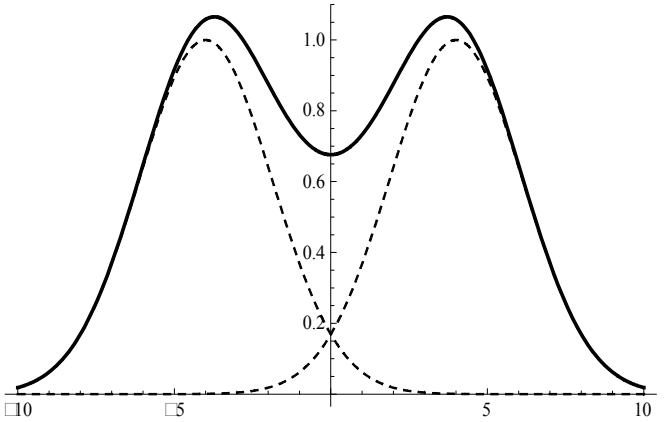

Fig.6.1 – Attraction. Resulting intensity plot of the two overlapping theta waves, solid line. Dashed line the initial theta waves.

The attraction rule describing approximately the behaviour of bosonic particles can then be written

$$I = |\theta|^2 = (|\theta_1| + |\theta_2|)^2 . \tag{6.2.8}$$

Therefore, since the intensity in the overlapping region increases the acrons, according to the principle of eurhythmy, tend to approach, that is, behave as if they attract each other.

This statement is no more than the enunciation of the coalescence theorem. The analytic proof of the coalescence theorem, even if possible, seems indeed very difficult nevertheless, a numeric "demonstration" of it has already been done by M. Gatta[24].

6.2.4. REPULSION

In this case, for repulsive particles, the nonlinear interaction process is such that the theta waves, which when well separated $\ell >> \sigma$ have, in line of principle, different phases, as they approach not only the phases tend to equalize but also a constant difference phase factor of π need to be established

$$\theta_1(x,t) = \theta_1 e^{-\frac{(x-\ell)^2}{2\sigma^2}+i(kx-\omega t+\delta)}, \quad \theta_2(x,t) = A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2}+i(kx-\omega t+\delta+\pi)} . \tag{6.2.9}$$

After this nonlinear stabilization of the waves the additive rule can be applied giving for the intensity

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = \\ = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} + 2A_1 A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-\ell)^2}{2\sigma^2}} \cos\pi \quad , \tag{6.2.10}$$

that is

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell)^2}{\sigma^2}} - 2A_1 A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-\ell)^2}{2\sigma^2}}$$

or,

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 = \\ = \left[A_1 e^{-\frac{x^2}{2\sigma^2}} - A_2 e^{-\frac{(x-\ell)^2}{2\sigma^2}} \right]^2 = (|\theta_1| - |\theta_2|)^2 \,, \tag{6.2.11'}$$

whose plot can be seen in Fig.6.2

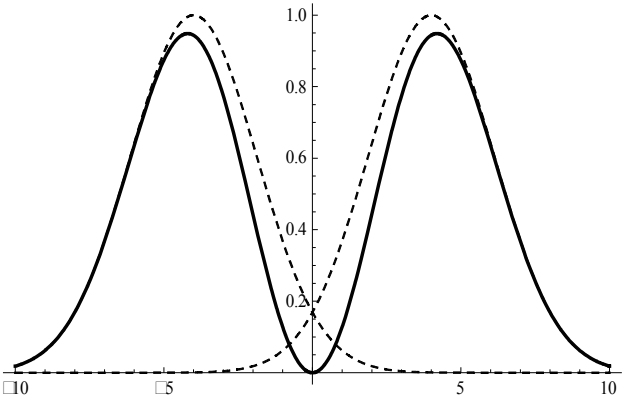

Fig.6.2 – Repulsion. Intensity plot of the two overlapping theta waves, solid line.

Therefore, the rule describing approximately the repulsive behaviour of fermionic particles can be written

$$I = |\theta|^2 = (|\theta_1| - |\theta_2|)^2 . \tag{6.2.11}$$

In this case of repulsion, the overlapping of the two theta waves is such that their intensity in the mixing region decreases. So, according to the principle of eurhythmy the two acrons tend to go away from each other, or in the old common language repel each other.

The last affirmation is the enunciation of the anti-coalescence theorem. Also in this case the analytic demonstration seems far away, yet a numeric "demonstration" has also been done by M. Gatta[24].

So far we have seen the individual basic interaction processes, usually called attraction and repulsion. Now, we can proceed with the study of the collective interaction processes.

6.3. INTERACTION LAW IN $1/R^2$ AND THE CONCEPT OF MASS

In a previous section it was studied the behaviour of a single or multiple independent acrons in a region where theta wave field intensity decreases with

$$I(r) = Ae^{-a\sqrt{r}}$$

which has an operational behaviour described by the concept of force field

$$I(r) = Ae^{-a\sqrt{r}} \Leftrightarrow F(r) \propto \frac{1}{r^2}$$

For this case the eurhythmic propensity for an acron to move, can be described as if a kind of pushing force was acting according with the inverse square of the distance. In this way the so-called universal attraction law is nothing more than a consequence of the principle of eurhythmy describing the average tendency for the acron to move when immersed in such a field.

However in the definition of the force enters also the concept of mass. In our case, in hyperphysis, the mass is to be understood as a function of the number of gravitic acrons,

$$m = m(\xi). \tag{6.3.1}$$

Where $m(\xi)$ stands for function depending on the number of acrons of the physical system. For large number of gravitic acrons whose behaviour can be approximately taken as independent we may write

$$m \propto N_{\xi} \tag{6.3.2}$$

where N_{ξ} represents the number of acrons. This expression means that the mass of a gravitic body is proportional to the number of acrons immersed in the theta wave field.

Moroever, in the present work, we have assumed that a free complex large particle, a large extended perturbation of the subquantum theta wave field, is to be described by a composed theta wave field approximately gaussian. Nevertheless we need to be aware that we are dealing basically with a three dimensional entity. This means that the global gravitic theta wave, an organized three dimensional perturbation of the chaotic subquantum field, has some kind of radial symmetry. In such circumstances it is reasonable to assume that the intensity of this perturbation when sufficiently far $r \geq r_0$ from the central region of the theta wave field corresponding to the operational gravitic field in $1/r^2$ approaches the gaussian. Only in the central region of the theta wave $0 \leq r \leq r_0$ this approximation is not valid.

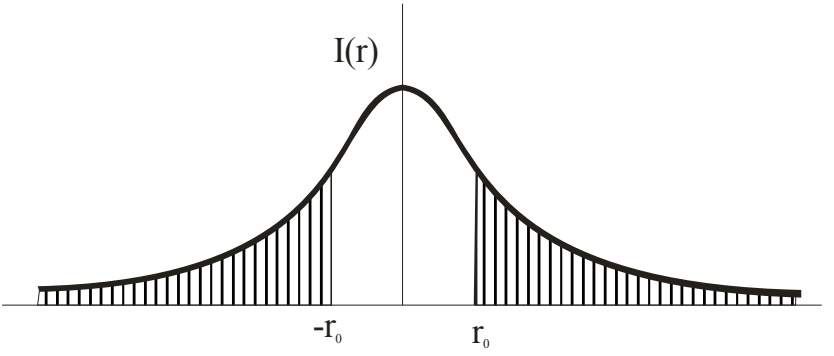

Fig.6.3 – From $r \geq r_0$ the intensity of the theta wave corresponding to the classical field in $1/r^2$ approaches the gaussian

6.4. PARTICLES MOVING IN THE SUBQUANTUM ORGANIZED FIELD

Till know we have discussed the cases where the particle was, in average, at rest relative to the subquantum organized field. Meaning that the composite theta wave does not appreciably move relative to the chaotic medium in which it constitutes a kind of steady organization. Naturally, the acron keeps moving incessantly in a chaotic motion but only inside the relatively still theta wave field. A more general approach needs to take into consideration the case where the theta wave moves relative to the subquantum medium. In this case, the motion of a coherent organized structure, the theta wave, in the chaotic medium also produces a sort of extended order in the subquantum medium. A kind of flow is produced, which can, in average, approximately be described by lines of current of subquantum fluid, as shown in Fig.6.4.

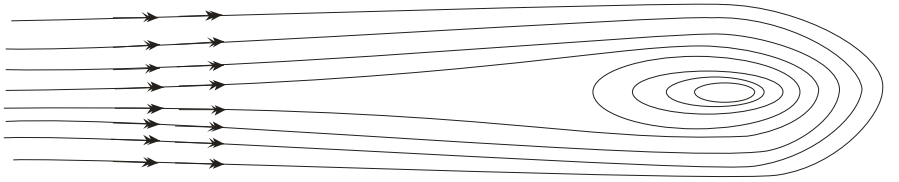

Fig.6.4 – Particle moving in the subquantum medium

Now if we consider the far perturbation in the subquantum field we have something like a jet of current with roughly a gaussian cross section. If it happens that two jets of current overlap, as shown in Fig. 6.5,

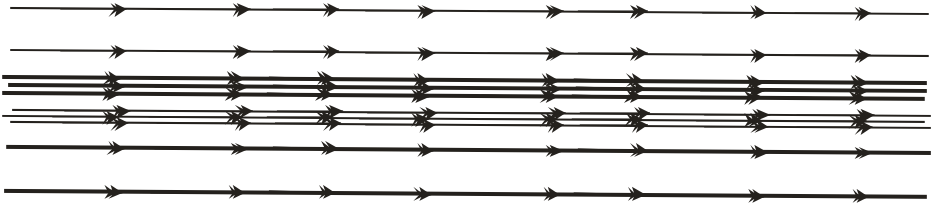

Fig.6.5 – Two jets of subquantum current
with the same direction overlap partially

their superposition gives rise to an increasing field intensity in the overlapping region. The resulting intensity cross section of the two gaussian jets of subquantum current can roughly be approached in one dimension by

$$\begin{aligned} I_T &= |\,\theta_T\,|^2 = |\,\theta_{T1} + \theta_{T2}\,|^2 = \\ &= |\,A_1 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}+i\alpha_1} + A_2 e^{-\frac{(x-\ell/2)^2}{2\sigma^2}+i\alpha_2}\,|^2 \\ &= A_1^2 e^{-\frac{(x+\ell/2)^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell/2)^2}{\sigma^2}} + \\ &\quad + 2A_1 A_2 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}} e^{-\frac{(x-\ell/2)^2}{2\sigma^2}} \cos\alpha_{12} \end{aligned} \tag{6.4.1}$$

where α_1 and α_2 are the angles of the lines of current relative to the vertical and $\alpha_{12} = \alpha_1 - \alpha_2$. In this case of currents with the same direction we have,

$$\alpha_1 = \alpha_2 = 0 \rightarrow \alpha_{12} = 0\,,$$

and consequently

$$\begin{aligned} I_T &= |\,\theta_T\,|^2 = |\,\theta_{T1} + \theta_{T2}\,|^2 = \\ &= A_1^2 e^{-\frac{(x+\ell/2)^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell/2)^2}{\sigma^2}} + 2A_1 A_2 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}} e^{-\frac{(x-\ell/2)^2}{2\sigma^2}}\,, \end{aligned}$$

or

$$\begin{aligned} I_T &= |\,\theta_T\,|^2 = |\,\theta_{T1} + \theta_{T2}\,|^2 = \\ &= \left[A_1 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}} + A_2 e^{-\frac{(x-\ell/2)^2}{2\sigma^2}} \right]^2 = (|\,\theta_1\,| + |\,\theta_2\,|)^2\,. \end{aligned} \tag{6.4.2}$$

The plot of the intensity of the cross section is shown in Fig.6.6.

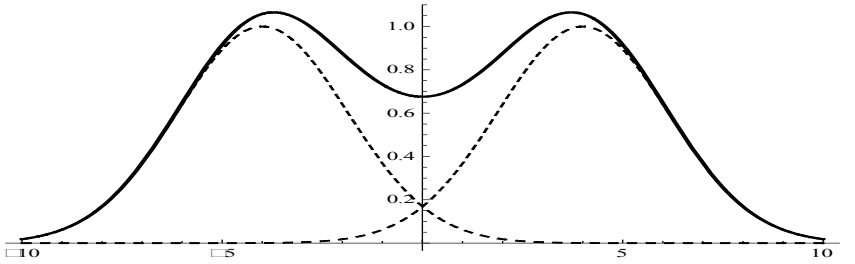

Fig.6.6 – Intensity cross section of the superposition
of two subquantum currents with the same direction

In this case, according to the principle of eurhythmy, the eventual acrons immersed in the jets tend naturally to approach or, in the common language to attract.

If the jets move in opposite directions, see Fig.6.7, their superposition is destructive.

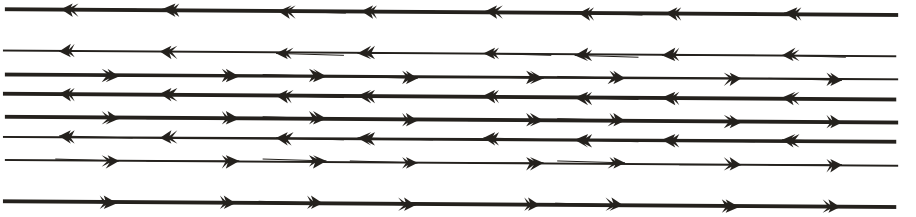

Fig.6.7 – Superposition of two subquantum currents
with opposite directions

In this case since $\alpha_{12} = \pi$ expression (6.5.1) became

$$I_T = |\theta_T|^2 = |\theta_{T1} + \theta_{T2}|^2 =$$
$$= A_1^2 e^{-\frac{(x+\ell/2)^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-\ell/2)^2}{\sigma^2}} - 2A_1 A_2 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}} e^{-\frac{(x-\ell/2)^2}{2\sigma^2}},$$

or

$$I_T = |\theta_T|^2 = |\theta_{T1} + \theta_{T2}|^2 =$$
$$= \left[A_1 e^{-\frac{(x+\ell/2)^2}{2\sigma^2}} - A_2 e^{-\frac{(x-\ell/2)^2}{2\sigma^2}} \right]^2 = (|\theta_1| - |\theta_2|)^2 . \qquad (6.4.3)$$

The cross section of the resultant overlapping of two subquantum jets moving in opposite directions are shown in Fig.6.8.

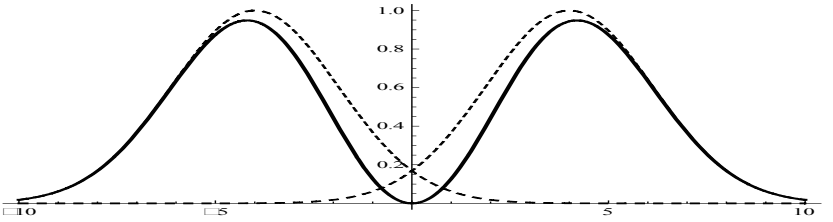

Fig.6.8 – Cross section of the superposition
of two subquantum currents with opposite directions

In this case, since there is negative overlapping, the resultant average intensity of the theta wave field decreases, instead of increasing. This process leads, following the principle of eurhythmy, to what is commonly called repulsion.

So this nonlinear interaction works like a kind of overall tendency, commonly called force, acting like pushing the corpuscles, the acrons. In this way, it is possible to write the average expression for the "attractive force" or "repulsive force", when the current jets are relatively far from each other. In such conditions, the intensity variation of the theta wave field can be approached to a variation in $1/r^2$

$$\vec{F} = k \frac{\vec{v}_1 \times \vec{v}_2}{r^2} , \qquad (6.4.4)$$

where r is distance between the jets of subquantum current and k is a proportionality constant and $\vec{v}_1$, $\vec{v}_2$ are the central velocity of the jets. These rules for the interaction of currents of subquantum in field are, in a way, very similar to the ones for steady theta waves.

7. FUNDAMENTAL NONLINEAR MASTER EQUATION

7.1. INTRODUCTION

In this chapter we shall try to find a basic differential equation that may describe, somewhat adequately, the behaviour of the theta wave and also the average motion of the acron, be it at the subquantum, quantum, gravitic, and other possible scales of observation and description of physical reality. This equation aims to describe the global average behaviour of the organized interacting regions of the subquantum medium both in time and space. In order to do that we start from the basic conservation equations, with the understanding that these are only approximate average relations between reciprocally interacting systems and not something more. They are devoid of any special ontological status. They describe only some aspects of the interacting processes that may, we hope, be said approximately constant under diverse scales of observation and description of reality.

Before deriving the master equation it is convenient first to derive, from the basic assumptions of Hyperphysis, the basic empirical relations of de Broglie and Planck, and only after that we shall arrive at the law of conservation of energy. In this way we not only show that the most basic concepts of the traditional physics like momentum and energy are not truly basic concepts, but mere convenient derivate representations of the frequency of the basic theta wave. The energy being a derived concept proportional to the temporal angular frequency, while the momentum is proportional to the spatial frequency.

7.2. DE BROGLIE AND PLANCK RELATIONS

We have already seen, section 5.3, that the average velocity of the acron, either single or complex, is proportional to the inverse of the wavelength, formula (5.3.9). On the other hand the mass of a particle, complex gravitic acron, depends on the number of the constituent gravitic acrons, that for a large number may be assumed as behaving independently.

$$m = c_m N_\xi, \tag{7.2.1}$$

where c_m is a proportionality constant and N_ξ is the number of acrons. In these conditions the classical momentum, the product of the mass by the velocity may be written

$$p = mv = c_m N_\xi \frac{1}{\lambda} \tag{7.2.2}$$

or, by relation (5.3.9)

$$p = \frac{h}{\lambda}, \tag{7.2.3}$$

which is de Broglie's well-known formula. From these formulas we gather that Planck´s constant can be written

$$h = c_m N_\xi. \tag{7.2.4}$$

Remembering that the spatial angular frequency is

$$k = \frac{2\pi}{\lambda} \tag{7.2.5}$$

de Broglie formula can also be expressed by

$$p = \hbar\, k, \tag{7.2.6}$$

In order to obtain Planck formula it is convenient to write the generic theta wave with a definite energy.

$$\theta = \theta_0 e^{-\frac{(x-v_g t)^2}{2\sigma^2} + i(kx-\omega t)} \tag{7.2.7}$$

$$\theta = \theta_0 e^{-\frac{(x-v_g t)^2}{2\sigma^2} + ik(x-v_f t)} \tag{7.2.7'}$$

where $v_g = v$, the so called group velocity is, in average equal to the average velocity of the acron is injected in the extended theta wave field. In common Fourier ontology it would be called group velocity. The other velocity term v_f is the phase velocity of the wave. In these conditions the phase of the theta wave can be written

$$\varphi = kx - kv_f t = kx - \frac{m}{\hbar} v_g v_f t = kx - \omega t,$$

now making

$$v_f = \frac{1}{2} v_g, \tag{7.2.8}$$

we have

$$\omega = \frac{1}{2}\frac{m}{\hbar}v_g^2, \tag{7.2.9}$$

or

$$\frac{1}{2}mv_g^2 = \hbar\,\omega, \tag{7.2.10}$$

that is

$$E = \hbar\,\omega, \tag{7.2.11}$$

Thus we have obtained the formula of Planck. Naturally we must be aware that we have not made a so called demonstration of Planck´s formula. What was done corresponds to a heuristic derivation showing that under certain conditions Planck´s formula is compatible with our assumptions. Break-grounding work conducing to great break-throughs in physics is like being on a razor's edge. Always trying not to fall, always trying to make things compatible with the empirical evidence in a global deeper picture. We are not playing a game with the rules defined by us just like in mathematics. In our case the Master of the game is Nature, whose rules we do not know. The best we can hope is to gather some more or less rather vague glimpses of the Truth. In such conditions we do not pretend to make rigorous demonstrations, we only hope to make reasonable derivations. In such conditions our formulas do not rule the phenomena, but rather describe them in an approximate way and even so only under certain conditions.

De Broglie and Planck formulas, in this more general approach to understand natural phenomena, are now generalized for any complex acron, and even for the case of absence of any acron, for different scales of description as we shall see. To do so it suffices to change the value of the proportionality constant. At the quantum level we have Planck, or quantum constant, at the subquantum scale, valid for the theta wave, the subquantum constant. In this case the velocity shown in the formula represents the global velocity of the small theta wave relative to another larger theta wave in which it is immersed. Naturally in this case the habitual notion of mass has no meaning and the momentum of the theta wave, simple or complex, is only related to its velocity and size. So we can write

$$p = \eta\, v, \tag{7.2.12}$$

where $p = \mathcal{A}/\sigma$ and $\eta \propto \mathcal{A}$ is a proportionality constant somehow playing the role of a generalized mass. For the case of the gravitic acron this constant is what is commonly called the mass

$$\eta = m = c_m N_\xi, \tag{7.2.13}$$

in the case of the theta wave, without acron, this proportionality constant is given, as we have seen previously, by

$$\mathcal{A} = \int_V |\theta(r)|^2 dr, \tag{7.2.14}$$

where V is the integration volume.

Thus we have seen that the most basic concepts of traditional physics, moment and energy may be seen as derived concepts from hyperphysics. The generalized momentum comes from the density of the mother theta wave $p = \mathcal{A}/\sigma$ and the energy as a function of the generalized momentum and the generalized mass, $E = p^2/2\eta$. Naturally, the common definitions $p = \nabla\varphi$ and $E = \varphi_t$ still hold for the mother theta wave.

7.3. The Master Equation

In this section we are going to derive, always following the habitual heuristic process, the nonlinear master equation. This process is very similar to the one developed in the book Towards a nonlinear quantum physics[1], only now has a much more general scope.

Since by now the concepts of mass, force, energy and momentum have been shown to be secondary concepts derived form the basic Hyperphysis assumptions it is possible to derive, in a simple way, the concepts of work and potential energy. Furthermore, as it is done in most text books of elementary mechanics, it is also possible to derive the expression for the energy conservation,

$$E_c + U = E, \tag{7.3.1}$$

saying the kinetic energy plus the potential energy is equal to the total energy. By the same token the continuity equation can also be derived

$$\nabla\vec{J} = \frac{\partial\rho}{\partial t}, \tag{7.3.2}$$

or, in one dimension

$$\frac{\partial J}{\partial x} = \frac{\partial \rho}{\partial t}, \tag{7.3.2'}$$

where

$$J = \rho v, \;\; \rho = |\theta|^2. \tag{7.3.3}$$

Now, writing the theta wave in the generic form

$$\theta(\vec{r}, t) = a(\vec{r}, t) e^{i\frac{1}{\hbar}\varphi(\vec{r},t)}, \tag{7.3.4}$$

we have

$$\begin{aligned} &\nabla\varphi = p; \quad \upsilon = \frac{\nabla\varphi}{\eta}; \quad J = \rho\upsilon; \quad \rho = a^2; \quad J = a^2\frac{\nabla\varphi}{\eta} \\ &E_c = \frac{1}{2}\eta\upsilon^2 = \frac{1}{2\eta}(\nabla\varphi)^2; \quad E = -\frac{\partial\varphi}{\partial t} = -\varphi_t \end{aligned} \tag{7.3.5}$$

and by substitution into the equation of conservation of energy and in the equation of continuity we have

$$\begin{cases} \dfrac{1}{2\eta}(\nabla\varphi)^2 + U = -\varphi_t \\ \dfrac{1}{\eta}\nabla(a^2\nabla\varphi) = -\dfrac{\partial a^2}{\partial t} \end{cases} \tag{7.3.6}$$

that can be written for the one-dimensional case

$$\begin{cases} \dfrac{1}{2\eta}\varphi_x^2 + U = -\varphi_t \\ \dfrac{1}{\eta}\partial_x(a^2\varphi_x) = -\partial_t a^2 \end{cases} \tag{7.3.7}$$

naturally, with $\partial_\xi = \partial / \partial \xi; \quad \varphi_x = \dfrac{\partial \varphi}{\partial x}$. Developing the last equation one has

$$\begin{cases} \dfrac{1}{2\eta}\varphi_x^2 + U = -\varphi_t \\ \dfrac{1}{2\eta}(2a_x\varphi_x + a\varphi_{xx}) = -a_t \end{cases} \tag{7.3.8}$$

now by multiplying the first of these equations by *a*, and the second by (-1) we get

$$\begin{cases} \dfrac{1}{2\eta}a\varphi_x^2 + Ua = -a\varphi_t \\ -\dfrac{1}{2\eta}(2a_x\varphi_x + a\varphi_{xx}) = a_t \end{cases} \tag{7.3.9}$$

and by adding and subtracting the same quantity $(\hbar^2 a_{xx} / 2\mu)$ to the first equation and multiplying the second by (i) they become

$$\begin{cases} -\dfrac{\hbar^2}{2\eta}a_{xx} + \dfrac{1}{2\eta}a\varphi_x^2 + \dfrac{\hbar^2}{2\eta}a_{xx} + Ua = -a\varphi_t \\ -\dfrac{i\hbar}{2\eta}(2a_x\varphi_x + a\varphi_{xx}) = i\hbar a_t \end{cases} \tag{7.3.10}$$

To get a single equation we need to add both equations

$$-\frac{\hbar^2}{2\eta}\left[a_{xx} - \frac{1}{\hbar^2}a\varphi_x^2 + \frac{i}{\hbar}(2a_x\varphi_x + a\varphi_{xx})\right] + \frac{\hbar^2}{2\eta}a_{xx} + Ua = i\hbar(a_t + \frac{i}{\hbar}a\varphi_t) \tag{7.3.11}$$

by multiplying and dividing by *a* the second term in a_{xx} and finally multiplying the two members of the expression by a generic phase factor $e^{\frac{i}{\hbar}\varphi}$ one arrives at

$$-\frac{\hbar^2}{2\eta}\left[a_{xx}-\frac{1}{\hbar^2}a\varphi_x^2+\frac{i}{\hbar}(2a_x\varphi_x+a\varphi_{xx})\right]e^{i\varphi}+ \\ +\frac{\hbar^2}{2\eta}\frac{a_{xx}}{a}a\,e^{i\varphi}+Ua\,e^{i\varphi}=i\hbar(a_t+\frac{i}{\hbar}a\varphi_t)\,e^{i\varphi} \tag{7.3.12}$$

Now recalling that

$$\theta = a\ e^{\frac{i}{\hbar}\varphi}$$
$$\theta_t = (a_t+\frac{i}{\hbar}a\varphi_t)\,e^{\frac{i}{\hbar}\varphi}$$
$$\theta_{xx}=\left[a_{xx}-\frac{1}{\hbar^2}a\varphi_x^2+\frac{i}{\hbar}(2a_x\varphi_x+a\varphi_{xx})\right]e^{\frac{i}{\hbar}\varphi} \tag{7.3.13}$$

by substitution into (7.3.12) we get finally

$$-\frac{\hbar^2}{2\eta}\,\theta_{xx}+\frac{\hbar^2}{2\eta}\frac{|\theta|_{xx}}{|\theta|}\theta+U\theta=i\hbar\theta_t\,, \tag{7.3.14}$$

standing for the master nonlinear equation. In the three dimensions we have

$$-\frac{\hbar^2}{2\eta}\nabla^2\theta+\frac{\hbar^2}{2\eta}\frac{\nabla^2(\theta\theta^*)^{\frac{1}{2}}}{(\theta\theta^*)^{\frac{1}{2}}}\theta+U\theta=i\hbar\frac{\partial\theta}{\partial t}\,. \tag{7.3.15}$$

When, in this equation, the nonlinear term is null or constant one gets a linear equation formally equal to the well known Schrödinger equation

$$-\frac{\hbar^2}{2\eta}\nabla^2\theta+U\theta=i\hbar\frac{\partial\theta}{\partial t} \tag{7.3.16}$$

But with the difference that now the constants η and $\hbar$ do not stand only for the mass and Planck's constants, but also for the generalized constants.

CONCLUSION

As a final note I wish to add that this work constitutes the preliminary version of the kernel of a more extended work which is still in progress. Some ideas need further clarification and above all it must be stressed that the formal part of the present work is still at an initial incipient phase. In order to describe properly the complex interacting nonlinear phenomena there is a need to develop a whole language, namely a new branch of mathematics assuming from the start the intrinsic deep interdependency of the physical systems.

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PRELIMINARY RESULTS FROM COMPUTER SIMULATIONS OF ELEMENTARY PARTICLE INTERACTIONS AS AN APPLICATION OF THE PRINCIPLE OF EURHYTHMY

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Summary: This Chapter deals with computer simulations reproducing the behavior of a single particle ("acron") inside its theta wave, and also the interaction between two such particles by way of their theta waves, in the general framework of a recently proposed "eurhythmycal principle", which takes into universal consideration the properties of stochasticity, nonlinearity, and locality. The historical heritage of the present attempt is briefly reviewed with the purpose of placing it clearly in the realist tradition.

Keywords: eurhythmy, coalescence, anticoalescence, subquantum medium, numerical simulation.

1. INTRODUCTION

It is probably safe to say that a vast number of physicists, of a mainstream bent or otherwise, feel uncomfortable with the heritage of the Copenhagen interpretation of quantum mechanics. Their particular reasons may vary, but some feel such a discomfort especially with the subjective character one is led to attribute to the wave function, either of a single particle or relative to a system of particles and existing only in a configuration space, along with a dubious ontology associated with the material particles themselves. In fact, our own position here is that it would be unacceptable not to attribute an objective, real existence to a wave that propagates along with a particle and interacts with macroscopic devices such as the two-slit diffraction grating, leading to the well known interference patterns.

It may well be asked, why bother, and try, not even to reinvent the wheel but merely to change its spokes? Doesn't it work so well (and

sometimes with such unsurpassed precision) in most of the practical situations?

The reply is, in our opinion, that there is an obligation to clarify, as completely as possible, the epistemological elements that support any physical theory, no matter how successful (under several criteria) and, in particular, to uncover, from the past and also from the present, different proposals that, if accepted, would have conduced us to a quite different view of the workings of Nature. Such is the case with the analysis of the conventionality elements in the special theory of relativity, initiated with Reichenbach, and utilized by Selleri, among others, to justify an equally empirically successful (or even more so) theory of relative motions, at least as it concerns the usual historical tests. Such is the case, also, of the deduction of Planck's radiation law without the need to presuppose the quantization of the energy of the matter oscillators. And we also know that, contrary to what is almost universally taught in quantum mechanics textbooks, one can explain the photoelectric effect without field quantization, i.e. photons. Besides, there exist in fact proposed experimental tests of some of the ideas underlying the approach herein presented, and their outcome would be decisive for its sustainability.

These, and other possible examples, serve to remind us that scientific progress is not linear and that it may be worthwhile, by itself and in face of present difficulties, to look back at the beginnings and wonder whether some other path would have possibly given us a different framework on which to develop a better description of the physical world and to tackle those difficulties. Or are we to accept the uniqueness and inescapable necessity of past scientific progress? Such an endeavor is, we believe, appropriate in itself, in spite of the common criticism regarding the sterility and futility of most of these efforts. Says Torretti, in this regard (Torretti, p.374): "In 70 years, theories countenancing such hidden variables have not led to the discovery of a single new physical effect. Indeed, their partisans are content if the consequences of their hypothesis agree with the predictions of QM"; and, from the note on same page: "...It is perhaps worth noting that, with the exception of Louis de Broglie (1956), Nobel prize winners have stood aloof of such efforts". This is not the place to argue at length on the possible reasons for this fact, suffice to say that it is not difficult to anticipate the increased risks associated with the undertaking of those same efforts. Which does not mean they have not been in fact undertaken or that observable discrepancies relative to the predictions of quantum mechanics, some of them resulting from the Fourier ontology, have not been anticipated. Besides, isn't it interesting having alternative theories with the same predictions as quantum mechanics, in particular for the philosophy of

science? And to seek a more nearly understandable and encompassing world view?

So, in studying the present approach, we take a decidedly realist stance, that is to say, we subscribe to scientific realism as the correct methodology in considering physical entities, be they directly observable or not, in spite of a possible more general anti-realism of no concern to us presently but which recognizes the complications associated with our understanding of "the true intrinsic nature of physical objects" -- both chairs and electrons. Accordingly, we take waves and particles as existing objectively, in time and space, independent of the observer and of his particular state of knowledge. Time and space, on the other hand, are the *principium individuationis* that allow for the very existence of particles, fields, and of waves, so that outside their realm physics has no purchase, and, consequently, a resolution of the well known paradoxes and quantum effects should be achieved without appealing to tools such as the configuration space, even if at the expense of special relativity. For example, T. Maudlin (Maudlin, 2002) concludes that violation of Bell's inequality requires both superluminal causal connections and superluminal information transmission, albeit not superluminal signaling nor matter/energy transport.

It must be added that a number of experimental results have in fact been presented as evidence of the limits of applicability of orthodox quantum mechanics. Some of these refer to the breakdown of the limits supposedly imposed by Heisenberg's uncertainty relations in the operation of scanning probe microscopes, or in the emission of light pulses from optical condensers (Croca, 2003). Furthermore, there are also a number of experimental results, some of them well established by different teams, that indicate the possibility of overcoming the speed limit of c established by the theory of special relativity and thus requiring a revision of its central tenets. One such phenomenon is the superluminal transmission of information-carrying photons through tunneling barriers; this and other examples are referred to by Selleri (Selleri, 2004), who proposes a new set of space and time coordinate transformations. We believe that, with the demise of the early 20th century positivism, supporting much of the philosophical prejudices operating in the development of special relativity and of quantum mechanics, new avenues of research are now open, eventually richer in their explanatory power.

An almost universal criticism leveled against all causal and local, hidden variables quantum theories is based on the experimental results realized in the context of the famous theorem of John Bell. It will be convenient, then, to refer to a possible support of these theories, beyond the more common consideration of eventual loopholes in the interpretations of the experimental results, based on the technical limitations of the

apparatuses. In fact, such a support is provided by the arguments of G. Lochak (Lochak, 1976) based, in turn, on de Broglie's theory of measurement in quantum systems. In this approach the essential role of the measuring device is recognized, in particular its influence on the probability densities of hidden variables.

In fact, one must distinguish between three different kinds of probabilities – present, predicted, and hidden, associated with the distributions of quantum measurement results and with the ensemble of hidden variables. Present probabilities are to be associated with the configuration of a quantum system prior to its preparation by some particular measuring device, and will most commonly correspond to position measurements in physical space (the prevalent measurement outcome), with a classical algebra of probabilities. In contrast, predicted probabilities are to be associated with the effective intervention of the particular measuring device (the "spectral analyzer"), and obey the usual rules of quantum mechanics, in particular the non-existence of joint probabilities for measurements associated with two non-commuting observables, such as the spin projections in a Stern-Gerlach apparatus.

Finally, hidden probabilities, obeying a classical probability algebra along with present probabilities, must be associated with the hidden parameters and the distribution of quantum variables which are, at this stage, not yet measured, because the very act of measurement will impose a state preparation that destroys the hidden distribution and generates, upon observation, the usual, expected quantum results.

Now, applying the conceptions in the context of Bell's theorem, Lochak observes that it presupposes the permanent validity of an initial classical probability distribution for the hidden parameters, independent of the kind of quantum state preparation and subsequent measurement one decides to perform (spin projections, usually) and which, therefore, loses its validity. Since one cannot expect to obtain the experimentally verified quantum results, upon measurement, employing along the whole process some initial (i.e., prior to state preparation by the spectral analyzer) classical probability distribution, which is independent of the particular measurement one is about to perform, one must conclude that Bell's reasoning is vitiated from the very beginning and is, in fact, non-applicable to at least some local, hidden-variables theoretical alternatives, such as de Broglie's double solution – and also to the proposal hereby presented.

2. SOME COMMENTS ON THE THEORY OF THE DOUBLE SOLUTION

Louis de Broglie proposed his theory of the Double Solution in an attempt to reconcile the undulatory and the particle aspects of the quantum particle, considering that the latter is to be represented by an extremely localized wave entity u_0, inside a much broader wave v which is responsible for all the diffraction, interference, and confinement effects one is used to consider. In fact, this physically existent wave v is to be linearly proportional to the usual, subjective, normalizable, Ψ wave of the Schrödinger equation, so that one can put $\Psi = C\ v$, where C is a constant. Most importantly for the interpretation of a measurement process, the v wave never collapses, but it branches out upon measurement, and the particle itself - represented by the u_0 wave - keeps on propagating on just one of those branches, the remaining becoming empty and decaying rapidly in time. The total physical, objective wave can be approximately expressed by writing $u= u_0 + v$, reducing to just the v part in most of its extension, since u_0 is extremely localized. Consequently, and even though it would be further assumed that the total wave function can be approximately described by a nonlinear differential equation, the v component above would still satisfy the same equation as the usual Ψ wave, i.e., the linear Schrödinger equation. In this way, most of the observable phenomena will conform to the usual quantum mechanical results and observable discrepancies, due to nonlinearity, should be obtained only in novel circumstances, yet to be explored. On the other hand, the nonlinear wave equation itself must reduce to the Schrödinger equation, having solutions Ψ and v, away from the localized u_0 wave.

It may be appropriate, at this point, to refer to some apparent discrepancies with regard to the meaning of the expression "Double Solution". In fact, initially, in 1927, de Broglie proposed that there would be two solutions to the linear wave equation, not only the continuous Ψ wave, with the usual statistical interpretation, but, in addition, an embedded, very narrow wave (in fact, a singularity, at least in the scale of the broader Ψ wave) representing the particle itself (see, for example, Jammer, pp.44-49). The particle would be guided by the probabilistic Ψ wave, such that its velocity would be given by the gradient of the phase of Ψ. To this version de Broglie called the "theory of the pilot wave" but, in fact, what he was publicizing then was a simplified version of his theory of the double solution, still very much underdeveloped due to foreseen mathematical difficulties; besides, de Broglie was by then fully conscious of the incongruence of having a purely probabilistic wave Ψ physically guiding the particle. Later, in 1971 (Louis de Broglie et João Andrade e Silva, 1971, p.35; Louis de Broglie, 1987), one reads a more complete version, to the effect that, on the physical side, there exists not only the singularity wave

(u_0, above), but also the v wave already mentioned; and, on the subjective, non-physical side, the traditional probabilistic Ψ wave. This extension of the original conception was due to the desire to keep the normalizable Ψ function, along with its manifestly indispensable statistical role, in spite of the fact that it definitely cannot be physical; its physical counterpart then becomes the v wave. Furthermore, and adopting some ideas from general relativity, even though the Ψ wave always described by a linear wave equation, along with the v wave, that is no longer so for the complete physical wave u, namely sufficiently close to the position where the wave function associated with the particle is non-zero.

For completeness, we now spell out the basic formalism of the linear theory of the Double Solution, so as to make clear the difference towards its more recent, nonlinear version. Since both the Ψ and the v waves satisfy the (linear) Schrödinger equation, we may write

$$-\frac{\hbar^2}{2m}\nabla^2 v(\mathbf{r},t) + U(\mathbf{r},t)v(\mathbf{r},t) = i\hbar\frac{\partial v(\mathbf{r},t)}{\partial t}$$

(**r**,t) being any spacetime point and U(**r**,t) the potential energy of a particle with mass m. Instead of v, one might as well put Ψ since they differ by a multiplying constant only. Being complex-valued, one may write v(**r**,t) in polar form,

$$v(\mathbf{r},t) = a(\mathbf{r},t)e^{i\varphi(\mathbf{r},t)}\ ,$$

where both a and φ are real-valued functions of (**r**,t). Inserting in the Schrödinger equation, we get

$$\frac{\partial a^2}{\partial t} + \nabla\cdot\left(a^2\frac{\nabla\varphi}{m}\right) = 0$$

and

$$\frac{\partial\varphi}{\partial t} + \frac{(\nabla\varphi)^2}{2m} + U - \frac{\hbar^2}{2m}\frac{\nabla^2 a}{a} = 0\ ,$$

which can be recognized as a fluid conservation equation where

$$a^2 d^3\mathbf{r} = |v|^2 d^3\mathbf{r}$$

will be the probability of having the particle effectively occupying the volume $d^3\mathbf{r}$ around point **r** at time t, followed by an Hamilton-Jacobi equation of motion with the addition of the term

$$(-\hbar^2/2m)(\nabla^2 a/a)$$

to the potential energy, the so-called "quantum potential" Q, which would not be present if it were $\hbar = 0$. By analogy with the classical Hamilton-Jacobi equation, one can identify the linear momentum of the particle **p** as given by

$$\mathbf{p} = \nabla\varphi$$

and the total energy by the l.h.s. of the second equation minus the first derivative term. Consequently, its velocity will be

$$\mathbf{v} = \nabla\varphi/m \ ,$$

the "guidance formula" that would allow the calculation of a precise trajectory if a precise initial position could be given. According to de Broglie, this would then constitute the basic laws of a new and more general mechanics. In passing, we note that the above system of equations also allows writing down a newtonian equation of motion of the form

$$d\mathbf{p}/dt = -\nabla(U + Q) \ ,$$

the total potential energy now having a quantum part Q, besides the classical part U. This would become the starting point for the so-called Bohmian mechanics.

Although this is not the place to review, in more detail, the theory of the Double Solution, we would like to make here just one further remark, since it crops up frequently and is often taken to be an insurmountable difficulty with the assumed objectivity of the wave function, propagating in the physical 3D space: if one accepts the objective reality of the quantum mechanical wave function, how can a system of particles, its quantum-mechanical state being described by a wave function propagating in 3n configuration space, be associated with another wave function, propagating in our real, 3D space?

As a matter of fact, the assumption of the existence of the many-particle wave function in physical three-dimensional space is one of the main criteria distinguishing de Broglie's theory from David Bohm's, for whom the many-particle wave function only exists in a 3n dimensional configuration space (another important difference being the existence, for de Broglie, of

the photon as a particle embedded in the guiding electromagnetic wave, whereas, for Bohm, there is no separate photon, but rather the possibility of a local concentration of energy (see Cushing, p.149). And, one must add, nonlocal effects would demand a profound review of either that possibility, or of special relativity - or both. The first route, of reviewing the quantum mechanical mathematical description of elementary particles, including the photon, has been explored by Croca and collaborators (see, for instance, Croca, 2003, and Araújo *et al.*, 2009).

In keeping with our realist prejudices, and recognizing that configuration space does not give all the information we believe should physically exist (after all, any quantum system, no matter how complex and how many particles it contains, exists in three-dimensional space), we review the simplest such system, formed by just two noninteracting and distinguishable particles. Following de Broglie (L. de Broglie et J. Andrade e Silva, Chap.8), we here write the corresponding extended v-waves as

$$v_1(\mathbf{r}_1, t) = a_1(\mathbf{r}_1, t)e^{i\varphi_1(\mathbf{r}_1,t)/\hbar} \;\; ; \;\; v_2(\mathbf{r}_2, t) = a_2(\mathbf{r}_2, t)e^{i\varphi_2(\mathbf{r}_2,t)/\hbar} \, ,$$

$\mathbf{r}_i$ being the (actual) position of the i-th particle. The trajectory of each one of the particles inside the respective wave is not affected by the wave of the other particle (since we are assuming noninteraction) -- they are determined by the respective guidance condition, given some initial physical configuration.

On the other hand, the configuration-space wave solution to the Schrödinger equation of this two-particle system will depend on both $\mathbf{r}_1$ and $\mathbf{r}_2$, as usual, in the form

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = a(\mathbf{r}_1, \mathbf{r}_2, t)e^{i\varphi(\mathbf{r}_1,\mathbf{r}_2,t)/\hbar} \, .$$

The guidance formula, as applied to each one of the particles, gives in real space the velocities

$$\mathbf{v_1} = \frac{1}{m_1}\nabla\varphi_1|_{\mathbf{r}=\mathbf{r}_1} \, , \mathbf{v_2} = \frac{1}{m_2}\nabla\varphi_2|_{\mathbf{r}=\mathbf{r}_2}$$

and, in 3X2 configuration space,

$$\mathbf{v}_i = \frac{1}{m_i}\nabla_i\varphi|_{i=1,2} \, .$$

Defining the configuration space wave function Ψ by the condition

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = v_1(\mathbf{r}_1, t)v_2(\mathbf{r}_2, t)$$

we extract the relations

$$a(\mathbf{r}_1, \mathbf{r}_2, t) = a_1(\mathbf{r}_1, t)a_2(\mathbf{r}_2, t)$$

and

$$\varphi(\mathbf{r}_1, \mathbf{r}_2, t) = \varphi_1(\mathbf{r}_1, t)\varphi_2(\mathbf{r}_2, t)$$

Here, the subjective $\Psi(\mathbf{r}_1, \mathbf{r}_2, t)$ wave has the usual probabilistic interpretation, that is to say, its modulus square gives the density of probability of the simultaneous presence of particle 1 at $\mathbf{r}_1$ and particle 2 at $\mathbf{r}_2$.

More complicated situations and, most importantly, systems in the presence of the random action of a subquantum medium, are described by de Broglie (Louis de Broglie et João Andrade e Silva, 1971), namely two different particles in interaction, two identical bosons, and two identical fermions. But our purpose here was basically to remind ourselves of the possibility of projecting the configuration space wavefunction onto a physical wavefunction in 3D real space.

3. A NONLINEAR WAVE EQUATION

It is possible to reverse the reasoning in the previous section and, starting from the same type of wave function and keeping the relations involving its modulus and phase, along with the conservation of probability and of total energy but without a quantum potential energy term, to arrive (Croca, pp.72/80) at the nonlinear wave equation below, generalizing Schrödinger's linear equation. In spite of this novel framework, the physical essentials of the theory and in particular its all important combination of particle and wave aspects are maintained, one wave being extended over space and enclosing another, very localized wave representing the particle, even though the underlying Fourier ontology is superseded in a fundamental way. From now on, and in order to emphasize this nonlinear framework, we will denote the extended wave function by the letter θ, the localized wave function by ξ, and their combination by ϕ. We further assume that we can write $\phi = \xi + \theta$ in the nonlinear equation

$$-\frac{\hbar^2}{2m}\nabla^2\phi(\mathbf{r},t)+\frac{\hbar^2}{2m}\frac{\nabla^2|\phi(\mathbf{r},t)|}{|\phi(\mathbf{r},t)|}\phi(\mathbf{r},t)+U(\mathbf{r},t)\phi(\mathbf{r},t)=i\hbar\frac{\partial\phi(\mathbf{r},t)}{\partial t}$$

Particular solutions to this equation in the form of wavelets, their properties, and some applications are presented by Croca (op. cit.), but that is of no concern to us presently since our starting point will be different (mathematically, but not physically!). Nevertheless, it is important to keep in mind that this assumed nonlinearity stands beneath the present description and that the supposed addition $\phi = \xi + \theta$ can only represent, at best, an approximation to the real total wave function ϕ sufficiently far away from the central region where the acron is located. Besides these limitations, there is another that results from the absence, in the above equations, of a random element in the process of guiding the acron inside the wave function, which reflects back on the evolution of this same wave function. Nevertheless, the general conceptual framework is still of an essentially nonlinear evolution, with a mutual interaction between the wave and the particle/acron.

4. THE PROPOSED SUBQUANTUM MEDIUM

We already mentioned another very important ingredient later introduced by de Broglie, the proposed existence of a subquantum medium, after the initial work of Bohm and Vigier (Bohm and Vigier, 1956). This subquantum medium acts as a hidden thermostat, filling all physical space and exchanging energy and momentum with each material particle and giving rise, in fact, to a "Thermodynamics of the isolated particle". More important to our present concerns, it will impart random motions on a quantum particle so that it no longer follows just one line given by the guidance condition in 3D space, but it will rather jump from line to line, in a sort of brownian motion. The overall motion will be a superposition of these two, leading to the $|\Psi|^2$ probability distribution. According to the ideas advanced by de Broglie, it would be the v wave that would act as the interface of energy and momentum exchanges between the particle and the subquantum medium. However, the new viewpoint to be introduced below is more fundamental, in the sense that not only is this mutual interaction nonlinear but, furthermore, the concept of mass, as well as laws of conservation of energy and momentum should result, on average, from the theory itself, following from a "principle of eurhythmy".

The idea of this subquantum medium is akin to the concept of vacuum in quantum electrodynamics which, as we well know, is far from empty. But also in other approaches, in particular in the so-called stochastic electrodynamics (de la Pena and Cetto), an underlying electromagnetic

vacuum is supposed to exist, bearing an invariant spectral density of energy (cubic in the frequency) and leading to an explanation of a number of phenomena hitherto only describable in the context of standard quantum mechanics. Some examples of successful descriptions in this theory are the Casimir effect, van der Walls forces, diamagnetism, Planck's radiation law, the Lamb shift, the harmonic oscillator and even the ground state of the hydrogen atom (in numerical simulations). A especially interesting effort was made with regard to the possible connection between the effect of the electron/vacuum interaction and the genesis of de Broglie's undulatory properties of matter.

The same physical concept of some underlying chaotic medium acting on the particle/singularity by way of the physical wave function θ must be transported to the proposed nonlinear approach and it becomes crucial to our present line of inquiry. In fact, instead of tackling the problem from the resolution of the nonlinear wave equation, in the present work we take a different point of departure, which is precisely the one that considers stochastic motions of the singularity, subject to the condition of increased probability presence in places where the intensity of the physical wave function θ is largest.

5. A NEW PROPOSAL

One of the aims of the above review of de Broglie's ideas was to bring forth the circumstance of the existence of a seldom mentioned research program in quantum mechanics which, in our view, has the great advantage of pretending to consider, in a fundamental way, only observer independent entities existing in time and 3D space. A number of obstacles have remained though, some of them due to an improper use of Fourier methods (in spite of the repeated warnings against infinite extension train waves), others to an insufficient development of the nonlinear description.

A new ontology for describing the physical world has been recently proposed by J. Croca, appealing to a unifying, general "principle of eurhythmy" and taking into consideration, *ab initio*, the essentially nonlinear characteristics of all elementary physical phenomena whose linearity becomes just an approximation which facilitates the calculations and is good enough in numerous situations, but ultimately failing to describe accurately all physical phenomena. Another key hypothesis in this proposal is the recognition that the use of nonlocal, extended mathematical functions of Fourier analysis (in which only infinite waves can have a single frequency), is bound to generate errors and paradoxes, such as the ones that we are all familiar with in quantum mechanics. Hence, considerations of nonlinearity,

objectivity and locality, already recognized but not sufficiently developed, must be kept in mind from the very beginning in studying whatever system, no matter how elementary at first glance -- for example, a single particle in otherwise empty space -- if one wishes to overcome the common explanations, originally due fundamentally to Niels Bohr and which remain, we believe, unsatisfactory.

Considering that an elementary particle is an extremely small spatial region of very high energy concentration always surrounded by an accompanying wave which contains a much smaller share of the total energy, this motion of this particle/wave system -- acron/theta wave system -- is the result of the mutual interaction between these two entities, in a nonlinear fashion since each one of them acts on the other and influences it. There is also an element of inescapable stochasticity and, even though the acron moves inside the theta wave to points where the intensity of the latter is greatest, these elementary displacements are not deterministic; the spatial distribution of the intensity field of the theta wave gives, more precisely, a spatial distribution of different displacement probabilities, duly simulated below.

Extending previous conceptions but in the framework of this new ontology, Croca modifies the basic principles, including in it all possible physical interactions and a treatment of nonlinearities in the assumed interaction of the interface particle/v-wave or, rather, acron/θ-wave. Due to the omnipresence of the subquantum medium, the acron moves inside its θ wave in a somewhat random fashion but not completely, since there is also the tendency to occupy positions where the intensity of θ is greatest, this being the present guiding condition, according to the general principle of eurhythmy. Furthermore, we must imagine that the acron concentrates almost all the energy in itself and that, without it, the θ wave decays rapidly to zero. As a consequence, as the acron moves, it leaves in its wake a decaying θ wave at the same time that it originates a new θ wave along the points it visits in 3-space. In general, these 'gross' motions are subluminal, but that is not necessarily so for the instantaneous, random motions (the “natural velocity”). Furthermore, the whole stochastic process is definitely non-markovian, since the θ wave keeps a tail, reminiscent of its previous configurations, which must be included in computing the subsequent motion of the acron.

Now, when an acron is considered by itself, not in the presence of others, it will not stand still because it is inescapably subject to the subquantum medium and it executes a random motion around its initial position. On the other hand, in the presence of another acron with its own accompanying θ wave, their interaction is described according to exactly the same principles, that is, the intensity of the resulting total θ wave determines

the spatial probability of position for the two acrons, explaining both attraction and repulsion, according to the phase differences between the θ waves of each one of the moving acrons and the resulting superposition, always with an element of stochasticity. Furthermore, only in limiting cases will that phase difference become a constant, and this is the situation we consider below. But, in reality, the general situation is more complex and richer in its consequences, as described elsewhere in this book.

This way of looking at elementary acron-acron interaction is to be extended to any number of them, according to the overall θ wave resulting from the composition of all the individual waves emanating from each individual acron, and it is taken to have an universal scope, ultimately unifying all known material interactions, according to the proposed principle of eurhythmy. In this regard, it is not dissimilar from Popper's aims with his model of changing fields of propensities as a scheme towards unification (Popper, p.202).

6. SIMULATIONS

Our simulation algorithm has, at its core, the calculation procedure for elementary transitions in physical space. Given that, from any initial coordinate (here in 1D, for simplicity), the transition probability is determined by the relative intensity of the total θ wave around the acron, which we take to be initially a gaussian of width $\sqrt{2}\,\sigma$, we may write, for the probability of remaining on the same spot (initial location zero, say)

$$p_{0,0} = \frac{I_0}{I_{-1} + I_0 + I_{+1}} = \frac{|\theta_0|^2}{|\theta_{-1}|^2 + |\theta_0|^2 + |\theta_{+1}|^2}$$

and the probabilities for moving one spatial unit to the left or to the right

$$p_{0,-1} = \frac{I_{-1}}{I_{-1} + I_0 + I_{+1}} = \frac{|\theta_{-1}|^2}{|\theta_{-1}|^2 + |\theta_0|^2 + |\theta_{+1}|^2}$$

$$p_{0,+1} = \frac{I_{+1}}{I_{-1} + I_0 + I_{+1}} = \frac{|\theta_{+1}|^2}{|\theta_{-1}|^2 + |\theta_0|^2 + |\theta_{+1}|^2}$$

respectively, where I_x is the intensity of the θ wave at position x along the axis. Of course, the three probabilities add up to a value of one. Now, in setting up the actual transition, with the three possible outcomes above, we imagine a straight segment of unit length divided in three unequal parts, each

one of them with a length given by the values of these three probability values. Next, a decision is taken as to the next step by using a uniform random number generator which produces a number in the interval from zero to one, falling in one of those three segments with a probability proportional to the respective length. The acron is then made to move to the next position just computed (which may happen to be the same it is starting from), originating a new θ wave there, at the same time that the intensity of the θ wave in the previous position decays to zero, exponentially, with a time constant that we call λ^{-1}, since that location became vacant, i.e., without the acron. The total θ wave intensity is no longer a pure gaussian, but rather the composition of the new gaussian with the "remains" of the previous one. From this new position, a new decision is taken as to the next motion of the acron (to the left, to the right, or standing still), a new θ wave is generated and the total θ wave is the sum of this last with the remains of all the precedent waves (composition of decaying waves). The same stochastic procedure was then extended to two spatial dimensions *x* and *y*, and could have been extended, obviously, to three dimensions, were it not for the forbiddingly long CPU times.

An important issue to be addressed at this point, and which underlines all the computer simulations, is the lack of precise, absolute scales for distances and time lapses, that is to say, for the parameters σ and for λ. Consequently, both the distances along the coordinate axes as well as the sampling ranges of the values of the intensities of the θ wave function, used to compute the next step in the acron motion, should be compared with the values of σ. The relative value of this probing distance is extremely important in all three simulations (single acron, attraction between two, and repulsion) because, if too small, the isolated acron barely moves and the two acrons never become "aware" of the presence of each other and, if too large, we get unphysical results, such as having the isolated acron move to regions where the σ wave function has too small an intensity. Clearly, more work needs to be done in setting up, from adequate physical considerations, a proper evaluation of the correct parameters. In the meantime, we chose a probing distance equal to whatever value we take for the standard deviation of the original gaussian. This simplification avoids excessively large computation times while insuring an appropriate dynamics. It should also be noted that the pictures below show the displacement of the envelope of the amplitude of the total wave function θ, not of its intensity. This is particularly obvious in the case of anticoalescence, in which the phase opposition is depicted by positive and negative envelopes.

7. A SINGLE ACRON

As just mentioned, even the "isolated" acron does not stand still due to the action of the subquantum medium, executing a kind of brownian motion, with constant average position. As a result, and as already anticipated by Croca elsewhere in this book, there will be in general an initial slight increase in height of the θ wave function due to its large spatial overlap with the previous function (purely gaussian, at time zero, by hypothesis). It is easy to show that this height would converge to $(1- e^{-\lambda})^{-1}$ times the initial value, which we take to be one, in case the acron stays always on the same coordinate -- which is not the actual case, of course, since it performs a random motion around the initial position, as already mentioned. However, and more realistically, this random motion was also simulated for the situation in which the acron starts from the origin but with some initial velocity, along one of the axes. The trajectory then becomes, as expected, linear on average, but with fluctuations around that average line. The corresponding initial θ wave itself is modeled by a running gaussian (here, along the *x*-axis)

$$\theta(x,t) = Ae^{-\frac{x^2}{2\sigma^2}+i(kx-\omega t+\delta)}$$

where δ is some initial phase and A a real amplitude.

The sequence of figures below depicts a series of instantaneous positions of a single acron, on the (*x,y*) plane, with a zero initial velocity. One can observe the expected random motion of the acron around its initial position at (0,0), the center of the figure, as well as the rapid and finite height increase, due to the superposition of each "new" θ wave with the remains of the previous ones, as well as the occurrence of some height decreases when the wave base spreads on the (*x,y*) plane for particularly long random steps.

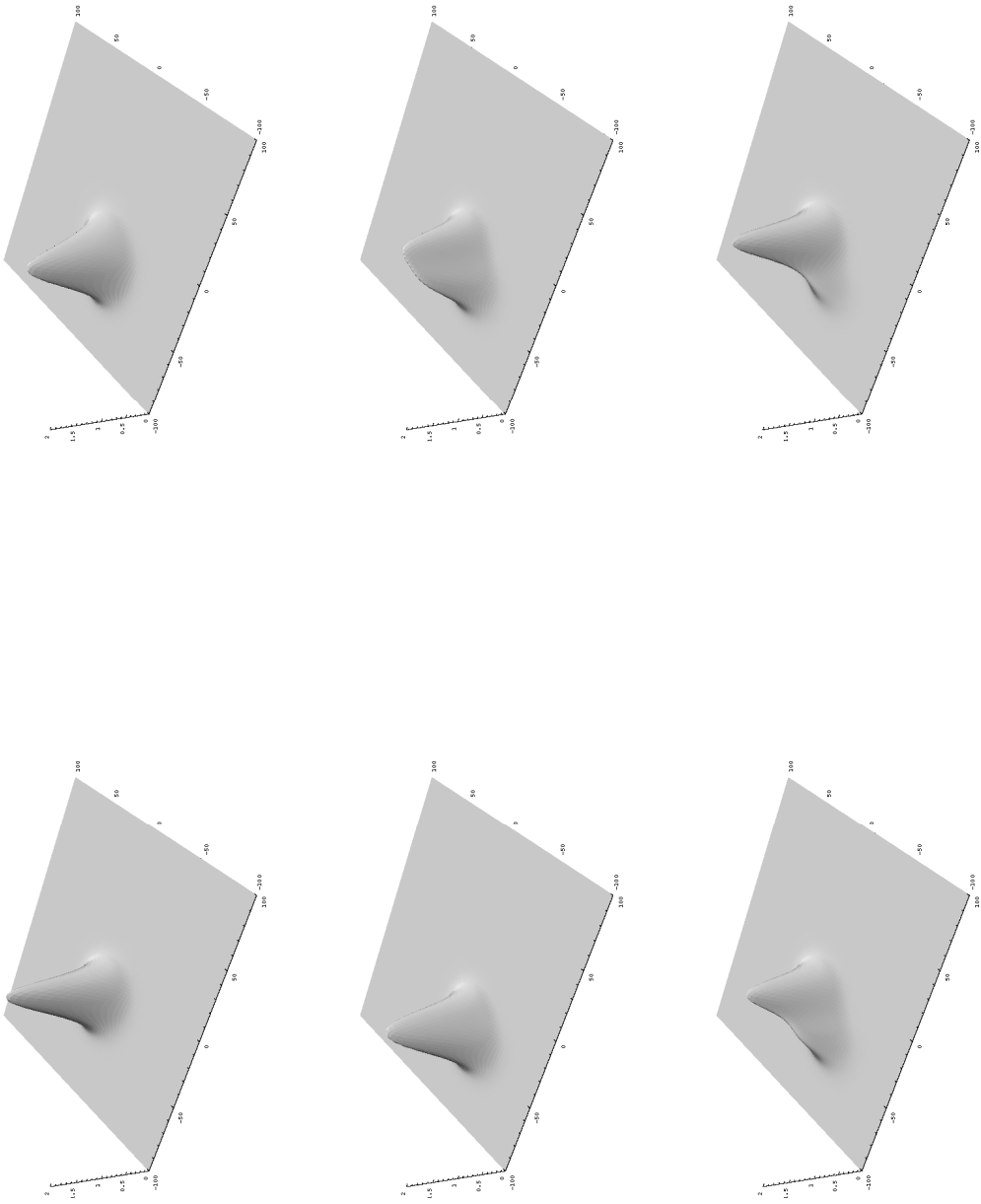

Fig.1 - One acron sequence, from left to right, top to bottom.
Zero initial velocity.

The basic computational procedure is fairly simple: one begins with an initial θ wave function with a gaussian envelope, centered at some initial position (x_0, y_0) with width σ such that $|\theta(x,y)|^2 = \mathrm{Exp}[(- (x - x_0)^2 - (y - y_0)^2)/\sigma^2]$, forming the first graph in the desired sequence. The intensity of this θ wave is next computed at three points along the *x*-axis, the initial (x_0, y_0) point, and the points ($x_0 + f\ \sigma$, y_0), ($x_0 - f\ \sigma$, y_0), where f is a number we took as 1, but which defines the distance, in terms of σ, at which the acron probes the surrounding θ wave function along the *x*-axis. These three values of intensities are combined and the three respective transition probabilities are computed, according to the expressions given above. This same procedure is repeated for transitions along the *y*-axis. So far, one only has the three values for the transition probabilities along each axis -- the probability to execute a jump of length $f\ \sigma$ to the left, or else to the right, or else of no jump. These three probabilities must of course add to one, but no decision has been made yet. That is achieved by generating a pseudo-random number, uniformly distributed in the interval 0 to 1. We divide this unit interval into three unequal (in general) subintervals, according to the numerical value of the three probabilities computed above. Whichever random number in the unit interval is generated for each jump, it must fall in one of these three subintervals and, in so doing, it automatically "chooses" the direction of the present jump, with the correct probability. Having done so for both axes, a new location is determined (out of a total of 9 possibilities, including the starting location) and, next, a new purely gaussian wave function is formed at this new location (with the same height and width as the initial one) and a new total θ wave function is also generated by adding this just formed gaussian with the previous θ wave function (no longer a gaussian, after the first transition) multiplied by the decaying factor $e^{-\lambda \Delta t}$ where we took unit time intervals. A new graph is now generated and the cycle recommences, through a previously established total number (usually 500 at most, due to hardware constraints).

8. TWO ACRONS IN INTERACTION

A system of two acrons of the same nature, with identical values for their σ and λ parameters, will be represented by the addition of the θ wave functions of the type represented above, each one centered at some initial positions $x = 0$ and $x = 1$, initial amplitudes A_1 and A_2, and initial phases δ_1 and δ_2:

$$\theta_1(x,t) = A_1 e^{-\frac{x^2}{2\sigma^2}+i(kx-\omega t+\delta_1)}$$

and

$$\theta_2(x,t) = A_2 e^{-\frac{(x-l)^2}{2\sigma^2}+i(k(x-l)-\omega t+\delta_2)}$$

The intensity of the total θ function becomes

$$I = |\theta|^2 = |\theta_1 + \theta_2|^2 =$$
$$A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-l)^2}{\sigma^2}} + 2A_1A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-l)^2}{2\sigma^2}} Cos(\delta_1 - \delta_2) .$$

The interaction between the two particles is here modeled essentially by assuming stable values for $(\delta_1-\delta_2)$: permanent attraction corresponds to the result $\delta_1 - \delta_2 = 0$ and permanent repulsion to the result $\delta_1 - \delta_2 = \pi$. Hence, in these particular circumstances, the total θ wave intensities become

$$I_{attraction} = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-l)^2}{\sigma^2}} + 2A_1A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-l)^2}{2\sigma^2}}$$
$$= \left(A_1 e^{-\frac{x^2}{2\sigma^2}} + A_2 e^{-\frac{(x-l)^2}{2\sigma^2}} \right)^2 = (|\theta_1| + |\theta_2|)^2 ,$$

and

$$I_{repulsion} = A_1^2 e^{-\frac{x^2}{\sigma^2}} + A_2^2 e^{-\frac{(x-l)^2}{\sigma^2}} + 2A_1A_2 e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{(x-l)^2}{2\sigma^2}} Cos(\pi)$$
$$= \left(A_1 e^{-\frac{x^2}{2\sigma^2}} - A_2 e^{-\frac{(x-l)^2}{2\sigma^2}} \right)^2 = (|\theta_1| - |\theta_2|)^2 .$$

In the computer simulations, we took the amplitudes A_1 and A_2 to have the same value of 1 and the *x* and *y* spatial dimensions are shown along with the numerical values chosen for the standard deviation of the initial gaussians. In the case of repulsion, the assumed phase opposition between the two waves is represented by a negative amplitude.

The computational procedure is basically an extension of the one already described above for the case of just one acron, with the proper inclusion of the fact that now the total θ wave function is, in each cycle, the algebraic sum of the wave functions due to each one of the two acrons, and that the jump probabilities are computed from the intensity of the total θ

wave function. In these simulations, some initial parallel velocities were imparted on the acrons in order to have them in motion but at such mutual distance that there is a reasonable chance of having them interact during the walk along the portion of the (x, y) plane chosen for the computation. At the same time, their initial positions are also chosen as not too larger than 2σ, for the overlap to have a chance of occurring during each run. Nevertheless, even with these deliberate choices, in a great number of runs the two acrons simply do not interact because they also have a large chance of moving away from each other in their spontaneous random motions. Of course, these previous preparations are in practice necessary because of the probabilistic nature of the motion and to the large running times on a common personal computer.

In the attraction simulation example, in Fig.2, one can appreciate an instance of a coalescence of the two acrons, which is not, of course, always obtained during each run. Many times the two acrons merely walk across the whole length without any interaction in the visible range of coordinates chosen since, due to the partial randomness in their motion, their wave functions do not always overlap. But when they do, there is typically a rapid transition to much smaller mutual distance regime, which is maintained, as shown in Fig.3, showing the mutual distance in terms of the value of σ as a function of the step order in the computation ("time"). The initial values taken were $(x_0, y_0)_1 = (-500.0, -120.0)$, $(v_{x0}, v_{y0})_1 = (10.0, 0.0)$ for acron 1, $(x_0, y_0)_2 = (-500.0, +120.0)$, $(v_{x0}, v_{y0})_2 = (10.0, 0.0)$ for acron 2, and $\sigma = 45$, $\lambda = 0.05$. Clearly, what is important is the relation between the distances and the value of σ.

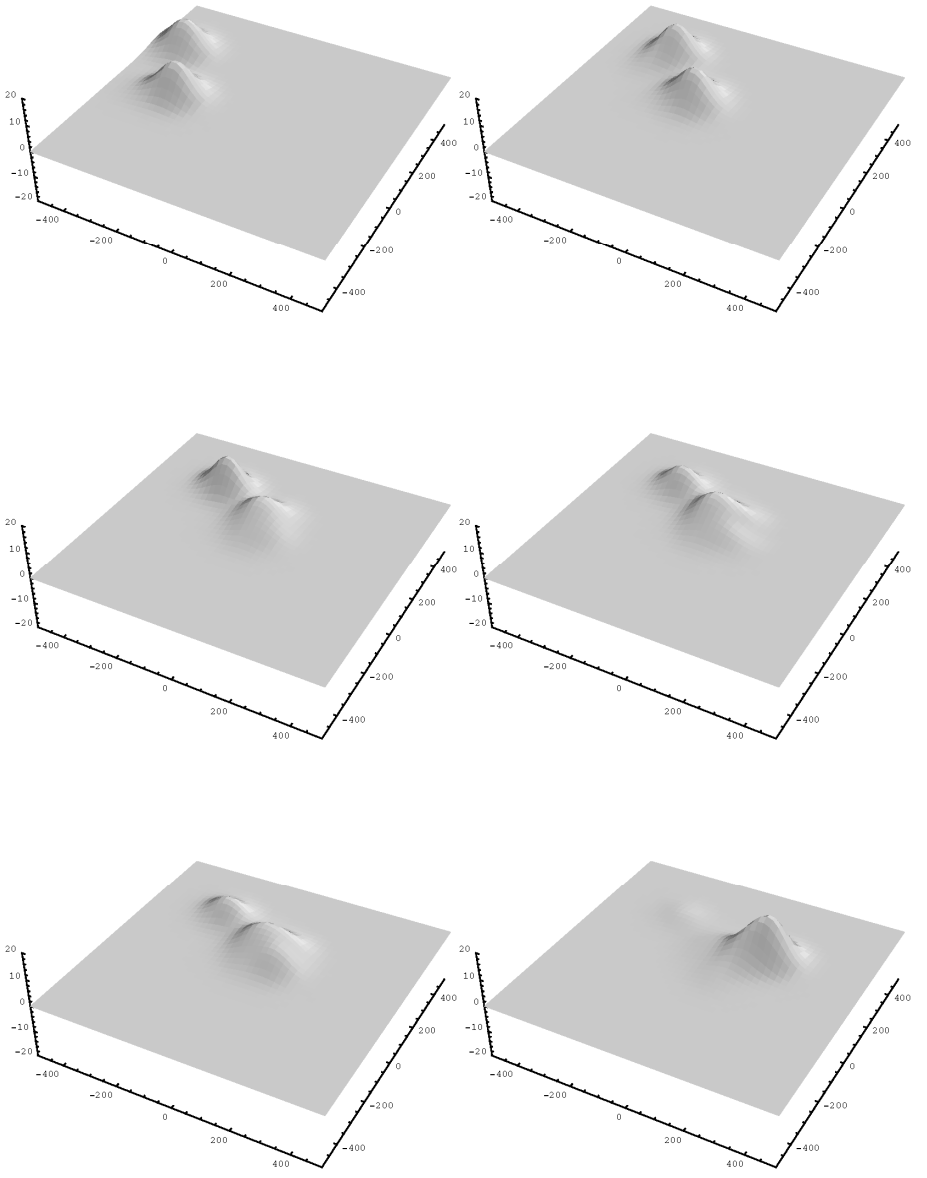

Fig.2 - Attraction of two acrons, sequence from left to right, top to bottom.
Equal initial velocities, both along the *x*-axis.

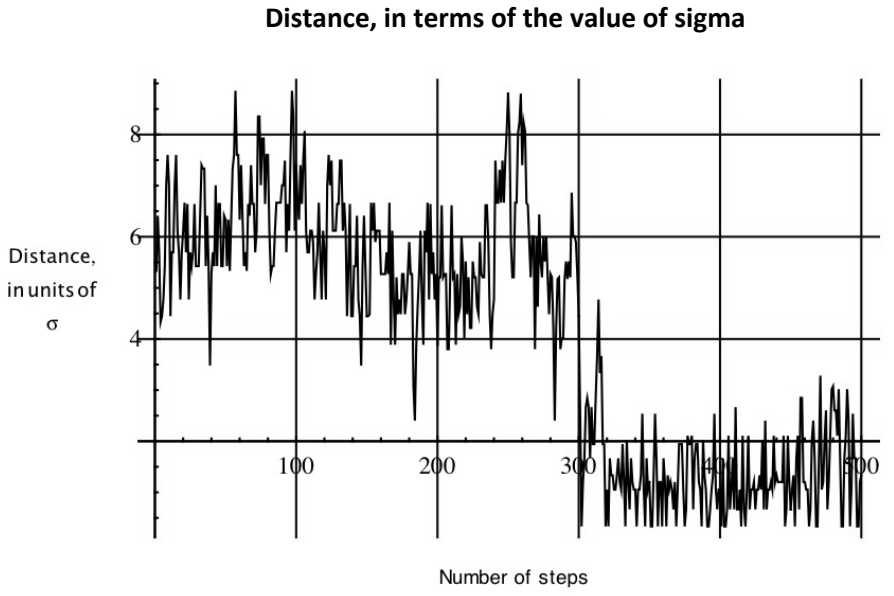

Fig.3 - Attraction of two acrons, same run as above.
Characteristic distance transition for coalescence, here at around the 300th step.

In the repulsion simulation shown in the example of Fig.4 below, on the other hand, one can observe how the two acrons seem to "avoid" each other when their mutual distance happens to decrease by chance in their stochastic walk, even to a very small value, as in the third and fourth pictures of the sequence. In this case of anticoalescence, larger jumps seem to occur generally immediately after a close encounter. Fig.5 depicts a typical evolution for the mutual distance, as time progresses. The initial values taken were $(x_0, y_0)_1 = (-500.0, -110.0)$, $(v_{x0}, v_{y0})_1 = (15.0, 0.0)$ for acron 1, $(x_0, y_0)_2 = (-500.0, +110.0)$, $(v_{x0}, v_{y0})_2 = (15.0, 0.0)$ for acron 2, and $\sigma = 45$, $\lambda = 0.05$.

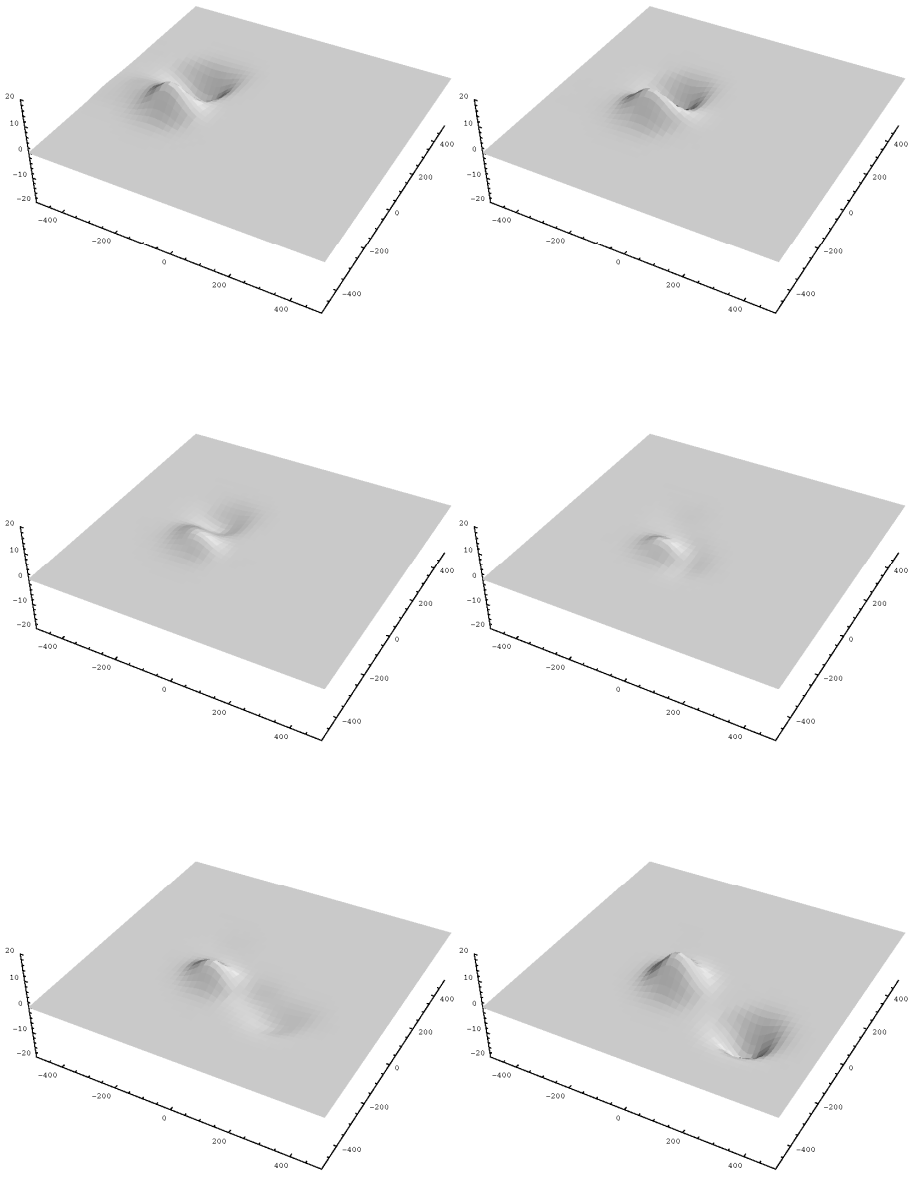

Fig.4 - Repulsion of two acrons, sequence from left to right, top to bottom.
Equal and parallel initial velocities.

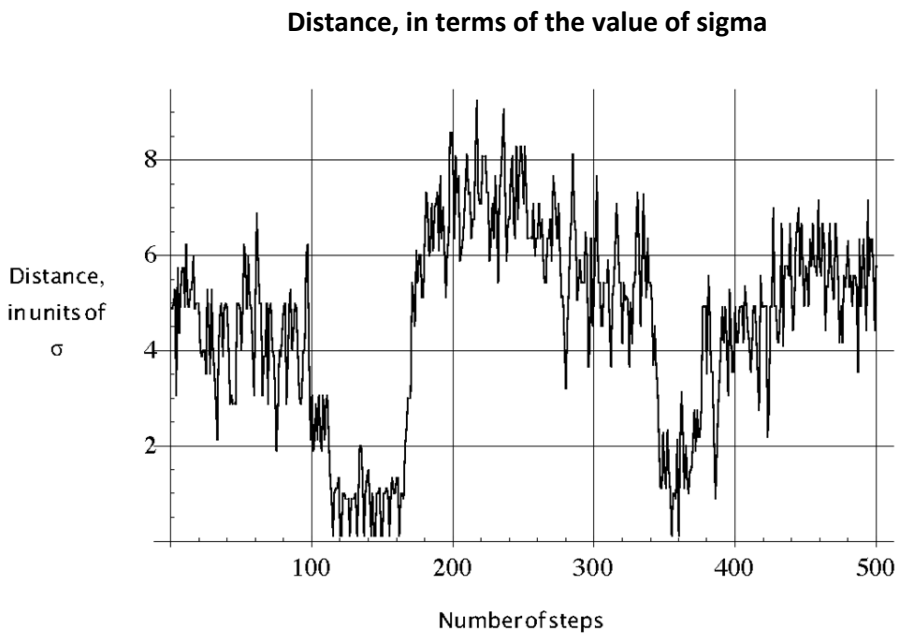

Fig.5 - Repulsion of two acrons, same run as above. Rapid increases in mutual distance may occur after accidental proximity.

9. CONCLUSIONS

One may say that the expected general trends described by the theoretical models developed under the eurhythmy principle seem to be adequately obtained in the simulations performed so far. Of course, being by essence stochastic, and non-markovian, a more refined appreciation of the results, for instance of the time evolution of the mutual distance between acrons in interaction, will demand much lengthier computations, with much larger time intervals and better resolution -- and a much more powerful computer hardware, allowing for an improvement of the preliminary results already obtained in this regard.

Also, one needs to obtain a sensible physical estimate for the physical parameters used in the present models, starting with the parameters σ and λ already introduced.

Still, it definitely seems that, in general, one not only observes the expected attractions and repulsions between two acrons, but also that, once two acrons attract to the point of coalescence, they remain attached even if without exact superposition for all times, and also that, in the opposite case of repulsion, they eventually separate from each other, even if at some point

in the simulation there occurs a large partial overlap of the respective θ wave functions.

In a final note, I would like to mention the striking similarity between aspects of the elementary quantum phenomena here considered and some of the results recently obtained by a group of french physicists (Couder *et al*., 2010), regarding the behavior of small silicone oil droplets maintained at a very small distance from a liquid surface of the same material (20 times more viscous than water) by means of vertical sinusoidal acceleration of the whole apparatus, and in which they went so far as observing the (wholly classical!) formation of interference patterns in a double slit set-up as well as quantized orbits among two such droplets. It may turn out that electrons are in fact more similar to chairs than what one is usually led to believe!

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ELEMENTARY NONLINEAR MECHANICS OF LOCALIZED FIELDS

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Summary: The notion of a field with localized, stable structure is studied here in several physical contexts (free, harmonic oscillator and gravitational or Coulombian potential). Starting from first principles, like the Hamilton's stationary action and the balance of conserved quantities, one generally obtains a description of motion based on the Hamilton-Jacobi equation for a phase field S, which is nonlinear, coupled with a continuity equation for an amplitude field e^{α}. This will be applied successfully to the description of the motion of coherent-type field structures that have an amplitude maximum along the expected classical path of a point particle for a particular potential. The existence of these structures is due to a choice of nonlinear solutions for the phase S contrarily to what is conventionally done in Quantum Mechanics which relies on the linearity of the theory to derive its amazing, undeniable successes in the description of the physics at the atomic scale. Nonlinearity of the equations dealt with here comes at a price of being unable to easily describe the interaction of fields or even of decomposing fields into simpler, basic structures. However it seems likely that linearity in the description of nature is the exception rather than the rule in the same way that conservative, integrable systems are a set of measure zero in the universe of dynamical systems. It seems therefore fitting that more attention should be dedicated to finding ways to describe nonlinear systems and interactions.

Keywords: Nonlinear physics, quantum potential, quantization, coherent states, Kepler problem.

*This work was developed in collaboration with J. R. Croca and R. A. N. Moreira at the Centro de Filosofia das Ciências da Universidadede Lisboa.

1 Introduction

In this work we explore a mathematical model based on a set of nonlinear equations deriving from fundamental physical assertions (Fermat-Huygens principles and Balance-Conservation equations) in a way that naturally embeds, as a subset of its solution space, some aspects of the standard quantum mechanics of simple systems associated with solutions of the usual Linear Schrödinger Equation (LSE). While the superposition principle is a key ingredient for the successes of Quantum Mechanics and cannot generally be reproduced with nonlinear equations with exception for some soliton equations, other aspects such as quantization of solutions can be successfully achieved for systems described with this approach.

The equations referred to here are the Hamilton-Jacobi equation for the phase $\varphi = S$ and the continuity equation for the amplitude squared $\rho = e^{2\alpha}$ of some complex function $\psi = \sqrt{\rho} e^{i\varphi}$. We note however that an appropriate change of space-time coordinates renders these equations scale-free which means that they can be used in principle at all scales of description. Using an analogy with standard Quantum Mechanics, one can see that these solutions could be integrated in a single complex wave-function $\psi = e^{\alpha + iS}$ which would become a solution to a non-linear Schrödinger-type equation with a field dependent quantum potential. In fact the equations can be combined into a particular class of Nonlinear Schrödinger Equation, henceforth designated NLSE, which reads

$$-\frac{1}{2}\Delta\psi + \left(V + \frac{1}{2}\frac{\Delta|\psi|}{|\psi|}\right)\psi = i\,\partial_t\psi\,. \tag{1}$$

This equation is similar to the conventional Linear Schrödinger Equation (LSE) but with an additional quantum potential term $Q(|\psi|) = \frac{1}{2}\frac{\nabla^2|\psi|}{|\psi|}$. The NLSE has a 10-dimensional symmetry algebra and is not generally linearizable.[1] In contrast, other treatments [2][3] where a term proportional (but not equal) to $Q(|\psi|)$ has been added onto the LSE provide an apparently nonlinear equation that is in fact geometrically equivalent to the linear one (i.e. has the same Lie algebra of symmetries and can be always reduced to an LSE by a phase rescaling). It should also be stressed that in order to accommodate the LSE several authors[4][5] have proposed that a quantum potential term should be added to the Hamilton-Jacobi equation in order to re-derive the standard quantum mechanical results, thus introducing a generalized Hamilton-Jacobi equation, a step which compromises a clear tie with the variational principles. In this work we will rather preserve the form of the classical principles and derive possible quantum mechanical consequences from the general conditions im-

posed on the solution space of the NLSE. In this sense our proposal cannot be confused with a "hidden variables" theory[6] such as Bohm advocated, and it also falls out of the range of the Bell's inequalities since a nonlinear quantization is considered in this model. Also, in the subset of solutions of the NLSE from which the linear quantum mechanical results are recovered, phases of type $\phi(\vec{\mathbf{q}},\, t) = \varphi_{\mathrm{o}}(\vec{\mathbf{q}}) - \mathcal{E}t$ are among many possible phase solutions for definite (stationary) energy states, thus giving a new meaning to the phase ambiguity concept and with possible deeper implications in the notion of gauge transformation.

Why would we want to move into a nonlinear setting given the amazing successes of the linear theory that the Schrödinger equation gave birth to? Well, these successes come at a cost of abandoning much of our holding on the objectivity of observations of the physical universe, and mostly this is due to our insisting that the description of nature be linear. But turning a blind eye to many awkward situations (such as plane wave expansions and the domination of non-local Fourier thinking, the ambiguities and frailties of the Heisenberg uncertainty/undecidability relations or the no-go theorems for canonical quantization and the observable-operator correspondence implied by the canonical commutation relations) does not mean we should refrain from seeking a theory that could embrace the successes of the linear methods but would not have these problems. In the XXth century, geometrization of physical theories helped us realize that the Lie algebra approach to dynamical equations is equivalent to a local, linear description of a nonlinear structure, the Lie group to which it is associated. In some cases (semi-simple groups) there is even a canonical structure for these algebras. That is the case for the Heisenberg algebra and what we now call the canonical commutation relations, but these also imply that the only dynamics they apply to lives in a flat manifold with a very restrict and specific transitive group action. Later in the century Souriau, Kostant and others started a Geometric Quantization approach to obtain the essential ingredients of a quantum mechanics on manifolds, followed by work on C^* and von-Neumann algebras by Emch, T.Ali and many others to address the issue of quantization of observables. In most cases advances were made when representation of groups was called in rather than that of its algebras, that is when we consider nonlinear global structures (even if in a linear representation of it). So the idea that the laws of nature obey linear equations is rather extraordinary, and we can do a lot with it, but maybe it is just an approximation of better equations, probably nonlinear. And if we can obtain a glimpse of what those might be by extending the actual models without compromising fundamental principles that served us well in the past then we should tackle the

problem of the non-linearity in the theory and maybe develop new methods to solve them.

2 Action Principles and Conservation Equations

The variational approach to mechanics states that the conservative dynamics of a point particle moving in a configuration space Q under a potential $V\left(\vec{\mathbf{q}}\right)$ can be described by the use of a Lagrangian $\mathcal{L}(\vec{\mathbf{q}},\ \vec{\mathbf{v}},\ \mathrm{t})$ whose integral

$$S(t,\ \vec{\mathbf{q}}\,;\, t_{\mathrm{o}},\ \vec{\mathbf{q}}_{o}) = \int_{t_{\mathrm{o}}}^{t} \mathcal{L}\left(\tau,\, \vec{\mathbf{q}}(\tau),\, \dot{\vec{\mathbf{q}}}(\tau)\right) d\tau\,, \tag{2}$$

called the Action, is (an) extreme precisely along the curve that describes the 'classical path' $\vec{\mathbf{q}}_{c}(\tau) = \gamma(\tau)$ from $\vec{\mathbf{q}}_{\mathrm{o}}$ to $\vec{\mathbf{q}}$.[7][8] The Action S determines the dynamics since it defines a partial differential equation, the Hamilton-Jacobi equation, which results from imposing such variational conditions of extreme over all possible paths $\gamma(\tau)$,

$$\delta_{\gamma} S = 0 \qquad \longrightarrow \qquad \frac{\partial S}{\partial t} + \frac{1}{2}\left(\nabla S\right)^{2} + V = 0\,, \tag{3}$$

From a Canonical Transformation point of view, this action is precisely the generating function $F(t,\ \vec{\mathbf{q}}\,;\, t_{\mathrm{o}},\, \vec{\mathbf{q}}_{\mathrm{o}})$ that maps every initial point in configuration space Q at time t_{o} to the position it would occupy at time t under the dynamics defined by the potential $V(\vec{\mathbf{q}})$.[9] The relation between this equation (3) and the classical particle equation of motion is visible when we use the notation $\vec{\mathbf{p}} = \nabla S$ and identify this, when restricted to the classical trajectory, with the classical particle momentum $\vec{\mathbf{q}}_{c}(\tau) = \gamma(\tau)$. In this case, a spatial derivation of the Hamilton-Jacobi equation (3) for a unit-mass particle yields

$$\left(\frac{\partial}{\partial t}\left(\nabla S\right) + \nabla S \cdot \Delta\, S = -\nabla\, V\right)\Big|_{\vec{\mathbf{q}}_{c}} \tag{4}$$

which means

$$\frac{\partial \vec{\mathbf{p}}_{c}}{\partial t} + \vec{\mathbf{p}}_{c} \cdot \nabla \vec{\mathbf{p}}_{c} \equiv \frac{d\vec{\mathbf{p}}_{c}}{dt} = \vec{\mathbf{F}}(\vec{\mathbf{q}}_{c}) \tag{5}$$

where $\vec{\mathbf{F}}(\vec{\mathbf{q}}_{c}) = -\nabla V(\vec{\mathbf{q}}_{c})$.

The variational approach started from an analytical mechanics version of 17th century Geometrical Optics Fermat's and Huygen's principles, where the Eikonal (or Optical Length of the path) $W(\vec{\mathbf{q}},\, \vec{\mathbf{q}}_{\mathrm{o}})$ plays the role of the Action S. According to Fermat, light in an inhomogeneous medium should propagate from

point $\vec{\mathbf{q}}_o$ to point $\vec{\mathbf{q}}$ in the shortest possible time, thus minimizing $W(\vec{\mathbf{q}}, \vec{\mathbf{q}}_o)$, and Huygen's principle states that light should travel along the normal to the level surfaces of $W(\vec{\mathbf{q}}, \vec{\mathbf{q}}_o)$. This "Transversality Condition" in the analytical mechanics context can be expressed in more contemporary terms as the exactness condition for the Cartan one-form $dS = \vec{\mathbf{p}} \cdot d\vec{\mathbf{q}} - \mathcal{H}\, dt$ which reads precisely

$$\vec{\mathbf{p}} = \nabla S \tag{6}$$

$$\mathcal{H}\left(t, \vec{\mathbf{q}}, \nabla S\right) = -\frac{\partial S}{\partial t} \tag{7}$$

the last being the Hamilton-Jacobi equation when we set

$$\mathcal{H}\left(t, \vec{\mathbf{q}}, \vec{\mathbf{p}}\right) = \vec{\mathbf{p}} \cdot \vec{\mathbf{v}} - \mathcal{L}\left(t, \vec{\mathbf{q}}, \vec{\mathbf{v}}\left(t, \vec{\mathbf{q}}, \vec{\mathbf{p}}\right)\right) \tag{8}$$

and $\vec{\mathbf{v}}\left(t, \vec{\mathbf{q}}, \vec{\mathbf{p}}\right)$ is obtained from inverting $\vec{\mathbf{p}} = \partial_{\vec{\mathbf{v}}} \mathcal{L}\left(t, \vec{\mathbf{q}}, \vec{\mathbf{v}}\right)$. A theorem of Jacobi will then guarantee that a solution $S\left(t, \vec{\mathbf{q}}; t_o, \vec{\mathbf{q}}_o\right)$ to this equation (7), where $\vec{\mathbf{q}}_o$ are n arbitrary parameters, will provide the Action (2) for the mechanical motion provided that

$$\det \left| \frac{\partial^2 S}{\partial \vec{\mathbf{q}}\, \partial \vec{\mathbf{q}}_o} \right| \neq 0$$

If we also assume a continuity law for $\rho(\vec{\mathbf{q}}, t) = e^{2\alpha(\vec{\mathbf{Q}}, t)}$ designating some conserved, extensive numeric density flowing with local current density $\vec{\mathbf{J}}(\vec{\mathbf{q}}, t) = \rho(\vec{\mathbf{q}}, t)\, \nabla S$, then we must have in general

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{J}} = 0 \quad \longrightarrow \quad \frac{\partial \rho}{\partial t} + \nabla \rho \cdot \nabla S + \rho\, \Delta\, S = 0 \tag{9}$$

i.e. in terms of α,

$$2\, \frac{\partial \alpha}{\partial t} + 2\, \nabla \alpha \cdot \nabla S + \Delta\, S = 0 \tag{10}$$

Notice that this is a condition on the "extended nature" of the solutions that we seek. In other words, besides specifying an identifier for the classical trajectory between two points (the extremes of S), we state that any "extended properties" of the particle described by the density field $\rho(\vec{\mathbf{q}}, t)$ should be conserved in balance during the motion. Now classical motion is but one of the possible paths in evolution space. There is no reason why (9) should have a solution ρ that is maximal along the classical path, or that on the neighborhood of the classical trajectory the flow should not tend to disperse away. But if such solutions exist, we may then attribute a familiar interpretation to the field ρ which is akin to the so-called coherent states in conventional quantum mechanics. In general however we may not immediately have an interpretation

for these fields, or their maxima. Making them probability distributions is one of an infinite number of options. We will therefore leave this interpretation open until further information specifies it.

3 The Nonlinear Schrödinger Equation

It was shown in previous work[1] that the two equations (3) and (10), together with a definition of $\psi = e^{\alpha + i\,\phi}$, with $\phi = S$, are the real and imaginary parts of a coordinate-rescaled nonlinear Schrödinger equation with quantum potential $Q(\vec{\mathbf{q}}, t) = \frac{\hbar^2}{2m}\frac{\Delta|\psi|}{|\psi|}$,

$$-\frac{\hbar^2}{2m}\Delta\psi + \left(V \;+\; \frac{\hbar^2}{2m}\frac{\Delta|\psi|}{|\psi|}\right)\psi \;=\; i\,\hbar\,\frac{\partial\psi}{\partial t} \tag{11}$$

when we make, in the former, the correspondence $\left\{t,\ \vec{\mathbf{q}}\right\} \rightarrow \left\{\frac{m\,c^2}{\hbar}\,t,\ \frac{m\,c}{\hbar}\,\vec{\mathbf{q}}\right\}$ and $V \rightarrow \frac{1}{m\,c^2}V$ to obtain (11).

Earlier work on stochastic models by Vigier and others[10]$^-$ [2] start from a Quantum Mechanical point of view and look at the generalized Schrödinger equation with a quantum potential

$$-\frac{\hbar^2}{2m}\Delta\psi + \left(V + \frac{\hbar^2}{2m}\,\gamma\,\frac{\Delta|\psi|}{|\psi|}\right)\psi = i\,\hbar\frac{\partial\psi}{\partial t} \tag{12}$$

where $\gamma \leq 1$ is a positive stochastic parameter which, when set to $\gamma = 0$, yields the linear Schrödinger equation.

We now show the relation of the two equations (3) and (10) to this seemingly more general equation (12), but first we should remark that, for all cases except $\gamma = 1$, equation (12) is related to a linear equation of Schrödinger type by a simple scaling of the phase ϕ. Take $\psi_{NL} = e^{\alpha + i\,\phi}$ a solution of the nonlinear equations (12). Then

$$\psi_L(t, \vec{\mathbf{q}}) = e^{\alpha(t,\,\vec{\mathbf{Q}}) \,+\, i\,\frac{1}{\sqrt{1-\gamma}}\,\phi(t,\,\vec{\mathbf{Q}})} \tag{13}$$

is a solution of the linear equation

$$-\tfrac{\hbar^2(1-\gamma)}{2\,m}\,\nabla^2\psi_L + V(t, \vec{\mathbf{q}})\,\psi_L = i\,\hbar\,\sqrt{1-\gamma}\,\partial_t\psi_L \tag{14}$$

which is a simple Schrödinger equation once we scale the coordinate system to substitute $\frac{1}{\sqrt{1-\gamma}}\,t \rightarrow t,\ \frac{1}{\sqrt{1-\gamma}}\,\vec{\mathbf{q}} \rightarrow \vec{\mathbf{q}}$.

It was shown by A. Rica da Silva et al. [1] that the reduction to the linear case is indeed intrinsic to (12) since this equation's Lie algebra of dynamical symmetries changes dimensionality in the limit $\gamma \to 1$. Equation (12) with $\gamma = 1$ is then truly nonlinear and we will show that only this nonlinear case can be considered fully compatible with the variational approach embodied in Fermat and Huygen's principles.

Starting from (12) and setting $\psi(\vec{\mathbf{q}},\ t) = e^{\,\alpha(\vec{\mathbf{Q}},\,t)+i\,\phi(\vec{\mathbf{Q}},\,t)}$ with α, ϕ real, and using the shorthand ∂_t for $\dfrac{\partial}{\partial t}$, then

$$\Delta\psi = \nabla\cdot(\psi\nabla(\alpha + i\,\phi)) = \Big(\Delta\alpha + (\nabla\alpha)^2 - (\nabla\phi)^2 + i\,(\Delta\phi + 2\nabla\alpha\cdot\nabla\phi)\Big)\psi \quad (15)$$

$$\partial_t\psi = (\partial_t\alpha + i\,\partial_t\phi)\psi \quad (16)$$

and from $|\psi(\vec{\mathbf{q}},\,t)| = e^{\,\alpha(\vec{\mathbf{Q}},t)}$ also

$$\Delta|\psi| = (\Delta\alpha + (\nabla\alpha)^2)|\psi| \quad (17)$$

Upon substitution on equation (12) and separating real and imaginary parts we obtain

$$\hbar\,\partial_t\phi = -\frac{\hbar^2}{2m}(\nabla\phi)^2 - V_R + (1-\gamma)\frac{\hbar^2}{2m}(\Delta\alpha + (\nabla\alpha)^2) \quad (18)$$

$$\hbar\,\partial_t\alpha = -\frac{\hbar^2}{2m}(\nabla^2\phi + 2\nabla\alpha\cdot\nabla\phi) + V_I \quad (19)$$

where we have admitted in all generality that $V = V_R + i\,V_I$, i.e the potential could exhibit a dissipative imaginary term. Rescaling coordinates and potential to

$$\left(\frac{m\,c^2}{\hbar}\,t,\ \frac{m\,c}{\hbar}\,\vec{\mathbf{q}},\ \frac{1}{m\,c^2}\,V\right) \to \big(t,\ \vec{\mathbf{q}},\ V\big), \quad (20)$$

these equations (19) will then read simply

$$\partial_t\phi = -\frac{1}{2}(\nabla\phi)^2 + \frac{\gamma-1}{2}\left(\Delta\alpha + (\nabla\alpha)^2\right) - V_R \quad (21)$$

$$\partial_t\alpha = -\frac{1}{2}\Delta\phi - \nabla\alpha\cdot\nabla\phi + V_I \quad (22)$$

Identifying the action with the phase, $S = \phi$, the density $\rho = e^{\,2\,\alpha}$ with $|\psi|^2$, and multiplying of the last equation in (22) by $e^{\,2\,\alpha}$ we obtain

$$\partial_t S + \frac{1}{2}(\nabla S)^2 = \frac{\gamma-1}{2}\,\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} - V_R \quad (23)$$

$$\partial_t\rho + \nabla\cdot(\rho\nabla S) = 2\,\rho\,V_I \quad (24)$$

Equation (24) is just the Continuity equation (10) with a source term proportional to the imaginary part of the complex potential V. As for equation (23), the assumption $\gamma \neq 1$ represents a significant departure from the Hamilton-Jacobi equation (3) and thus compromises the connection to the classical Variational Principles. It has been the approach of Bohm and others to introduce the quantum potential Q in a way to preserve the linearity of the theory and therefore comply with the results of the conventional quantum wisdom to which their theory is mathematically equivalent strictly choosing $\gamma = 0$.

Remaining faithful to the original form of the Hamilton-Jacobi equation requires $\gamma = 1$ and we thus choose to work with a truly nonlinear Schrödinger equation (1), or equivalently with its decomposition in real and imaginary components for $\psi = e^{\alpha + i\phi}$

$$\partial_t \phi + \frac{1}{2}(\nabla\phi)^2 + V_R = 0 \tag{25}$$

$$\partial_t \alpha + \nabla\alpha \cdot \nabla\phi + \frac{1}{2}\Delta\phi = V_I \tag{26}$$

4 Standard Quantization as a Particular Case

We now would like to obtain conditions on the general solutions $\alpha(\vec{\mathbf{q}}, t)$, $\phi(\vec{\mathbf{q}}, t)$ that lead to quantization conditions similar to the ones that appear in the linear theory. Recall that in the linear case special types of states $\psi(\vec{\mathbf{q}}, t)$ are sought that turn the Linear Schrödinger Equation into an eigenvalue problem for complex functions $f_k(\vec{\mathbf{q}})$

$$\left(-\frac{\hbar^2}{2m}\Delta + V(\vec{\mathbf{q}})\right) f_k(\vec{\mathbf{q}}) = \mathcal{E}_k f_k(\vec{\mathbf{q}}) \tag{27}$$

by setting $\psi_k(\vec{\mathbf{q}}, t) = f_k(\vec{\mathbf{q}})\, e^{-\frac{i}{\hbar}\mathcal{E}_k t}$. These states are "stationary" in the sense that they have a well defined energy $\mathcal{E}_k$ and their amplitude $|\psi_k(\vec{\mathbf{q}}, t)|^2 = \rho(\vec{\mathbf{q}})$ is time-independent. Furthermore they define normalized orthogonal bases from which all other states may be reconstructed by linear superposition.

In the case of the NLSE (1), the non-linearity associated with the quantum potential

$$Q = \frac{1}{2}\frac{\Delta|\psi|}{|\psi|} = \frac{1}{2}\left(\Delta\alpha + (\nabla\alpha)^2\right) \tag{28}$$

means that the field interacts with itself and affects the quantum potential it sees. This is not an uncommon situation, although one always resorts to some

kind of artifice to render this self interaction negligible in order to be able to solve the field equations. This is in particular the case of the Born-Infeld-Mie theory of Electrodynamics[12] where variational considerations and covariance requirements on the observables leads to a field theory in a manifold with metric where the sources are themselves functions of the fields. This would provide a seamless integration between fields and charges, and was intended to probe the inner structure of the electron and other particles. Unfortunately the equations prove themselves too difficult to handle, and the resulting classical electrodynamic theory (Maxwell) is an hybrid were the sourceless Mie theory is superimposed on charge distributions that are independent of the fields, thus linearizing the equations. We will not go so far here, but will look for solutions such that the Quantum Potential is time-independent and set to verify

$$Q = \frac{1}{2}\left(\Delta\alpha + (\nabla\alpha)^2\right) = V(\vec{\mathbf{q}}) - \mathcal{E} \tag{29}$$

Notice that this choice of Q implies that the NLSE seems to turns into a linear Schrödinger equation, with a classical potential $\tilde{V}(\vec{\mathbf{q}}) = 2V(\vec{\mathbf{q}}) - \mathcal{E}$

$$-\frac{1}{2}\Delta\psi + \tilde{V}(\vec{\mathbf{q}})\,\psi = i\,\partial_t\psi\,. \tag{30}$$

The equivalence is not complete however because the solutions to equation (30) are to be taken as the subset of solutions of (26) where the initial condition $\alpha(\vec{\mathbf{q}}, 0) = \alpha_{\mathrm{o}}(\vec{\mathbf{q}})$ is one of the solutions of (29). The $\alpha_{\mathrm{o}}(\vec{\mathbf{q}})$ solutions to the quantum potential constraint (29) are in fact the amplitudes for the eigenstates of a conventional Linear Schrödinger Equation when the phase of ψ is chosen to be $\phi(\vec{\mathbf{q}}, t) = \varphi(\vec{\mathbf{q}}) - \mathcal{E}t$. In general however we need not make this imposition on the form of the phase $\phi(\vec{\mathbf{q}}, t)$, contrary to the usual practice. The phase solutions for (30) are obtained from the Hamilton-Jacobi equation (3) and may have different forms which can influence the time evolution of the amplitude solution starting from an eigenstate (i.e. these may no longer be 'stationary').

4.1 Example 1: The Harmonic Oscillator

In the one dimensional Harmonic Oscillator case with $V(x) = \frac{1}{2}\,k\,x^2$, Equation (29) is a Ricatti-type equation which is solved by setting

$$\alpha(x) = \int_0^x \frac{w'(\xi)}{w(\xi)}\,d\xi \tag{31}$$

with which we obtain from (29) a linear ODE equation

$$w''(x) = 2\ (V(x) - \mathcal{E})\ w(x) \tag{32}$$

Setting $\omega^2 = k$, when $\frac{2\mathcal{E}-\omega}{2\,\omega} = n \in \mathbb{N}$ (i.e. $\mathcal{E} = \left(n + \frac{1}{2}\right)\ \omega$) we have the general solution,

$$w(x) = e^{-\frac{x^2\,\omega}{2}}\ \left(A\,H_n(x\,\sqrt{\omega}) + B\ {}_1F_1\left(-\frac{n}{2}, \frac{1}{2}; x^2\,\omega\right)\right) \tag{33}$$

where the H_n are Hermite polynomials, and the ${}_1F_1$ confluent hypergeometric functions. Since

$${}_1F_1\left(-\frac{n}{2}, \frac{1}{2}; x^2\,\omega\right)\ e^{-\frac{x^2\,\omega}{2}}$$

is bounded only for $\frac{n}{2} \in \mathbb{N}$ and is then proportional to

$$(-1)^{\frac{n}{2}} H_n\left(x\,\sqrt{\omega}\right)\ e^{-\frac{x^2\,\omega}{2}},$$

we may then always choose $B = 0$ for $n = 0, 1, 2, \ldots$. Thus

$$\alpha_n(x) = -\frac{\omega}{2}\,x^2 + \log\left(H_n\left(x\,\sqrt{\omega}\right)\right) + c_1 \tag{34}$$

These are the amplitudes for the stationary states of the usual Linear Schrödinger Equation. We can now use the Hamilton-Jacobi equation (3) to find phase solutions. If we do require that $\varphi(x,t) = \phi(x) - \mathcal{E}\,t$ in analogy with common practice, we obtain phases of the form

$$\varphi(x,t) = \pm\ \frac{\sqrt{\mathcal{E} - V(x)}\sqrt{V(x)} + \mathcal{E}\ \tan^{-1}\left(\frac{\sqrt{V(x)}}{\sqrt{\mathcal{E}-V(x)}}\right)}{\omega} - \mathcal{E}\,t \tag{35}$$

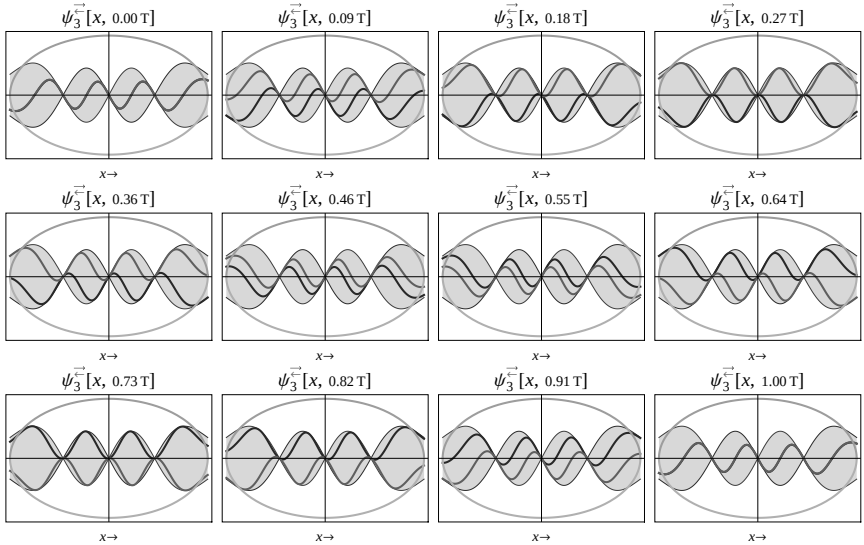
or

$$\varphi(x,t) = \pm\frac{x}{2}\sqrt{2\,\mathcal{E} - x^2\omega^2} - \mathcal{E}\left(t \mp \frac{1}{\omega}\ \tan^{-1}\left(\frac{x\,\omega}{\sqrt{2\,\mathcal{E} - x^2\omega^2}}\right)\right) \tag{36}$$

Restriction to $\mathcal{E}_n = (n + \frac{1}{2})\,\omega$ values sets the possible phases $\varphi_n(x,\ t)$ for the eigenstates of the harmonic oscillator. Contrarily to the usual Quantum Mechanical view, these states are not "stationary" in the sense that they have zero momenta, since here

$$p_n(x) = \nabla\varphi_n(x,\ t) = \pm\sqrt{\mathcal{E}_n - V(x)} \tag{37}$$

There is however a degeneracy in each level with states of opposing momenta. Still, the envelope of the propagating state remains static, as well as the nodes of each eigenstate, just as in the standard case.



The continuity equation (10) can then be used to determine the evolution in time of a given state. In the particular case of the eigenstates $\psi_n = \sqrt{\rho_n} e^{i\varphi_n}$ it gives the additional information about the imaginary potential V_I that is needed to implement such standard eigenstates

$$V_{In} = \frac{1}{2\rho_n} \nabla \cdot \left(\rho_n \nabla \varphi_n\right) \tag{38}$$

These will exhibit source-sink singularities precisely at the nodes location, as needed to produce the necessary wave behavior.

4.2 Example 2: The Coulomb Potential

We will now look for solutions of the Quantum Potential condition with $V(r) = \frac{k}{r}$ the Coulomb Potential, for instance the case of the Hydrogen atom.

$$Q = \frac{1}{2} \frac{\Delta|\psi|}{|\psi|} = V(r) - \mathcal{E} \tag{39}$$

using spherical coordinates $\{r, \theta, \varphi\}$. In these coordinates the Laplacian of a function $f(r, \theta, \varphi)$ is

$$\frac{1}{r^2}\partial_r\left(r^2\partial_r f\right) + \frac{1}{r^2}\left(\frac{1}{\sin(\theta)}\partial_\theta\left(\sin(\theta)\partial_\theta\, f\right) + \frac{1}{\sin(\theta)^2}\partial_{\varphi,\varphi} f\right) = \\ = \frac{1}{r^2}\partial_r\left(r^2\partial_r f\right) - \frac{1}{r^2}\mathbb{L}^2 f \tag{40}$$

Notice that here $|\psi| = \pm\sqrt{|\psi|^2}$, since it is $|\psi|^2$ that has physical significance: hence the sign choice for $|\psi|$ is unimportant in the combination $\frac{\Delta|\psi|}{|\psi|}$ and we may relax the positivity condition on $|\psi|$. Actually, we might argue in favor of a description of the wave function as $\psi = \mathcal{A}e^{i\mathcal{S}}$, with $\mathcal{A}$ and $\mathcal{S}$ real-valued functions, and then the quantum potential should read $\mathcal{Q} = \frac{1}{2}\frac{\Delta\mathcal{A}}{\mathcal{A}}$. We restrict the search to solutions that have separable coordinates in the form $\mathcal{A} = \mathcal{R}(r)\mathcal{Y}(\theta,\varphi)$. Setting $\mathcal{E} = -\frac{k^2}{2n^2}$ we obtain as solution the standard hydrogen atom quantized solutions, with $\mathcal{Y}(\theta,\varphi) = \mathcal{R}e(Y_l^m(\theta,\varphi))$[1] and

$$\mathcal{R}(r) = \frac{2\,k^{\frac{3}{2}}}{n^2}\sqrt{\frac{(n-l-1)!}{(n+l)!}}\;e^{-\frac{kr}{n}}\left(\frac{2\,k\,r}{n}\right)^{l} L_{n-l-1}^{2l+1}\left(\frac{2\,k\,r}{n}\right) \tag{44}$$

depending on the generalized Laguerre polynomials $L_{n_r}^{2l+1}(r)$, with $n_r = n-l-1$ the radial quantum number, n the principal quantum number, $l = 0, 1, \ldots, n-1$ the azimuthal quantum number and $m = -l, \ldots, l$ the magnetic quantum number.

When we restrict further to $\mathcal{R}(r) = e^{\rho(r)}$ and $\mathcal{Y}(\theta,\varphi) = e^{\upsilon(\theta,\varphi)}$ we may deduce the same solutions from the amplitude equation $\frac{1}{2}(\Delta\alpha + (\nabla\alpha)^2) = V(r) - \mathcal{E}$ with $\alpha = \rho(r) + \upsilon(\theta,\varphi)$.

$$-\left(\frac{l\,(l+1)}{r^2}\right) + \rho'(r)^2 + \frac{2\,\rho'(r) + r\,\rho''(r)}{r} = -2\,\mathcal{E} + \frac{2\,k}{r} \tag{45}$$

The phases for this motion should now be determined independently from the

[1] The Angular Momentum operator $\mathbb{L}$ in Spherical Coordinates $\{r, \theta, \varphi\}$ is

$$\begin{aligned}
\mathbb{L}_x &= \;\; i\left(\sin(\varphi)\partial_\theta + \cot(\theta)\cos(\varphi)\partial_\varphi\right)\\
\mathbb{L}_y &= -i\left(\cos(\varphi)\partial_\theta - \cot(\theta)\sin(\varphi)\partial_\varphi\right)\\
\mathbb{L}_z &= -i\,\partial_\varphi\\
\mathbb{L}^2 &= -\left(\frac{1}{\sin(\theta)}\partial_\theta\left(\sin(\theta)\partial_\theta\right) + \frac{1}{\sin(\theta)^2}\partial^2_{\varphi,\varphi}\right)
\end{aligned} \tag{41}$$

and has eigenfunctions $Y_l^m(\theta,\varphi)$ which verify

$$\begin{aligned}
\mathbb{L}^2 Y_l^m(\theta,\varphi) &= l(l+1)Y_l^m(\theta,\varphi)\\
\mathbb{L}_z Y_l^m(\theta,\varphi) &= m\,Y_l^m(\theta,\varphi)
\end{aligned} \tag{42}$$

The Spherical Harmonic Functions $Y_l^m(\theta,\varphi)$ are related to the Legendre Polynomials $P_l^m(\cos(\theta))$ by

$$Y_l^m(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\,\frac{\Gamma(l-m+1)}{\Gamma(l+m+1)}}\,P_l^m(\cos(\theta))\,e^{i\,m\,\varphi} \tag{43}$$

Hamilton-Jacobi equation in spherical coordinates:

$$\partial_t \phi + \frac{1}{2}\left(\left(\partial_r \phi\right)^2 + \frac{1}{r^2}\left(\csc(\theta)^2\left(\partial_\varphi \phi\right)^2 + \left(\partial_\theta \phi\right)^2\right)\right) - \frac{k}{r} = 0 \tag{46}$$

If we look at phase solutions with no θ or φ dependence, we find in the standard case of choosing the time dependence as $\mathcal{E}t = -\frac{k^2}{2n^2}t$ that a solution is

$$\phi(r,t) = \frac{kr}{n}\sqrt{1+\frac{2n^2}{kr}} + n\log\left(4kr\left(1+\sqrt{1+\frac{2n^2}{kr}}\right)^2\right) - \frac{k^2}{2n^2}t \tag{47}$$

while the bound states phase is

$$\phi(r,t) = -\frac{kr}{n}\sqrt{\frac{2n^2}{kr}-1} - 2n\tan^{-1}\left(\frac{1}{\sqrt{\frac{2n^2}{kr}-1}}\right) + \frac{k^2}{2n^2}t \tag{48}$$

We could then proceed to find the time-evolution for solution with these phases using (26) and the appropriate initial states in a manner similar to the previous example. Again one finds that stationary state solutions corresponding to the standard energy and angular momentum quantum numbers[13] have momenta according to the ansatz $\vec{\mathbf{p}} = \nabla\phi$.

Notice that in both examples quantization arises from solvability conditions on the amplitude solutions $\alpha(\vec{\mathbf{q}})$ of quantum potential equation (29). The question now arises as to whether quantization is strictly connected to this restriction on the quantum potential, or can there be other types of quantization that do not require these conditions? Additionally, given the equivalence of the NLSE with classical dynamical equations, can there be quantization of classical motion? In astrophysics, for example, there is a remarkable rule called Bode's Law which finds astounding regularities in the sequence of orbital radii for planetary systems, and this ranging from the solar scale to the satellite systems of smaller planets. There is as yet no formal explanation for these regularities.

5 The NLSE in Two Dimensions: the Kepler Problem

We will now explore the Nonlinear Schrödinger equation solutions in a general setting for the classical Kepler-type problem $V = -\frac{k}{r}$. We can use cylindrical coordinates (r,θ) in two dimensions, after the appropriate reduction to center of mass coordinates. For a system of masses m_1, m_2 we will use the notation

$\mu = \frac{m_1 m_2}{m_1+m_2}$ for the reduced mass. For the gravitational case $k = Gm_1m_2 = \kappa\mu$. This means that the amplitude (10) and phase equations (3) will be rewritten as

$$\partial_t \alpha + \frac{1}{2\mu}\left(\nabla^2\phi + 2\,\nabla\alpha\cdot\nabla\phi\right) = 0 \tag{49}$$

$$\partial_t \phi + \frac{1}{2\mu}(\nabla\phi)^2 + V(r) = 0 \tag{50}$$

i.e.

$$\partial_t \alpha + \frac{1}{2\mu}\left(\partial^2_{r,r}\,\phi + \partial^2_{\theta,\theta}\,\phi + \frac{1}{r}\partial_r\,\phi\right) + \frac{1}{\mu}\left(\partial_r\alpha\,\partial_r\,\phi + \frac{1}{r^2}\,\partial_\theta\,\alpha\,\partial_\theta\,\phi\right) = 0 \tag{51}$$

$$\partial_t \phi + \frac{1}{2\mu}\left(\left(\partial_r\phi\right)^2 + \left(\frac{1}{r}\partial_\theta\,\phi\right)^2\right) - \frac{k}{r} = 0 \tag{52}$$

Since by definition we must have $\partial_r\phi = p_r$ along the classical path, we may equate this to the radial momenta which must obey the identity

$$\mathcal{E} = \frac{1}{2\mu}\left(\partial_r\phi\right)^2 + \frac{1}{2\mu}\frac{{L_o}^2}{r^2} - \frac{k}{r} \tag{53}$$

From this we get

$$p_r = \sqrt{2\,\mu\left(\mathcal{E} + \frac{k}{r}\right) - \frac{{L_o}^2}{r^2}} \tag{54}$$

We therefore guess that $\phi(r,\theta,t)$ must bare some similarity with the function obtained from

$$\phi(r,\theta,t) = \int p_r\,dr + L_o\,\theta\, - \mathcal{E}\,t \tag{55}$$

where the θ and t dependency were deduced as consequence of separability condition for the action

$$\phi(r,\theta,t) = \phi_r(r,t) + \phi(\theta,t) \tag{56}$$

Using the notation

$$\xi(r) = \frac{r\,p_r}{L_o} = \sqrt{\frac{2\,\mu\,\mathcal{E}}{{L_o}^2}r^2 + \frac{2\,\mu\,k}{{L_o}^2}r - 1} = \frac{\sqrt{r_{\max}-r}\sqrt{r-r_{\min}}}{\sqrt{r_{\max}}\sqrt{r_{\min}}} \tag{57}$$

we actually get

$$\begin{aligned}\phi(r,\theta,t) = L_o\,\xi(r) + \tfrac{k\sqrt{2\mu\mathcal{E}}}{2\mathcal{E}}\,\log\left(\tfrac{\sqrt{2\mu\mathcal{E}}}{\mathcal{E}L_o}(2\,\mathcal{E}\,r + k) + 2\,\xi(r)\right) + \\ + L_o\left(\theta + \tan^{-1}\left[\tfrac{{L_o}^2 - r\,k\,\mu}{{L_o}^2\,\xi(r)}\right]\right) - \mathcal{E}\,t\end{aligned} \tag{58}$$

$$\alpha(r,\theta,t) = \tfrac{1}{2}\,\log[\xi(r)]+ \\ +\mathcal{F}\left[\theta + \tan^{-1}\left[\frac{{L_o}^2 - r\,k\,\mu}{{L_o}^2\,\xi(r)}\right],\; t + \frac{k\,\mu^2}{(2\,\mu\,\mathcal{E})^{3/2}}\log\left[\frac{\sqrt{2\,\mu\,\mathcal{E}}\,(2\,r\,\mathcal{E}+k)}{\mathcal{E}\,L_o} + 2\,\xi(r)\right] - \frac{L_o}{2\,\mathcal{E}}\,\xi(r)\right] \tag{59}$$

where $\mathcal{F}$, is an arbitrary function.

On the other hand, we can write the following first integrals defining the classical motion [14]:

$$\beta_1 = \partial_{\mathcal{E}}\,\phi = -t + \int\left(\partial_{\mathcal{E}}\sqrt{2\mu\left(\mathcal{E} + \tfrac{k}{r}\right) - \tfrac{L^2}{r^2}}\right)dr \tag{60}$$

$$\beta_2 = \partial_L\phi \;=\; \theta + \int\left(\partial_L\sqrt{2\mu\left(\mathcal{E} + \tfrac{k}{r}\right) - \tfrac{L^2}{r^2}}\right)dr \tag{61}$$

which means that along the classical orbit solutions $\overline{r}(t),\ \overline{\theta}(t)$ with initial angular momentum $L = L_o$ we have

$$\beta_1 = -t - \frac{k\,\mu^2}{(2\mu\mathcal{E})^{3/2}}\,\log\left[\frac{\sqrt{2\mu\mathcal{E}}\,(2\mathcal{E}\,\overline{r}(t)+k)}{\mathcal{E}\,L_o} + 2\,\xi\,(\overline{r}(t))\right] + \frac{L_o}{2\,\mathcal{E}}\,\xi\,(\overline{r}(t)) \tag{62}$$

$$\beta_2 = \overline{\theta}(t) + \tan^{-1}\left(\frac{{L_o}^2 - \overline{r}(t)\,k\,\mu}{{L_o}^2\,\xi(\overline{r}(t))}\right) \tag{63}$$

which by the way is the same as the classical trajectory equation ahead (71). [2]

It is therefore tempting to define two new functions

$$\beta_1(r,\,t) \;\;= -t - \frac{k\,\mu^2}{(2\mu\mathcal{E})^{3/2}}\,\log\left[\frac{\sqrt{2\mu\mathcal{E}}(2\mathcal{E}\,r+k)}{\mathcal{E}\,L_o} + 2\,\xi(r)\right] + \frac{L_o}{2\,\mathcal{E}}\,\xi(r) \tag{64}$$

$$\beta_2(r,\,\theta) \;\;= \theta + \tan^{-1}\left(\frac{{L_o}^2 - r\,k\,\mu}{{L_o}^2\,\xi(r)}\right) \tag{65}$$

[2] In this form equation (63) is troublesome since θ changes in the range $[0,\ 2\pi]$, whereas $\tan^{-1}$ gives only values between $\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$. In calculations we must therefore implement a version in which we replace θ by $\sin^{-1}\left(\sin\left(\theta - \gamma + \frac{\pi}{2}\right)\right)$. This will now mean that the new values

$$\beta_2 = \sin^{-1}\left(\sin\left(\theta - \gamma + \frac{\pi}{2}\right)\right) + \tan^{-1}\left(\frac{r\,k\,\mu - L^2}{L^2\,\xi(r)}\right) = 0$$

or

$$\beta_2 = \sin^{-1}\left(\sin\left(\theta - \gamma + \frac{\pi}{2}\right)\right) - \sin^{-1}\left(\frac{1}{e} - \frac{s}{r}\right) = 0$$

should then be used.

with which we can express the general solutions (58), (59) as

$$\phi(r,\theta,t) = L\left(2\xi(r)+\beta_2(r,\theta)\right) - \mathcal{E}\left(3t+2\beta_1(r,t)\right) \tag{66}$$

$$\alpha(r,\theta,t) = \frac{1}{2}\log\left[\xi(r)\right] + \mathcal{F}\left(\beta_2(r,\,\theta),\beta_1(r,\,t)\right) \tag{67}$$

These are therefore general solutions to the NLSE with the assumption that the phase must be of the form (55). This will serve as a model in the construction of the particular quantized orbit solutions in the following.

5.1 Particular Solutions and Quantization

We start by recalling the formal solution of the classical equations of motion for the Kepler potential $V(r) = -\frac{k}{r}$ with the purpose of obtaining a solution of the NLSE involving these classical motions as parametrized by $\hat{\vartheta}(r)$ and $\hat{\tau}(r)$. For that, one must make use of several simplifying arguments. Notice that as they stand, these solutions are only valid within a half of the motion, either way between r_{min} and r_{max}. We will try to use Pars parametrization of r to achieve the full motion.[15] It should be noticed however that we may choose to parametrize θ periodically in $\theta \in [0, 2\pi]$ or indefinitely $\theta \in \mathbb{R}$, but must then do the same for our classical motion $\hat{\vartheta}(r)$, because in the difference $\theta - \hat{\vartheta}(r)$ it matters.

5.1.1 The Classical Trajectory

According to Newton's equations of motion for the Kepler potential $V(r) = -\dfrac{k}{r}$ in the reduced mass system

$$\frac{d^2\vec{\mathbf{r}}}{dt^2} = -\frac{k}{\mu\, r^3}\vec{\mathbf{r}} \tag{68}$$

(with $\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$, $\mu = \frac{m_1 m_2}{m_1+m_2}$ and $k = G m_1 m_2$), given the initial point coordinates $\{r_o,\, \vartheta_o\}$ and initial Energy-Angular Momenta $\{\mathcal{E},\, L\}$, we are able to define an orbit characterized by its eccentricity e and distance s from a focus F to a directrix,

$$r(\vartheta) = \left(\frac{-1+e\,\cos(\vartheta_o-\beta_2)}{-1+e\,\cos(\vartheta-\beta_2)}\right) r_o \qquad \text{where} \qquad \begin{cases} e = \dfrac{r_o}{s+r_o\,\cos(\vartheta_o-\beta_2)} \\ s = \dfrac{r_o}{e}\left(e\,\cos(\vartheta_o-\beta_2)-1\right) \end{cases} \tag{69}$$

In (69) the constant β_2 defines the angle of inclination of the major axis, and the angle $\vartheta - \beta_2$ is Kepler's *True Anomaly*. It can be shown that the following relations hold:

$$e = \sqrt{1 + \frac{2\mathcal{E}L^2}{k^2\mu}} \quad ; \quad s = \frac{L^2}{e\,k\,\mu} \quad ; \quad \mathcal{E} = \frac{(e^2-1)\,k}{2\,e\,s} \tag{70}$$

Conversely, we can express

$$r(\vartheta) = \frac{s\,e}{1 - e\,\cos(\vartheta - \beta_2)} \tag{71}$$

from which it follows that r will have maxima and minima (in the case $e < 1$) or just minima ($e \geq 1$)

$$r_{min} = \frac{s\,e}{1+e} \quad ; \quad r_{max} = \frac{s\,e}{1-e} \tag{72}$$

For $e \in [0,\,1[$ the conic is an ellipse with semi-axis

$$a = -\frac{k}{2\,\mathcal{E}} = \frac{e\,s}{1-e^2} \quad \text{and} \quad b = a\sqrt{1-e^2}$$

and focal distance $s = a\,e$ to the center. The apogee and perigee radius are $r_a = a(1-e) = r_{min}$ and $r_p = a(1+e) = r_{max}$. If $e = 1$ the conic is a parabola and for all $e > 1$ we have hyperbolas with asymptote angle $\theta_\infty = \cos^{-1}(-\frac{1}{e})$. We make the option to express quantities in terms of the coefficients a and b, which are the lengths of the semi-major and minor axis for closed orbit solutions. Inversion of the parametric $r = r(\vartheta)$ equation yields

$$\hat{\vartheta}(r) = \pm\cos^{-1}\left(\pm\frac{r\,a - b^2}{r\sqrt{a^2-b^2}}\right) \tag{73}$$

which can be set into different formulas for different ϑ domains.

The derivation of a direct relation between $\hat{\vartheta}$ and r has an ambiguity because two possible values of $\hat{\vartheta}$ can occur for the same r. The choice

$$\hat{\vartheta}(r) = \cos^{-1}\left(\frac{b^2 - r\,a}{r\sqrt{a^2-b^2}}\right) \tag{74}$$

for $\hat{\vartheta} \in [0,\,\pi]$ must be complemented with

$$\hat{\vartheta}(r) = \cos^{-1}\left(-\frac{b^2 - r\,a}{r\sqrt{a^2-b^2}}\right) + \pi \tag{75}$$

for $\hat{\vartheta} \in [\pi,\,2\pi]$. The motion will then occur from $r_{\min} \to r_{\max} \to r_{\min}$ in a continuum matter. (When we use Pars parametrization $r \to \hat{r}(\rho) = a\,(1 + e\,\cos(\rho))$ we can associate to each of these functions a full orbit (see ahead).)

5.2 Quantized Phase Solution

We are thus looking for solutions of the Hamilton-Jacobi equation in cylindrical coordinates

$$\partial_t \phi + \frac{1}{2}(\nabla\phi)^2 - \frac{k}{r} = \partial_t \phi + \frac{1}{2}\left(\partial_r \phi\right)^2 + \frac{1}{2\,r^2}\left(\partial_\theta \phi\right)^2 - \frac{k}{r} = 0 \tag{76}$$

of the form $\phi(r,\theta,t) = -\mathcal{E}t + \mathcal{L}\theta + \mathcal{F}(r)$, where $\mathcal{L} = \frac{b\sqrt{k}}{\sqrt{a}}$ and $\mathcal{E} = -\frac{k}{2a}$.

In the range $\theta \in [0,\pi]$ the solution of the Hamilton-Jacobi equation (76) is

$$\phi_1(r,\theta,t) = 2\zeta(r) + \frac{b\sqrt{k}}{\sqrt{a}}\left(\theta - \hat{\vartheta}_1(r)\right) + \frac{k}{2a}\left(t - 2\hat{\tau}_1(r)\right) \tag{77}$$

with the functions

$$\begin{aligned}
\hat{\vartheta}_1(r) &= \cos^{-1}\left(\frac{b^2 - ra}{r\sqrt{a^2 - b^2}}\right) \\
\hat{\tau}_1(r) &= \frac{a^{3/2}}{\sqrt{k}}\,\tan^{-1}\left(\frac{(a-r)\,\sqrt{k}}{\sqrt{a}\,\zeta(r)}\right) + \frac{a}{k}\,\zeta(r) + \frac{\pi\,a^{3/2}}{2\sqrt{k}} \\
\zeta(r) &= \sqrt{\frac{k}{a}\left(2\,r\,a - r^2 - b^2\right)} = \sqrt{2\,k\,\frac{\left(r_{\max} - r\right)\left(r - r_{\min}\right)}{r_{\max} + r_{\min}}}
\end{aligned} \tag{78}$$

In the range $\theta \in [\pi, 2\pi]$ the phase solution to (76) is

$$\phi_2(r,\theta,t) = -2\zeta(r) + \frac{b\,\sqrt{k}}{\sqrt{a}}\left(\theta - \hat{\vartheta}_2(r)\right) + \frac{k}{2a}\left(t - 2\hat{\tau}_2(r)\right) \tag{79}$$

where now, setting $T = 2\pi a^{3/2}\sqrt{\frac{\mu}{k}}$, the auxiliary functions are $\zeta(r)$ as above and

$$\begin{aligned}
\hat{\vartheta}_2(r) &= \cos^{-1}\left(\frac{ra - b^2}{r\sqrt{a^2 - b^2}}\right) + \pi \\
\hat{\tau}_2(r) &= T - \hat{\tau}_1(r)
\end{aligned} \tag{80}$$

Requiring that along the classical path $\theta = \hat{\vartheta}(r)$ both parts of the orbit meet at $r_{\max}$ (where $\zeta(r_{\max}) = 0$) means imposing the condition that the two phase parametrizations differ by a multiple of 2π

$$\phi_2(r_{\max},\pi,t) - \phi_1(r_{\max},\pi,t) = 2\,\pi\,\sqrt{a\,k} = 2\,n\,\pi \tag{81}$$

so the quantization condition is

$$a = \frac{n^2}{k} \tag{82}$$

We can check that along the classical path the classical momenta does coincide with the gradient of the phase as expected:

$$\nabla\phi\left(r, \hat{\vartheta}(r), \hat{\tau}(r)\right) = -\frac{\zeta(r)}{r}\,\vec{\mathbf{e}}_r + \frac{b\sqrt{k}}{r\sqrt{a}}\,\vec{\mathbf{e}}_\theta = \dot{r}\,\vec{\mathbf{e}}_r + r\,\dot{\vartheta}\,\vec{\mathbf{e}}_\theta \equiv \vec{p}_{cl} \tag{83}$$

since from the classical equations $\dot{r} = \sqrt{\frac{e\,k}{s}}\sin\left(\hat{\vartheta}(r)\right) = -\frac{\zeta(r)}{r}$ and $r\,\dot{\vartheta} = \frac{\mathcal{L}}{r} = \frac{b\sqrt{k}}{r\sqrt{a}}$.

Besides imposing a condition on the phase values along the classical paths, we must also require that, for this periodic motion, the phase returns overall to the same values after one orbit is complete. This matching will provide the final quantization of the phase but in order to avoid the multiplicity of solutions due to the r-parametrization limitations, we will use instead [15]

$$\hat{r}(\rho) = a(1 \pm e\cos(\rho)) \tag{84}$$

where the angular parameter $\rho \in [0,\ 2\pi]$. This will then cover the parametrization of the closed direct and retrograde orbits according to the sign in (84). A solution is thus

$$\phi(\rho, \theta, t) = \frac{k}{2\,a}\,t\ + \sqrt{a}\,\sqrt{k}\ (\rho\ - e\sin(\rho)) + \frac{b\,\sqrt{k}}{\sqrt{a}}\left(\theta - \hat{\vartheta}(\rho)\right) \tag{85}$$

or

$$\phi(\rho, \theta, t) = \frac{k}{2a}(t - 2\hat{\tau}(\rho)) + \frac{b\sqrt{k}}{\sqrt{a}}(\theta - \hat{\vartheta}(\rho)) \tag{86}$$

where, for $\theta \in [0,\ +\infty]$,

$$\hat{\vartheta}(\rho) = 2\tan^{-1}\left(\frac{\sqrt{1-e}}{\sqrt{1+e}}\,\tan\left(\frac{\rho}{2}\right)\right) + \pi\left(1 - sgn(\pi - \rho \bmod (2\pi)) + 2\lfloor\frac{\rho}{2\pi}\rfloor\right) \tag{87}$$

(or drop the $2\lfloor\frac{\rho}{2\pi}\rfloor$ term if you just want $\theta \in [0,\ 2\pi]$) and

$$\hat{\tau}(\rho) = \frac{a^{3/2}}{\sqrt{k}}\,(\rho - e\ \sin(\rho)) \tag{88}$$

Given this phase, and in order that we have closed orbits and closed phase surfaces we require that, for any ρ and t, we must have

$$\phi(\rho,\ 0,\ t) - \phi(\rho,\ 2\,\pi,\ t) = 2\,l\,\pi \quad \Longrightarrow \quad k = \frac{a}{b^2}\,l^2 \tag{89}$$

But from (82) we must then have a relation that specifies

$$e = \frac{\sqrt{n-l}\sqrt{n+l}}{n} \tag{90}$$

in which case we have the quantization

$$\begin{aligned}
&\mathcal{E} = -\frac{k^2}{2n^2} && ; && \mathcal{L} = l \\
&r_{\min} = \frac{n^2}{k} - \frac{n\sqrt{n-l}\sqrt{n+l}}{k} && ; && r_{\max} = \frac{n^2}{k} + \frac{n\sqrt{n-l}\sqrt{n+l}}{k} \\
&a = \frac{n^2}{k} && ; && b = \frac{n\,l}{k} \\
&e = \frac{\sqrt{n-l}\sqrt{n+l}}{n} && ; && s = \frac{n l^2}{k\sqrt{n-l}\sqrt{n+l}}
\end{aligned} \tag{91}$$

The resulting evaluation of the amplitude equation

$$\partial_t\,\alpha + \partial_r\,\alpha\,\partial_r\,\phi + \frac{1}{r^2}\partial_\theta\,\alpha\,\partial_\theta\,\phi + \frac{1}{2r^2}\left(\partial_{\theta,\theta}\,\phi + r\,\partial_r\,\phi + r^2\,\partial_{r,r}\,\phi\right) = 0 \tag{92}$$

can now be done if we express equation (92) in terms of the new coordinates $\{\rho, \theta, t\}$, in which case we can substitute in the phase $\phi(\rho, \theta, t)$ as determined above. Using the composition law $\partial_\rho f(r(\rho)) = \partial_r f(r)\partial_\rho r(\rho)$ we obtain the derivative transforms

$$\begin{aligned}
\partial_r f(r) &= \frac{\partial_\rho f(r(\rho))}{\partial_\rho r(\rho)} \\
\partial^2_{r,r} f(r) &= \frac{1}{\left(\partial_\rho r(\rho)\right)^2}\,\partial^2_{\rho,\rho} f(r(\rho)) - \frac{\partial^2_{\rho,\rho} r(\rho)}{\left(\partial_\rho r(\rho)\right)^3}\,\partial_\rho f(r(\rho))
\end{aligned} \tag{93}$$

with which the amplitude equation reads, after simplification,

$$\begin{aligned}
\frac{a^{3/2}}{\sqrt{k}}\,\partial_t\,\alpha - \frac{\left(2 - e^2\left(2 - \sin^2(\rho)\right)\right)\csc^2(\rho)}{e^2(e\cos(\rho)+1)}\,\partial_\rho\,\alpha + \frac{\sqrt{1-e^2}}{(e\cos(\rho)+1)^2}\,\partial_\theta\,\alpha = \\
= -\frac{\left(2 - e^2\left(2 + \sin^2(\rho)\right)\right)\cot(\rho)\csc^2(\rho)}{2e^2(e\cos(\rho)+1)}
\end{aligned} \tag{94}$$

This has the complete solution, with $\mathcal{F}$ an arbitrary function,

$$\alpha(\rho,\theta,t)=\frac{1}{4}\log\left(\left(-\frac{\sin(\rho)}{2\left(2-e^2\left(2-\sin^2(\rho)\right)\right)}\right)^2\right)+ \\ +\mathcal{F}\left(t-\hat{\tau}(\rho)+\tau_1(\rho)+\frac{2\,a^{3/2}}{\sqrt{k}}\,\rho\,,\ \theta-\hat{\vartheta}(\rho)+\vartheta_1(\rho)\right) \quad (95)$$

with

$$\tau_1(\rho)=\frac{a^{3/2}(1-e)^{3/2}\sqrt{e+1}}{\sqrt{2-e^2}\sqrt{k}}\left(\frac{\left(2+e-\sqrt{2-e^2}\right)\tan^{-1}\left(\frac{\sqrt{1-e^2}\tan\left(\frac{\rho}{2}\right)}{\sqrt{1+e\sqrt{2-e^2}}}\right)}{\sqrt{1+e\sqrt{2-e^2}}}- \right. \\ \left. -\frac{\left(2+e+\sqrt{2-e^2}\right)\tan^{-1}\left(\frac{\sqrt{1-e^2}\tan\left(\frac{\rho}{2}\right)}{\sqrt{1-e\sqrt{2-e^2}}}\right)}{\sqrt{1-e\sqrt{2-e^2}}}\right) \quad (96)$$

$$\vartheta_1(\rho)=\frac{e+1}{\sqrt{2-e^2}}\left(\frac{\left(2-e+\sqrt{2-e^2}\right)\tan^{-1}\left(\frac{\sqrt{1-e^2}\tan\left(\frac{\rho}{2}\right)}{\sqrt{1+e\sqrt{2-e^2}}}\right)}{\sqrt{1+e\sqrt{2-e^2}}}- \right. \\ \left. -\frac{\left(2-e-\sqrt{2-e^2}\right)\tan^{-1}\left(\frac{\sqrt{1-e^2}\tan\left(\frac{\rho}{2}\right)}{\sqrt{1-e\sqrt{2-e^2}}}\right)}{\sqrt{1-e\sqrt{2-e^2}}}\right) \quad (97)$$

The choice for $\mathcal{F}$ depends obviously on the initial conditions imposed on the shape of the wave.

6 Motion in 1D Under a Constant Potential

We will now look for solutions of the Hamilton-Jacobi equation other than the standard $S=\phi(\vec{\mathbf{x}})-\mathcal{E}t$. We first assume the case where $\mathcal{V}_R=\mathcal{V}$ constant, and $\mathcal{V}_I=0$. In this case the phase and amplitude equations are

$$\begin{cases} \partial_t\phi+\frac{1}{2}(\nabla\phi)^2+\mathcal{V} & =0 \\ \partial_t\alpha+\frac{1}{2}\Delta\phi+\nabla\alpha\cdot\nabla\phi & =0 \end{cases} \quad (98)$$

and furthermore we may reduce the phase equation to the free case by the simple transformation

$$\varphi(t,x) = \phi(t,x) + \mathcal{V}\left(t + t_0\right) \tag{99}$$

in which case we are left to find solutions to the free Hamilton-Jacobi equation

$$\partial_t \varphi + \frac{1}{2}(\nabla\varphi)^2 = 0 \tag{100}$$

6.1 Similarity Method for the Hamilton-Jacobi Equation

We recognize that similarity solutions exist for this particular equation, two of which are the linear and gaussian phase solutions we will explore ahead. In fact, if we introduce the one-parameter group of (similarity) transformations

$$\begin{cases} \varphi &= \lambda\ \tilde{\varphi} \\ t &= \lambda^m \tilde{t} \\ x &= \lambda^n \tilde{x} \end{cases} \tag{101}$$

then (100) will read in these new coordinates

$$\lambda^{2-2n}\left(\lambda^{2n-m-1}\,\partial_{\tilde{t}}\tilde{\varphi} + \frac{1}{2}(\partial_{\tilde{x}}\tilde{\varphi})^2\right) = 0 \tag{102}$$

This means that as long as $m = 2n - 1$, equation (100) is invariant under the transformations (101), and thus its solutions also should not display any dependence on λ under the same transformations. We therefore introduce a new variable

$$\xi = t\,x^{-2+\frac{1}{n}} \tag{103}$$

and look for solutions of the form

$$\varphi_n(t,\ x) = \frac{x^2}{t}\,\varpi(t\,x^{-2+\frac{1}{n}}) \tag{104}$$

Introducing this into (100) we aim at obtaining a ODE depending solely on ξ, $\varpi(\xi)$ and its derivative $\varpi'(\xi)$. In fact, we do get

$$2\left(\xi\,\varpi'(\xi) - \varpi(\xi)\right) + \left(2\left(\xi\,\varpi'(\xi) - \ \varpi(\xi)\right) - \frac{1}{n}\,\xi\,\varpi'(\xi)\right)^2 = 0 \tag{105}$$

The solution to this equation is given implicitly (for $n \neq \frac{1}{2}$) as

$$\left(2-\frac{1}{n}\right)\left(\frac{\arctan\left(\frac{-1\pm 2\sqrt{\beta_n(\xi)}}{\sqrt{-(1-2n)^2}}\right)}{\sqrt{-(1-2n)^2}}+\frac{1}{2}\log\left((1-n)\,n-\sqrt{\beta_n(\xi)}+\beta_n(\xi)\right)\right)= \\ = c+\log(\xi) \quad (106)$$

with $\varpi = \dfrac{n^2-\beta_n(\xi)}{2\,(-1+2n)}$. In practice these involve algebaric equations of high order which are not analytically solvable except for a few n. Besides the obvious $\varphi = c$ constant solution (which actually is a case of $n^{-1} = 0$ solution) we mention the next three [3]

$n^{-1}=0$	$\varphi = \dfrac{x^2}{2t}+c$
$n^{-1}=1$	$\varphi = c\,x-\frac{1}{2}c^2\,t$
$n^{-1}=2$	$\varphi = \dfrac{x^2}{2t+c}$
$n^{-1}=3$	$\varphi = \dfrac{x^2}{2t}-\dfrac{2\,c\,x}{t^2}+\dfrac{4\,c^2}{3\,t^3}\left(1+\sigma\left(1-\dfrac{t\,x}{c}\right)^{\frac{3}{2}}\right)$

We can see from above that, apart from the constant solution, two solutions are worth mentioning, the linear and the gaussian (quadratic) phase solutions.

[3] For a complex unimodular number $|\mathbb{z}| = 1$, we can write $\mathbb{z} = \sqrt{1-\zeta^2}+i\,\zeta = e^{i\theta}$. Hence

$$\theta = -i\,\log\left(e^{i\,\theta}\right) = \arctan\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = -i\log\left(\sqrt{1-\zeta^2}+i\,\zeta\right)$$

Conversely, given $\beta \in \mathbb{R}$ we can always create a unimodular complex number $\mathbb{z} = \frac{1+i\beta}{\sqrt{1+\beta^2}}$ such that

$$\arctan(\beta) = -i\,\log\left(\frac{1+i\,\beta}{\sqrt{1+\beta^2}}\right)$$

The expression $\arctan(\beta) = \frac{i}{2}\log(1-i\,\beta)-\frac{i}{2}\log(1+i\,\beta)$ is a consequence of this setting $\mathbb{u} = 1+i\,\beta$ with $1+\beta^2 = \mathbb{u}\,\bar{\mathbb{u}}$.

Choosing a constant $c = \mathcal{P} = p\sqrt{1 - \frac{2\mathcal{V}}{p^2}}$

$$\phi_1 = \varphi_1(t,\, x) - \mathcal{V}\,(t + t_o) = p\sqrt{1 - \tfrac{2\mathcal{V}}{p^2}}\,\left(x + x_o\right) - \frac{p^2}{2}\,\left(t + t_o\right) \tag{107}$$

(when $p^2 \geq 2\mathcal{V}$) and

$$\phi_2 = \varphi_2(t,\, x) - \mathcal{V}\,(t + t_o) = \frac{\left(x + \xi_o\right)^2}{2\,\left(t + \tau_o\right)} - \mathcal{V}\,\left(t + t_o\right) \tag{108}$$

Notice that the constants x_o, t_o, p are not all independent: in fact the general solution of type 1 depends on only two constants $\mathcal{P}$, φ_o and is of the form

$$\varphi_1(t,\, x) = \mathcal{P}\,x - \frac{\mathcal{P}^2}{2}\,t + \varphi_o \tag{109}$$

We have used $\varphi_0 = \mathcal{P}\,x_0 - \dfrac{\mathcal{P}^2}{2}\,t_0$ to produce the previous expression for ϕ_1, where we exhibit the quantity p which corresponds to the moment in free motion.

The corresponding amplitudes are, using $\zeta_o = t_1 + \dfrac{x_1}{\mathcal{P}}$,

$$\begin{aligned}
\alpha = \alpha_1(t,\, x) &= f\left(t + \frac{x}{\mathcal{P}} + \zeta_o\right) = f\left(t + t_1 - \frac{x + x_1}{p\sqrt{1 - \frac{2\mathcal{V}}{p^2}}}\right) \\
\alpha = \alpha_2(t,\, x) &= g\left(\frac{t + \tau_o}{x + \xi_o}\right) - \frac{1}{2}\,\log(x + \xi_o)
\end{aligned} \tag{110}$$

where $f(z)$ and $g(z)$ are arbitrary functions. We readily notice that the first type solution $\psi_1(t,\, x) = e^{\,\alpha_1 + i\,\varphi_1}$ is of the traveling kind, i.e. where the envelope shape $e^{\,f(z)}$ translates itself with time in x with 'speed' $\mathcal{P} = p\sqrt{1 - \frac{2\mathcal{V}}{p^2}}$. We also see that the second type of solution doesn't depend at all on momentum, but it does have a term which is constant along lines of constant slope $\frac{t+\tau_o}{x+\xi_o} = s$. This means that a boundary condition $\alpha = f(t(\sigma), x(\sigma))$ determines the solution uniquely (as long as $\{t(\sigma), x(\sigma)\}$ goes through the singularity $\left\{-\tau_o,\, -\xi_o\right\}$. The appearance of this is indication that a boundary between two constant potentials exists at a finite distance. The reason for the singular behavior is simple enough: for constant potentials there are no forces changing $\mathcal{P} = \nabla\phi$, however there will be a reflection at the boundary to a higher constant potential. It must

be then that an infinite acceleration has occurred at some point, for the moment to change instantaneously to $-\mathcal{P}$.

The third type if solution also gives an integral for the amplitude equations. The phase $\varphi(x,t)$ can really be expressed in a simpler form by noting that $\varphi(x,t) = \tilde{\varphi}(\xi(x,t),t)$ with $\xi(x,t) = 1 - \frac{t\,x}{c}$. In this case we can write

$$\tilde{\varphi}(\xi,t) = \frac{c^2}{2\,t^3}\left(\xi^2 + \frac{8}{3}\,\sigma\,\xi^{3/2} + 2\,\xi - \frac{1}{3}\right) \tag{111}$$

where $\sigma = \pm 1$. In order to express the equations in these new variables one must replace

$$\partial_{\,x} f(x,t) = \partial_x \tilde{f}(\xi(x,t),t) = \partial_\xi \tilde{f}(\xi,t)\partial_x \xi(x,t) \tag{112}$$

whereas

$$\partial_{\,t} f(x,t) = \partial_t \tilde{f}(\xi(x,t),t) = \partial_\xi \tilde{f}(\xi,t)\partial_t \xi(x,t) + \partial_t \tilde{f}(\xi,t) \tag{113}$$

The resulting equation for the amplitude is

$$\partial_t \tilde{\alpha}(\xi,t) = -\frac{(\xi^{1/2} + \sigma)}{2\,t\,\xi^{1/2}}\left(1 + 4\,\xi\,\partial_\xi \tilde{\alpha}(\xi,t)\right) \tag{114}$$

Anyway, the amplitude solution in terms of these variables is

$$\tilde{\alpha}(\xi,t) = \mathcal{F}\left(\frac{t}{\sqrt{\xi} + \sigma}\right) - \frac{\log(\xi)}{4} \tag{115}$$

with $\mathcal{F}$ an arbitrary function. Substituting back $\xi(x,t)$ in the resulting solution we obtain

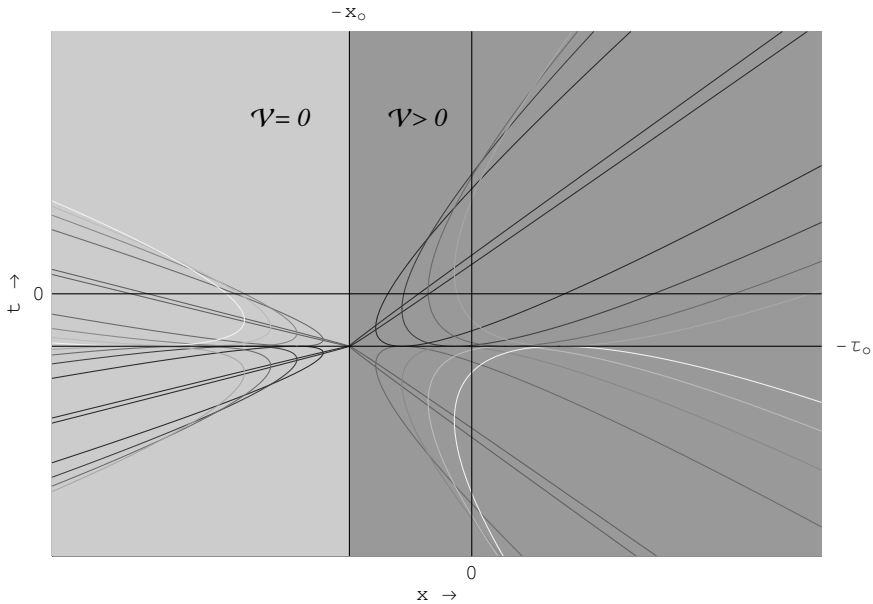
$$\alpha(x,t) = \mathcal{F}\left(\frac{t}{\sqrt{1 - \frac{t\,x}{c}} + \sigma}\right) - \frac{1}{4}\,\log\left(1 - \frac{t\,x}{c}\right) \tag{116}$$

6.2 Equal Phase Condition and Matching of Solutions

We now wish to match these two types of solution in the presence of a potential barrier $\mathcal{V}$ at $-x_o$. We start by requesting that the phases φ_1 and φ_2 be the same on the matching region. This means that $\varphi_1(t,\,x) = \varphi_2(t,\,x)$ which determines a geometric location $x = x(t)$ where this is possible (notice $p^2 = 2\mathcal{E}$ where the

total energy is $\mathcal{E} = \frac{1}{2}\mathcal{P}^2 + \mathcal{V}$).

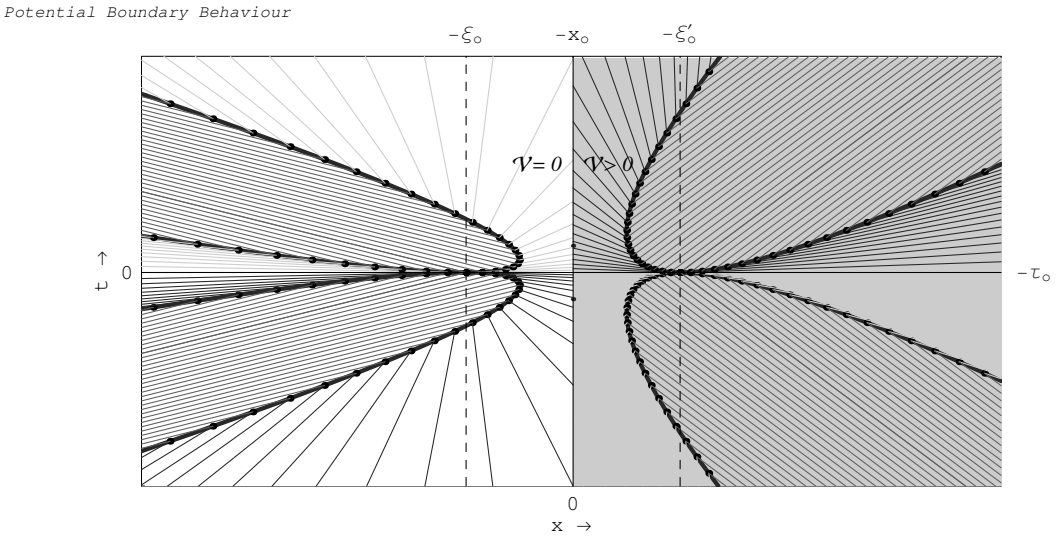
$$x(t) = \mathcal{P}\left(t + \tau_0\right) \pm \sqrt{\lambda\left(t + \tau_0\right)} - \xi_0$$
$$\lambda = 2\mathcal{P}\left(x_0 - \xi_0\right) - 2\mathcal{E}\left(t_0 - \tau_0\right) \quad ; \qquad \mathcal{P} = p\sqrt{1 - \frac{2V}{2\mathcal{E}}} \tag{117}$$



The previous figure shows matching curves for incoming and outgoing solutions, in both regions, arriving at $\left\{-\xi_o,\ -\tau_o\right\}$ in the neighborhood of the potential step on $-x_o$, and color-coding different values of ξ_o. We notice that only very focused solutions actually get close to the potential step ($\xi_o - x_o \approx 0$), and the transmitted solution gets diffracted in a less focused solution starting from a symmetrical position ($\xi_o' - x_o = x_o - \xi_o$). The region inside the U-shaped curves is where the linear-phase solution φ_1 can live. The remaining region is the domain of the Gaussian-type phase φ_2.

The following picture illustrates this idea for a particular case. The parallel lines with slope $\frac{dx}{dt} = \mathcal{P}$ inside the linear-phase regions are in fact characteristics for the amplitudes α_1 associated with these phases φ_1, i.e. the α_1 is constant along each of these lines. One should keep in mind that the lines of constant phase φ_1 have half that $\frac{dx}{dt} = \frac{1}{2}\mathcal{P}$, so $\psi = e^{\alpha_1 + i\varphi_1}$ will not be constant

along either lines. The fact that these characteristics are parallel also indicates that the momentum $\mathcal{P} = \nabla \varphi_1$ is constant in each of these regions.



The lines in the Gaussian-phase regions are not characteristics of the phase, but they all converge on the critical point $\{-\xi_o, -\tau_o\}$. This is in a sense the 'return point' for the *center of mass* of a more or less diffused distribution. Since along the boundaries $x(t)$ the two types of phase coincide $\varphi_1(t, x(t)) = \varphi_2(t, x(t))$, we must conclude that, in order to have continuity of the solutions $\psi_1(t, x(t)) = \psi_2(t, x(t))$, we must obtain

$$\alpha_1(t,\ x(t)) = f\left(t + \frac{x(t)}{\mathcal{P}} + \zeta_o\right) = g\left(\frac{t + \tau_o}{x(t) + \xi_o}\right) - \frac{1}{2}\ \log\left(x(t) + \xi_o\right) = \alpha_2(t,\ x(t)\Big) \tag{118}$$

We therefore see the meaning of the lines in the Gaussian-phase regions: along these lines the value of $\alpha_2(t',\ x')$ is determined by the value of $\alpha_1(t,\ x(t))$ at the boundary point $\{t,\ x(t)\}$ corresponding to the line that goes through $\{t',\ x'\}$. That is, given $\{t,\ x(t)\}$, at any of the points $\{t',\ x'\}$ verifying

$$\frac{t' + \tau_o}{x' + \xi_o} = \frac{t + \tau_o}{x(t) + \xi_o} \tag{119}$$

in the Gaussian-phase regions, the value

$$\alpha_2(t',\ x') = f\left(t + \frac{x(t)}{\mathcal{P}} + \zeta_o\right) + \frac{1}{2}\ \log\left(\frac{x(t) + \xi_o}{x' + \xi_o}\right) \tag{120}$$

In fact, (119) determines implicitly $t = t(t',\ x')$ as

$$t + \tau_0 = \frac{2\mathcal{P}\left(x_0 - \xi_0\right)\left(t' + \tau_0\right)^2}{\left(x' + \xi_0 - \mathcal{P}\left(t' + \tau_0\right)\right)^2} \tag{121}$$

and therefore also $x(t) = \tilde{x}(t',\ x')$, and so

$$\alpha_2(t',\ x') = f\left(t(t',\ x') + \frac{\tilde{x}(t',\ x')}{\mathcal{P}} + \zeta_o\right) + \frac{1}{2}\log\left(\frac{\tilde{x}(t',\ x') + \xi_o}{x' + \xi_o}\right) \tag{122}$$

is well defined.

6.3 Reflection

Having determined the nature of the matching of a linear-phase (traveling) part of the solution with a gaussian phase (non-traveling) solution, we see how an incoming wave of momentum $\mathcal{P}$ approaches the potential at $-x_o$ has a center-of-mass reflection at $\left\{-\xi_o,\ -\tau_o\right\}$, and the resulting motion is uniquely determined by the gaussian-phase lines that go through this point, and along which the amplitude is determined to be a constant minus $\frac{1}{2}\log\left(x + \xi_o\right)$. As we expect, these lines carry information for the reflection, since their prolongation into the future should obey the same rule as before the time $-\tau_o$. Thus we see that the lines in the frontal region of the incident wave all have influence only after a finite time has passed. Those that correspond to small negative slopes $> -\mathcal{P}$ will determine the frontal region of the reflected motion, while the large negative slopes $< -\mathcal{P}$ will affect only the long time wake of the traveling packet. Those that correspond to positive slopes $> \mathcal{P}$ will dominate the region near the reflection point after a while, and it is the lines with small positive slopes $< \mathcal{P}$ that determine the motion just after the reflection in the interval $\left[-\xi_o,\ -x_o\right]$. We see thus that the wake of the incoming wave is left to linger in the wake after the reflection, whilst the frontal crest gets to remain in the frontal crest.

6.4 Potential Step

In order to have continuity of the amplitude as we go across the potential step, (since we already know that there can be no continuity in the gaussian phase along $x = -x_o$ since we cannot maintain for all times $\frac{(\xi_o - x_o)^2}{2(t+\tau_o)} = \frac{(\xi_o' - x_o)^2}{2(t+\tau_o)} - V(t+\tau_o)$

unless $V = 0$) we must have that

$$\alpha_2\left(x' = -x_o, t'\right) = g\left(\frac{t' + \tau_o'}{-x_o + \xi_o'}\right) - \frac{1}{2}\log\left(-x_o + \xi_o'\right) \tag{123}$$

which determines g along the lines

$$\frac{t'' + \tau_o'}{x'' + \xi_o'} = \frac{t' + \tau_o'}{-x_o + \xi_o'} \tag{124}$$

through the dependence

$$t' + \tau_o{}' = -\frac{(x_o - \xi_o{}')(\tau_o{}' + t'')}{\xi_o{}' + x''} \tag{125}$$

so that now

$$\alpha_2\left(x'', t''\right) = \alpha_2\left(-x_o, -\tau_o{}' - \frac{(x_o - \xi_o{}')(\tau_o{}' + t'')}{\xi_o{}' + x''}\right) + \frac{1}{2}\log\left(\frac{-x_o + \xi_o'}{x'' + \xi_o'}\right) \tag{126}$$

We should ponder on what the situation might be when the potential region is of finite extent.We can understand that on either side of the potential region there should be propagation with the same momenta (energy conservation) even though the amplitudes might be differ. Remember that we are not considering dissipation in the potential region, or else we would have to include an imaginary term to the potential $V = V_r + i V_i$. That does not happen! Although one might think that elliptical isophase lines could exist, from the nature of these phases we can show that there can be only parabolic solutions. This means that there will be no symmetry in time history as a wave goes through a potential wall of finite depth and height. The incoming wave might be partially reflected and refracted, but there is no incoming or outgoing reflection within the potential region. But this might be as it should, for the idea of reflections at a potential drop and incoming ghost waves are a linear-world fantasy that we cannot afford here. The same goes for the temptation to consider a variable potential as a sequence of slices of constant potential. There is no going around the resolution of the complete equations: that is why they are called Nonlinear, no superposition principle applies. We will try our hand at one of these cases in the next section.

7 COHERENT-TYPE SOLUTIONS FOR THE HARMONIC OSCILLATOR

7.1 TRAVELING 'LINEAR'-PHASE SOLUTIONS

We will now address the case of a particular potential $V(x) = \frac{\omega^2}{2}(x+\xi_o)^2$ with ω constant.

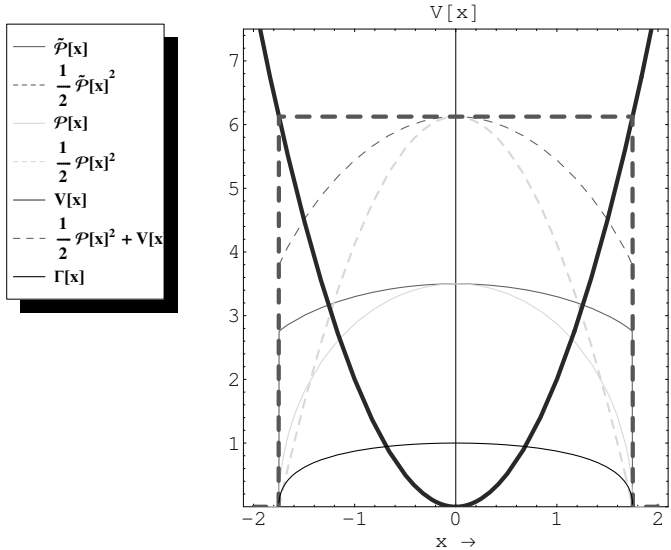
Using again the designation $\mathcal{P}(x) = p\sqrt{1-\frac{2V(x)}{p^2}}$ and $\mathcal{E} = \frac{1}{2}\mathcal{P}(x)^2 + V = \frac{1}{2}p^2$, we obtain a version of the linear-phase solution

$$\varphi_1(t,\ x) = \left(\frac{1}{2} + \frac{\Gamma(x)}{\mathcal{P}(x)^2}\,\mathcal{E}\right)\mathcal{P}(x)\,(x+\xi_o) - \mathcal{E}\,\left(t+\tau_0\right) \tag{127}$$

where

$$\Gamma(x) = \left(\frac{\omega\,(x+\xi_o)}{\mathcal{P}(x)}\right)^{-1}\ \arctan\left(\frac{\omega\,(x+\xi_o)}{\mathcal{P}(x)}\right) \tag{128}$$

is a sort of characteristic of the interval where the solution is real, i.e. positive pure-number function, bounded by 1, which obeys $\lim_{\omega\to 0}\Gamma(x) = 1$.



Explicitly

$$\varphi_1(t,\,x) = \frac{1}{2}\left(\mathcal{P}(x) + \frac{p^2}{\omega\,(x+\xi_o)}\,\arctan\left(\frac{\omega\,(x+\xi_o)}{\mathcal{P}(x)}\right)\right)(x+\xi_o) - \frac{p^2}{2}\,\left(t+\tau_0\right) \tag{129}$$

Remarkably, the physical moment associated with this phase is still

$$\mathcal{P}(x) = \partial_x\,\varphi_1(t,\,x) = p\,\sqrt{1-\frac{2\,V(x)}{p^2}} \tag{130}$$

(hence the reason for our notation above) and not the multiplier of $x+\xi_o$, which we denote by $\tilde{\mathcal{P}}(x) = \frac{1}{2}\left(1+\frac{\Gamma(x)\,p^2}{\mathcal{P}(x)^2}\right)\mathcal{P}(x)$. Physically this makes sense since $\mathcal{P}(x)$ changes between $\mathcal{P}(-\xi_o) = p$ and $\mathcal{P}(\pm\frac{p}{\omega} - \xi_o) = 0$, in the domain where φ_1 is real, while the quantity

$$\frac{\pi}{4}\,p < \tilde{\mathcal{P}}(x) \leq p \tag{131}$$

has a lower bound $\neq 0$, and thus does not behave like a localized physical momentum.

The associated amplitude to this phase is now easily determined as

$$\alpha_1(t,\,x) = -\frac{1}{4}\,\log(p^2-\omega^2\,(x+\xi_o)^2) + \\ + f\left(t+t_0-\frac{1}{\omega}\,\arctan\left(\frac{\omega\,(x+\xi_o)}{\sqrt{p^2-\omega^2\,(x+\xi_o)^2}}\right)\right) \tag{132}$$

where f is an arbitrary function, or else

$$\alpha_1(t,\,x) = -\frac{1}{4}\,\log\left(\mathcal{P}(x)^2\right) + +f\left(t+t_0-\frac{\Gamma(x)}{\mathcal{P}(x)}\,(x+\xi_o)\right) \tag{133}$$

If therefore at time $-t_0$ we know of the initial shape of the wave $\alpha_1(-t_0,\,x) = \mathcal{F}_0(x)$, then we can determine f by solving $\frac{1}{\omega}\arctan\left(\frac{x\,\omega}{\mathcal{P}(x)}\right) = \zeta$ and substituting $x = -\xi_0 + \frac{p}{\omega}\,\sin(\zeta\,\omega)$ in the initial condition to obtain

$$f(\zeta) = \frac{1}{2}\,\log(p\,\cos(\zeta\,\omega)) + \mathcal{F}_0\left(\frac{p}{\omega}\,\sin(\zeta\,\omega)\right) \tag{134}$$

Introducing this back into (132) we obtain the particular solution

$$\alpha_1(t,x) = \frac{1}{4}\,\log\left(\frac{p^2}{\mathcal{P}(x)^2}\cos\left(\omega\,(t+t_0-\frac{\Gamma(x)}{\mathcal{P}(x)}\,(x+\xi_0))\right)^2\right) + \\ + \mathcal{F}_0\left(\tfrac{p}{\omega}\,\sin\left(\omega\,(t+t_0-\tfrac{\Gamma(x)}{\mathcal{P}(x)}\,(x+\xi_0))\right)\right) \tag{135}$$

7.2 Gaussian Phase

A Gaussian phase solution also exists for this potential.

$$\varphi_2(x,t) = \frac{\omega\left(x+\xi_0\right)^2}{2\,\tan(\omega\,\left(t+\tau_0\right))} \tag{136}$$

to which corresponds the amplitude

$$\alpha_2(x,t) = g\left(\frac{1}{\omega}\,\log\left(\frac{\sin(\omega\,\left(t+\tau_0\right))}{x+\xi_0}\right)\right) - \frac{1}{2}\,\log(x+\xi_0) \tag{137}$$

We should now look for the condition of equal phase between the two type of solutions. This now becomes a transcendental equation

$$\frac{\omega\left(x+\xi_0\right)^2}{2\,\tan(\omega\,\left(t+\tau_0\right))} = \left(\frac{1}{2} + \frac{\Gamma(x)}{\mathcal{P}(x)^2}\,\mathcal{E}\right)\mathcal{P}(x)\,(x+\xi_o) - \mathcal{E}\,\left(t+\tau_0\right) \tag{138}$$

which we can only hope to solve approximately. But in principle this will determine the isophase curves. One thing is immediately apparent, and its the fact that this equation has many disconnected solutions, which is as well for we expect to find, under the right initial conditions, that there are 'periodic' bounded motions included in these solutions. In a previous work we did find that these same equations possess a nonlinear version of the harmonic oscillator coherent state, and we could find these solutions explicitly.

7.3 Coherent States of the Linear Schrödinger Equation.

As is widely known, exact solutions to a **LINEAR** Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi \tag{139}$$

exist for the case of the harmonic potential $V(x) = \frac{1}{2}m\,\omega^2 x^2$ which evolve without spread and oscillate back and forth within the confines of the potential. These types of solutions are not stationary states (meaning they are not eigenfunctions of the Hamiltonian operator). They are called **COHERENT STATES** (see e.g. [13]) on account of their non-dispersive nature, and have the form

$$\psi(x,t) = \left(\frac{m\,\omega}{\hbar\pi}\right)^{\frac{1}{4}} e^{\,-\frac{m\,\omega}{2\hbar}(x-\overline{x}(t))^2 + \frac{i}{\hbar}\left(\overline{p}(t)(x\;-\frac{1}{2}\,\overline{x}(t)) - \frac{\hbar}{2}\omega\,t\right)} \tag{140}$$

where $\bar{x}(t)$ is a solution of the classical harmonic oscillator $\dfrac{d^2\bar{x}(t)}{dt^2} = -\omega^2\,\bar{x}(t)$ and $\overline{p}(t)$ the classical momentum. In fact, given initial position $\bar{x}(0) = \bar{x}_o$ and energy $\mathcal{E}_o = \dfrac{1}{2m}\overline{p}_o{}^2 + \dfrac{1}{2}m\,\omega^2\bar{x}_o{}^2$,

$$\left\{\begin{array}{l}\bar{x}(t) = \alpha\ \cos(\omega\, t + \beta)\\ \\ \overline{p}(t) = -m\,\omega\,\alpha\ \sin(\omega\, t + \beta)\end{array}\right. \quad \left\{\begin{array}{l}\alpha\ = \sqrt{\frac{2\,\mathcal{E}_o}{m\ \omega^2}}\\ \\ \beta\ = -\arctan\left(\frac{\overline{p}_o}{m\,\omega\,\bar{x}_o}\right)\end{array}\right. \quad \left\{\begin{array}{l}\bar{x}_o = \alpha\ \cos(\beta)\\ \\ \overline{p}_o = -m\,\omega\,\alpha\ \sin(\beta)\end{array}\right. \tag{141}$$

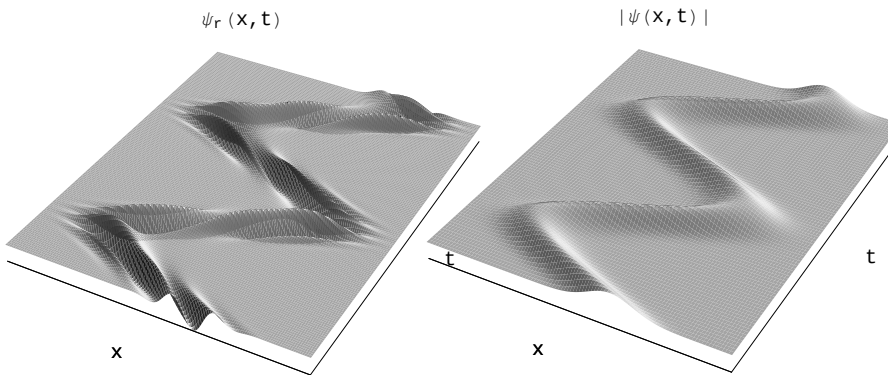


Figure 1: Coherent Sate of the Harmonic Oscillator for the LSE

Actually, in the free case $V(x) \equiv 0$, no such free coherent states can exist for the LSE: in fact, any localized initial structure will spread over time due to the linearity of the superposition. However, a simple calculation shows that a solution of the **Free Nonlinear Schrödinger equation** (11) does exist which is coherent:

$$\begin{array}{rl}\psi_{free}(x,t) & = c\,e^{\,-a\,(x\,-\,b\,-\,t\,e)^2\,+\,i\,(e\,(x\,-\,b\,-\,t\,e)\,+\,\frac{1}{2}\,e^2\,t\ \ )}\\ & = c\,e^{\,-a\,(x\,-\,\bar{x}(t))^2\,+\,i\,(\overline{p}_o\,(x\,-\,\bar{x}(t))\,+\,\frac{1}{2}\overline{p}_o^2\,t)}\end{array} \tag{142}$$

In this case, we can identify the constants as $e \equiv \overline{p}_o$, $b = \bar{x}_o$ and then $\bar{x}(t) = b + t\ e$, $\mathcal{E}_o = \frac{1}{2}\overline{p}_o^2$. Note that this can also be represented as

$$\psi_{free}(x,t) = c\,e^{\,-\frac{1}{2}i\,\bar{x}_o\overline{p}_o}\ e^{\,-a\,(x\,-\,\bar{x}(t))^2\,+\,i\,\overline{p}_o\,(x\,-\,\frac{1}{2}\bar{x}(t))} \tag{143}$$

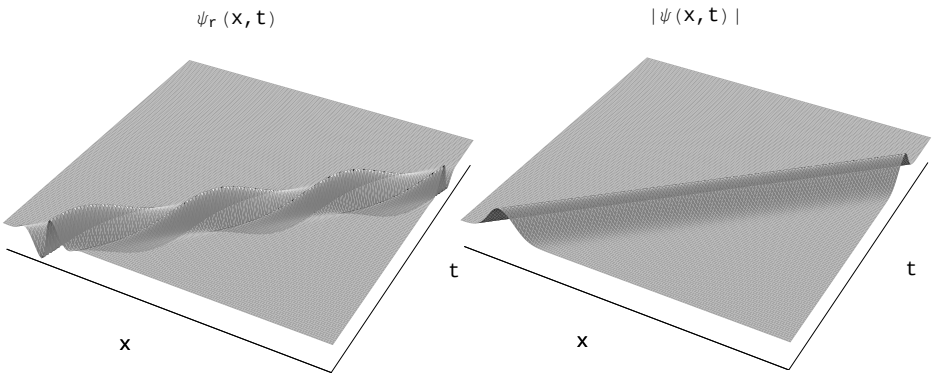


Figure 2: Coherent Sate for the free NLSE

7.4 Coherent-type Solutions of the NLSE

Solutions of the non-linear equation also exists when $V(x) = \frac{1}{2}\omega^2 x^2$ in the form

$$\psi(x,t) = \left(\frac{1}{\cos^2(b-\omega t)}\right)^{\frac{1}{4}} e^{-\frac{(x-c\,\cos(b-\omega t))^2}{\cos^2(b-\omega t)}\left(a-\frac{1}{4}i\,\omega\,\sin(2(b-\omega t))\right)}\times \\ \times e^{i\,c\,\omega\sin(b-\omega t)\left(x-\frac{1}{2}c\,\cos(b-\omega t)\right)} \tag{144}$$

Using the notation

$$c\,\cos(b-\omega t) = \bar{x}(t) \quad ; \qquad c\,\omega\,\sin(b-\omega t) = \bar{p}(t) = \frac{d\bar{x}(t)}{dt} \tag{145}$$

we obtain again something similar to (140)

$$\psi(x,t) = \left(\frac{1}{\cos^2(b-\omega t)}\right)^{\frac{1}{4}} \quad e^{-\frac{\left(a-\frac{1}{4}i\,\omega\,\sin(2(b-\omega t))\right)}{\cos^2(b-\omega t)}(x-\bar{x}(t))^2 + i\,\bar{p}(t)\left(x-\frac{1}{2}\bar{x}(t)\right)} \tag{146}$$

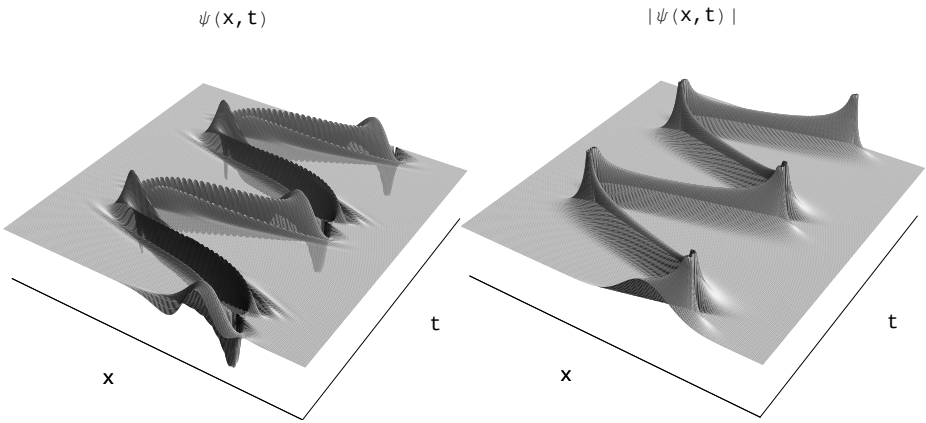


Figure 3: Coherent Sate for the Harmonic Oscillator NLSE

Remarkably, if we notice that the phase $\phi = S(x,t)$ is

$$\begin{aligned} S(x,t) &= \overline{p}(t)\left(x - \tfrac{1}{2}\,\overline{x}(t)\right) + \tfrac{1}{2}\,\omega\,\tan(\hbar - \omega\,t)\,(x - \overline{x}(t))^2 \\ \nabla S(x,t) &= \overline{p}(t) + \omega\,\tan(\hbar - \omega\,t)\,(x - \overline{x}(t)) \end{aligned} \tag{147}$$

then, for the classical motion $x \equiv \overline{x}(t)$, we still have that $\nabla S(\overline{x}(t),t) = \overline{p}(t)$. [1]

[1] In terms of these classical variables we can write the phase and amplitude of the wave function ψ as

$$\begin{cases} \phi \equiv S(x,t) = \overline{p}(t)\left(x - \frac{1}{2}\overline{x}(t)\right) + \frac{1}{2}\frac{\overline{p}(t-\delta t)}{\overline{x}(t-\delta t)}\,(x - \overline{x}(t))^2 \\ \alpha \equiv \log|\psi| = \frac{1}{4}\log\left(\frac{\mathcal{E}_o}{V(\overline{x}(t-\delta t))}\right) - \frac{\mathcal{E}_o\,a}{V(\overline{x}(t-\delta t))}(x - \overline{x}(t))^2 \end{cases}$$

where we have used, for $c \neq 0$,

$$\begin{cases} \tan(\hbar - \omega\,t) = \frac{\overline{p}(t) + \omega\,\tan(g)\,\overline{x}(t)}{\omega\,\overline{x}(t) - \tan(g)\,\overline{p}(t)} = \frac{\overline{p}(t-\delta t)}{\omega\,\overline{x}(t-\delta t)} \\ \sec^2(\hbar - \omega\,t) = \frac{\left(\overline{p}(t)^2 + \omega^2\,\overline{x}(t)^2\right)\sec^2(g)}{\left(\omega\,\overline{x}(t) - \tan(g)\,\overline{p}(t)\right)^2} = \frac{\mathcal{E}_o}{V(\overline{x}(t-\delta t))} \end{cases}$$

and the notation $g = \omega\,\delta t = \hbar - b$. In the case $c = 0$ we obtain a non-traveling solution of the NLSE with harmonic potential $V(x) = \frac{1}{2}\,\omega^2\,x^2$

$$\psi_{ntr}(x,t) = \left(\frac{1}{\cos^2(\hbar - \omega\,t)}\right)^{\frac{1}{4}} e^{-\frac{a\,x^2}{\cos^2(\hbar - \omega\,t)} + \frac{1}{2}\,i\,\omega\,\tan(\hbar - \omega\,t)\,x^2} \tag{148}$$

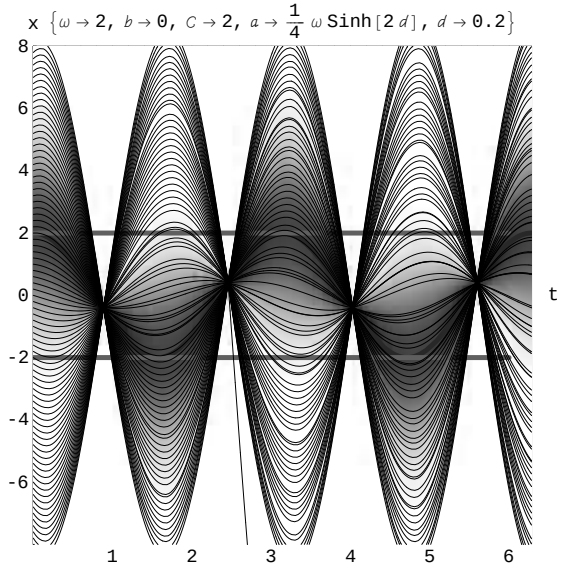
The general solution for amplitude and phase is

$$\begin{aligned}\alpha(x,\,t) &= -\frac{a}{\cos^2(h-\omega\,t)}\,(x-\bar{x}(t))^2 - \frac{1}{2}\log[\cos(h-\omega\,t)] \\ \phi(x,\,t) &= \overline{p}(t)\left(x-\tfrac{1}{2}\,\bar{x}(t)\right) + \tfrac{1}{2}\,\omega\,\tan(h-\omega\,t)\,(x-\bar{x}(t))^2\end{aligned} \tag{149}$$

The problem with this solution is that it periodically blows up, when its variance

$$z(t) = \frac{\cos^2(h-\omega\,t)}{a}$$

shrinks to zero. This happens at the instants that make $\cos(h-\omega\,t) = 0$, as is shown in the following image representing field lines for the $\{1,\,\nabla\phi\}$ evolution in $\{t,\,x\}$ space. Nevertheless, we can see that the areas of greater amplitude (darker) follow the particular field lines that are closest to the classical trajectory.)



7.5 Complex Phase Solutions

When one chooses complex h one changes the relation between amplitude and phase. In order to have well behaved solutions when $h = i\,d$ we must use $d > 0$.

Let us consider the substitution $z(t) = \frac{1}{a}\cos^2(h - \omega\,t)$ in the solution (146). With this notation

$$\frac{a - \frac{1}{4}\,i\,\omega\,\sin(2\,(h - t\,\omega))}{\cos(h - \omega\,t)^2} = \frac{1 - \frac{1}{4}\,i\,z'(t)}{z(t)} \tag{150}$$

$$\psi(x,\,t) = \left(\frac{1}{a\,z(t)}\right)^{\frac{1}{4}}\,e^{-\frac{\left(1-\frac{1}{4}\,i\,z'(t)\right)}{z(t)}\,(x - \overline{x}(t))^2 + i\,\overline{p}(t)\left(x - \frac{1}{2}\,\overline{x}(t)\right)} \tag{151}$$

If we replace this in the expression found for the NLSE solution $\psi(x,\,t)$, with the appropriate identification of $\xi(x,\,t) = x - \overline{x}(t)$ and $\zeta(x,\,t) = x - \frac{1}{2}\overline{x}(t)$, we obtain

$$\psi(x,\,t) = e^{-\frac{1}{4}\log\left(a\,z(t)\right) - \frac{1}{z(t)}\xi(x,\,t)^2 + i\left(p(t)\,\zeta(x,\,t) + \frac{1}{4}\frac{z'(t)}{z(t)}\xi(x,\,t)^2\right)} \tag{152}$$

In a sense we can read out 'amplitude' and 'phase' factors:

$$\begin{aligned}\alpha(x,\,t) &= -\frac{1}{4}\log\left(a\,z(t)\right) - \frac{1}{z(t)}\xi(x,\,t)^2\\ \phi(x,\,t) &= p(t)\,\zeta(x,\,t) + \frac{1}{4}\frac{z'(t)}{z(t)}\xi(x,\,t)^2\end{aligned} \tag{153}$$

but if we complexify $z(t)$ with $h = i\,d$, and set

$$z(t) = \frac{\cos(i\,d - \omega\,t)^2}{a} = \sigma(t)\,e^{i\,\theta(t)} \tag{154}$$

we must now reformulate the amplitude and phase of the wave function $\psi = e^{\alpha + i\,\phi}$ to $\psi = e^{\mathcal{A} + i\,\mathcal{S}}$ where $\mathcal{A} = \mathrm{Re}[\alpha + i\,\phi]$ and $\mathcal{S} = \mathrm{Im}[\alpha + i\,\phi]$. This

leads to[2]

$$\mathcal{A} = -\frac{1}{4}\log(a\,\sigma(t)) - \Big(\cos(\theta(t)) + \frac{1}{4}\theta'(t)\sigma(t)\Big)\frac{\xi(x,\,t)^2}{\sigma(t)}$$
$$\mathcal{S} = -\frac{1}{4}\theta(t) + p(t)\zeta(x,\,t) + \Big(\sin(\theta(t)) + \frac{1}{4}\sigma'(t)\Big)\frac{\xi(x,\,t)^2}{\sigma(t)} \tag{155}$$

where

$$\sigma'(t) = -\frac{\omega\,\sin(2\,t\,\omega)}{a} \qquad \text{and} \qquad \theta'(t)\,\sigma(t) = \frac{\omega\,\sinh(2\,d)}{a} \tag{156}$$

[2]Recalling that, for $h = i\,d$, we have

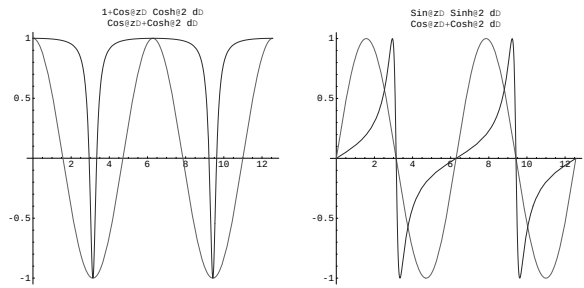
$$\cos(h - \omega\,t) = \cosh(d + i\,\omega\,t) \quad ; \quad i\,\sin(h - \omega\,t) = -\sinh(d + i\,\omega\,t)$$

we note that a new variable $\theta(t)$ is useful:

$$\theta(t) \approx \int \Big(\frac{2\,\omega\,\sinh(2\,d)}{\cos(2\,t\,\omega)+\cosh(2\,d)}\Big)dt = 2\arctan(\tan(t\,\omega)\,\tanh(d))$$

and we can indeed verify that this is the correct variable for obtaining the usual derivatives of the functions $\cos(\theta(t))$ and $\sin(\theta(t))$.

$$\cos(\theta(t)) = \frac{\cos(2\,t\,\omega)\,\cosh(2\,d) + 1}{\cos(2\,t\,\omega) + \cosh(2\,d)} \quad ; \qquad \sin(\theta(t)) = \frac{\sin(2\,t\,\omega)\,\sinh(2\,d)}{\cos(2\,t\,\omega) + \cosh(2\,d)}$$



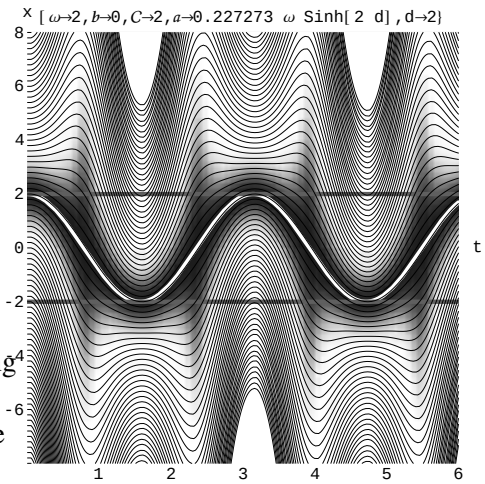
By introducing now a time-dependent variance

$$\sigma(t) = \frac{\cos(2\,\omega t) + \cosh(2\,d)}{2\,a} = \frac{\cos(\omega t)^2 + \sinh(d)^2}{a} = \frac{1}{2\,a}\,\frac{\sinh(2\,d)^2}{\cosh(2\,d) - \cos(\theta(t))} \geq 0$$

we notice that with the new angle $\theta(t)$

$$\cos(\theta(t)) = \frac{\cos(2\,t\,\omega)\,\cosh(2\,d) + 1}{2\,a\,\sigma(t)} \quad ; \qquad \sin(\theta(t)) = \frac{\sin(2\,t\,\omega)\,\sinh(2\,d)}{2\,a\,\sigma(t)}$$

Notice that even though these new functions $\mathcal{A}$ and $\mathcal{S}$ do not obey corresponding Hamilton-Jacobi and Continuity equations (3) and (10), $\psi(x,\, t)$ is still formally a solution of the NLSE due to a nonstandard grouping of real and imaginary components $\mathcal{A}+i\,\mathcal{S} = \alpha+i\,\phi$ with complex $z(t)$. This indicates that maybe equations (3) and (10) are more fundamental than the NLSE. Well-behaved (physical) solutions of this kind only exist for complex solutions α and ϕ, defined by a choice of h pure imaginary. Given $\mathcal{A}$ and $\mathcal{S}$, we can consider the current density $J = e^{2\mathcal{A}}\nabla\mathcal{S}$. With these choices we obtain a coincidence between the classical momenta $\overline{p}(t)$ along the classical trajectory and $\nabla\mathcal{S}(t,\, \overline{x}(t))$. The following picture illustrates how the 'field' should move away from the classical initial condition, and where the amplitude of the field is highest. At $t = 0$ we have $\sigma_o = \frac{\cosh[d]^2}{a}$. For a well-behaved solution we must have bounded



$$a < \frac{\omega\, \sinh(2\, d)}{4} \tag{157}$$

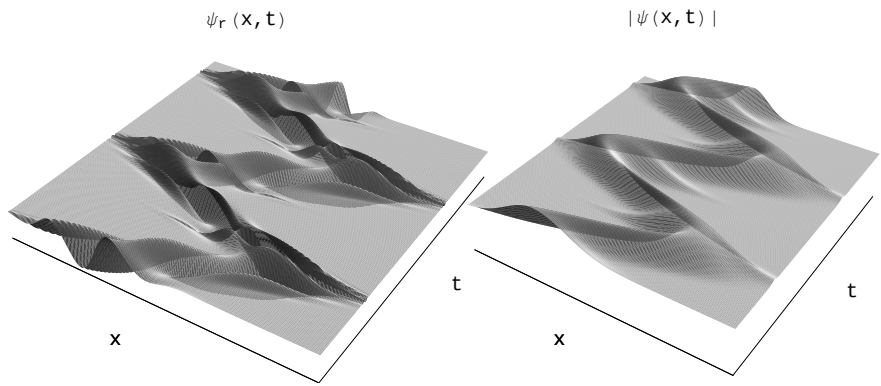


Figure 4: Complex extension of Coherent Sate Solution

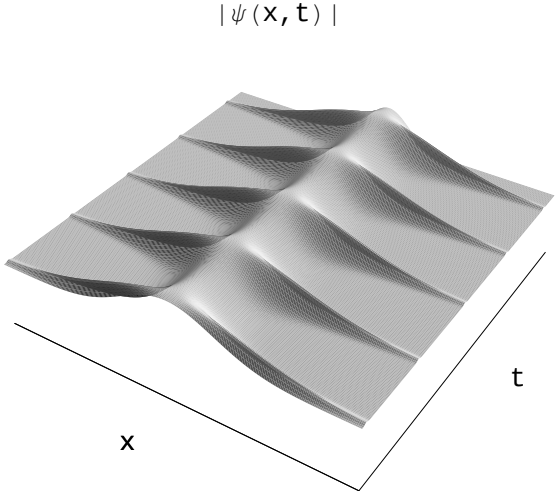


Figure 5: Stationary (i.e. non traveling) Solution

8 CONCLUSION

We have shown that preservation of time-honored physical principles like Fermat and Huygen's can be compatible with the conventional quantization of simple quantum mechanical systems, in contrast to the standard Linear Schrödinger Equation which requires a departure from Hamilton-Jacobi equations and variational principles. Preserving these together with balance equations for some extended field naturally yields a Nonlinear Schrödinger Equation which incorporates an additional Quantum Potential. When restricted to particular forms of the Quantum Potential, the solutions include the quantization levels for standard quantum mechanical problems like the Harmonic Oscillator or the Hydrogen atom, with a difference in the choice of phases which no longer represent null-moment states but rather degenerate into opposing moment states.

Our goal however was to obtain quantizations different from the standard ones. With this purpose in mind we sought solutions of the Kepler problem in the form of a complex field obeying the NLSE without a priori knowledge of the Quantum Potential, or equivalently the solution of the Hamilton-Jacobi equation with the continuity equation. This is in line with a train of thought that considers the auto interactions of the field with itself, expressed here by the Quantum Potential contribution, as legitimate expressions for source terms of the field. General form solutions for the NLSE equations could be found, but

attention was here given to solutions that expressed a field whose amplitude would be maximized along a curve that represented the classical path. The concept is akin to the derivation of the harmonic oscillator's coherent states. While doing that, the closure conditions for the single-valuedness of the phase along the classical trajectory provided a first quantum number n associated with the energy of the orbit. In consequence, we observed that the phase part of the wave $e^{i\varphi(r,\theta,t)}$, although being a double cover, is closed only if a second quantum number $-n < l < n$ is introduced in relation to quantization of the angular momentum of the orbit. As a consequence we obtain a quantization of the orbital elements for Keplerian orbits described by solutions of the NLSE. Since we worked a two dimensional example it comes as no surprise that the azimuthal quantum number m is missing, and only the total angular momentum l intervenes.

In conclusion, we would like to remark that imposing particular forms on the phase solutions maybe too restrictive on the possible solutions describing physical states of the systems. For the harmonic oscillator we have found solutions of the Hamilton-Jacobi equations with Gaussian type phases. Even for the free particle in one dimension we have found both traveling and Gaussian phase solutions that may shed new light on the propagation of particles in a potential well. Most interesting is the fact that traveling coherent structures can exist as solutions to the NLSE but not in the linear theory. We have found these both in the free and the harmonic oscillator case. This will be the subject of a future paper.

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SYMMETRY GENERATED SOLUTIONS OF A NONLINEAR SCHRÖDINGER EQUATION

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Summary: We study the symmetry generated solutions of the pair of equations that classically correspond to the Hamilton-Jacobi equation for an action $S = \phi(\mathbf{r}, t)$ and the continuity equation for an extended density field $\rho = e^{\alpha(\mathbf{r},t)}$. This pair was already shown in [1] to be equivalent to a type of nonlinear Schrödinger equation with a quantum potential term. The geometry of these solutions is probed towards an insight for their physical/geometrical meaning in the 1 + 1 and 2 + 1 dimensional cases. We use prolongation methods to find the infinitesimal generators of symmetries for these equations and study their flows and some possible solutions.

Keywords: Nonlinear PDEs; Symmetry Algebras ; Quantum Potential; Schrödinger-type Equations.

1 INTRODUCTION

We have shown previously in [1], using symmetry arguments as can be found in Olver [2] and Ovsiannikov [3], that the stochastically motivated nonlinear Schrödinger-type equation introduced by Vigier others [4]-[6]

$$\left(-\frac{\hbar^2}{2\,m}\,\nabla^2 + V + \gamma\,\frac{\hbar^2}{2\,m}\,\frac{\nabla^2|\psi|}{|\psi|}\right)\psi = i\hbar\,\partial_t\psi$$

where γ is some friction coefficient related to vacuum dissipative effects, is equivalent to a linear Schrödinger equation unless $\gamma = 1$ exactly. Their free symmetry algebras in 1 + 1 dimensions were shown to fundamentally differ from that of the standard linear free Schrödinger equation only for the particular value $\gamma = 1$ of the quantum potential coupling constant. Our calculation also showed that for $\gamma \neq 1$ or $\gamma = 0$ (i.e. the regular, linear Schrödinger equation) the same infinite-dimensional abelian sub-algebra of the full symmetry

algebra of the equation exists in both cases, and how we can associate a linear superposition law to symmetry generators in this sub-algebra. The case with $\gamma = 1$ is different in that respect, and we will show here that its symmetry algebra is finite-dimensional, and that no such linear superposition principle exists as expected. Furthermore, only for $\gamma = 1$ are these nonlinear Schrödinger-type equations mathematically equivalent to a pair of familiar equations in classical physics, namely the Hamilton-Jacobi equation for an action functional $\phi(\mathbf{x}, t)$ and a coupled continuity equation for a fluid of density $\rho(\mathbf{x}, t)$ and current density $\mathbf{J} = \rho\,\nabla\phi$.

We proceed to study the transformations generated by the symmetry algebra for the free case of this nonlinear equation viewed as a pair of classical equations referred above, and obtain through them new solutions from known initial ones.

In the following, for readability we will rescale the coordinates $\{t, \mathbf{x}\}$ to the dimensionless variables $\{t' = \frac{mc^2}{\hbar}t,\ \mathbf{x}' = \frac{mc}{\hbar}\mathbf{x}\}$ and use dimensionless $V' = \frac{V}{mc^2}$ for potential, dropping primes in the process. We will also eventually use subscripts to denote derivation with respect to the variables, i.e. $\alpha_t = \partial_t\alpha$, $\phi_x = \partial_x\phi$, etc.

We quickly review here the correspondence between the set of classical equations mentioned above and nonlinear Schrödinger-type equations we consider here.

The Hamilton-Jacobi equation resulting from minimizing a variational principle for an Action Functional $S = \phi(t, \mathbf{x})$ in dimensionless variables is

$$\partial_t\phi + \frac{1}{2}(\nabla\phi)^2 + V = 0 \tag{1}$$

Usually one assumes that locality is a characteristic of classical potentials, meaning that the potential function $V = V(t, \mathbf{x})$ is a function in configuration space. Additionally we consider that the momentum per-unit-mass $\mathbf{p} \equiv \nabla\phi$ is coupled with a medium of some "density" $\rho(t, \mathbf{x})$, which means that a current $\mathbf{J} = \rho\,\nabla\phi$ and possibly too localized sources and sinks $\mathcal{F} = \mathcal{F}(t, \mathbf{x})$ are to be restricted to obey a balance or "Continuity" equation

$$\partial_t\rho + \nabla\cdot(\rho\nabla\phi) = \mathcal{F} \tag{2}$$

If we are to assume that $\rho > 0$ always, then we can write $\rho(t, \mathbf{x}) = e^{2\alpha(t,\mathbf{x})}$ for α a real-valued function which would obey

$$\partial_t\alpha + \nabla\alpha\cdot\nabla\phi + \frac{1}{2}\Delta\phi = \frac{1}{2}\frac{\mathcal{F}}{\rho} \tag{3}$$

Casting both (1) and (3) as evolution equations

$$\begin{cases} \partial_t \phi = -\dfrac{1}{2}(\nabla\phi)^2 - V \\ \\ \partial_t \alpha = -(\nabla\alpha)\cdot(\nabla\phi) - \dfrac{1}{2}\Delta\phi + \dfrac{1}{2}\dfrac{\mathcal{F}}{\rho} \end{cases} \tag{4}$$

and multiplying the first equation by $\psi = e^{\alpha + i\phi}$ and the second by $i\psi$ while noting that $\rho = |\psi|^2$, we obtain a nonlinear Schrödinger-type equation for the wave-function ψ

$$i\,\partial_t \psi = \Big(-\frac{1}{2}\Delta + V + \frac{1}{2}\frac{\Delta|\psi|}{|\psi|} + \frac{1}{2}\frac{i\mathcal{F}}{|\psi|^2}\Big)\psi \tag{5}$$

Even when there are no sources or sinks ($\mathcal{F} = 0$), the final theory obtained from the classical Hamilton-Jacobi and the Continuity equations corresponds to a nonlinear field theory for the wave-function ψ, the nonlinearity being due to the appearance of a so-called Quantum Potential

$$Q(\psi) = \frac{1}{2}\frac{\Delta|\psi|}{|\psi|} \tag{6}$$

in the corresponding Schrödinger-type equation.

We must notice at this point that our procedure is totally different from David Bohm's approach to the Quantum Potential [7]-[8] which is equivalent to admitting a subtle, yet fundamental, change in the classical Hamilton-Jacobi equation at (4), introducing there a "generalized" potential $V\,(t, \mathbf{x}, \rho(t, \mathbf{x}))$ that contains a classical potential $U(t,\ \mathbf{x})$ and a quantum potential through the association

$$V\,(t, \mathbf{x}, \rho(t, \mathbf{x})) = U(t,\ \mathbf{x}) - \frac{1}{2}\frac{\Delta\sqrt{\rho(t,\ \mathbf{x})}}{\sqrt{\rho(t,\ \mathbf{x})}} \tag{7}$$

thus actually making the correspondence of a non-local, new and mostly unknown field "mechanics" with the standard Quantum Mechanics of the Schrödinger equation

$$i\,\partial_t \psi = \left(-\frac{1}{2}\Delta + U(t,\ \mathbf{x})\right)\psi \tag{8}$$

at the expense of the classical variational principle.

2 The Free Hamilton-Jacobi Symmetries

In order to study the symmetry algebra of the general free ($V = 0$) nonlinear Schrödinger-type (NLS) equation

$$i\,\partial_t\psi = \frac{1}{2}\Big(-\Delta + \frac{\Delta|\psi|}{|\psi|}\Big)\psi \tag{9}$$

we will use in the 1+1-dimensional case the equivalent equations with $\psi = e^{\alpha+i\,\phi}$:

$$\begin{cases}\phi_t = & -\frac{1}{2}\phi_x^2 \\ \alpha_t = & -\frac{1}{2}\phi_{xx} - \alpha_x\phi_x\end{cases} \tag{10}$$

We will now look for symmetry generated solutions of the autonomous equation $\phi_t = -\frac{1}{2}\phi_x^2$, which is the Hamilton-Jacobi free equation, using prolongation theory to get at the maximum number of symmetries of the solution sub-manifold $\mathcal{S} \subset \mathbb{R}^5$, defined as the locus of points which verify the algebraic equation

$$F(t, \mathbf{x}, \phi, \phi_x, \phi_t) = \phi_x^2 + 2\phi_t = 0 \tag{11}$$

The tangent vector fields to the manifold $\phi = \phi(\mathbf{x}, t)$ which is a solution to this equation is

$$\mathbf{X} = X^t(t, \mathbf{x}, \phi)\partial_t + X^x(t, \mathbf{x}, \phi)\partial_x + Z(t, \mathbf{x}, \phi)\partial_\phi \tag{12}$$

with first prolongation

$$\mathbf{X}^{(1)} = \mathbf{X} + Z^t(t, \mathbf{x}, \phi, \phi_x, \phi_t)\partial_{\phi_t} + Z^x(t, \mathbf{x}, \phi, \phi_x, \phi_t)\partial_{\phi_x} \tag{13}$$

which must verify $\mathbf{X}^{(1)}F = 0$, i.e.

$$Z^x\phi_x + Z^t = 0 \tag{14}$$

Here

$$\begin{cases}Z^x & = \partial_x Z + \phi_x\partial_\phi Z - \phi_t\cdot\big(\partial_x X^t + \phi_x\partial_\phi X^t\big) - \phi_x\cdot\big(\partial_x X^x + \phi_x\partial_\phi X^x\big) \\ Z^t & = \partial_t Z + \phi_t\partial_\phi Z - \phi_t\cdot\big(\partial_t X^t + \phi_t\partial_\phi X^t\big) - \phi_x\cdot\big(\partial_t X^x + \phi_t\partial_\phi X^x\big)\end{cases} \tag{15}$$

Thus, substituting whenever possible ϕ_t by $-\frac{1}{2}\phi_x^2$ we get explicitly:

$$X^{(1)}F = 2Z_t + 2(Z_x - X_t^x)\phi_x + (X_t^t - 2X_x^x + Z_\phi)\phi_x^2 + (X_x^t - X_\phi^x)\phi_x^3 + \frac{1}{2}X_\phi^t\phi_x^4 = 0 \tag{16}$$

which is a polynomial equation of 4-th degree in ϕ_x whose coefficients must all vanish separately to give the system of PDE's :

$$\begin{cases} Z_t &= 0 \\ Z_x &= X^x_t \\ Z_\phi &= 2X^x_x - X^t_t \end{cases} \qquad \begin{cases} X^t_x &= X^x_\phi \\ X^t_\phi &= 0 \end{cases} \tag{17}$$

The solution to this set of PDE's can be seen to be

$$\begin{cases} X^t &= a_1 + 2\,t\,a_4 + x\,a_5 + t\,a_7 + x^2\,a_8 + 2\,t^2\,a_9 + 2\,t\,x\,a_{10} \\ X^x &= a_2 + x\,a_4 + \phi\,a_5 + t\,a_6 + 2\,\phi\,x\,a_8 + 2\,t\,x\,a_9 + \left(2\,\phi\,t + x^2\right)\,a_{10} \\ Z &= a_3 + x\,a_6 - \phi\,a_7 + 2\,\phi^2\,a_8 + x^2\,a_9 + 2\,\phi\,x\,a_{10} \end{cases} \tag{18}$$

where the a_i are constants. These equations generate a 10-dimensional Lie algebra of symmetries for the equation $\phi_x^2 = -2\phi_t$ whose infinitesimal generators are:

$\mathbf{X}_1 =$	∂_t		
$\mathbf{X}_2 =$		∂_x	
$\mathbf{X}_3 =$			∂_ϕ
$\mathbf{X}_4 =$	$2\,t\,\partial_t$	$+x\,\partial_x$	
$\mathbf{X}_5 =$	$x\,\partial_t$	$+\phi\,\partial_x$	
$\mathbf{X}_6 =$		$t\,\partial_x$	$+x\,\partial_\phi$
$\mathbf{X}_7 =$	$t\,\partial_t$		$-\phi\,\partial_\phi$
$\mathbf{X}_8 =$	$x^2\partial_t$	$+2\,\phi\,x\,\partial_x$	$+2\,\phi^2\partial_\phi$
$\mathbf{X}_9 =$	$2\,t^2\partial_t$	$+2\,t\,x\,\partial_x$	$+x^2\partial_\phi$
$\mathbf{X}_{10} =$	$2\,t\,x\,\partial_t$	$+(x^2 + 2\,t\phi)\,\partial_x$	$+2\,x\,\phi\,\partial_\phi$

(19)

which in turn generate the following one-parameter groups :

$\Phi^{1}_{\varepsilon}(t,x,\phi)$	$=($	$t+\varepsilon$	,	x	,	ϕ	$)$
$\Phi^{2}_{\varepsilon}(t,x,\phi)$	$=($	t	,	$x+\varepsilon$	,	ϕ	$)$
$\Phi^{3}_{\varepsilon}(t,x,\phi)$	$=($	t	,	x	,	$\phi+\varepsilon$	$)$
$\Phi^{4}_{\varepsilon}(t,x,\phi)$	$=($	$e^{2\varepsilon}t$	,	$e^{\varepsilon}x$	,	ϕ	$)$
$\Phi^{5}_{\varepsilon}(t,x,\phi)$	$=($	$t+\varepsilon(x+\frac{1}{2}\varepsilon\phi)$	,	$x+\varepsilon\phi$	,	ϕ	$)$
$\Phi^{6}_{\varepsilon}(t,x,\phi)$	$=($	t	,	$x+\varepsilon t$	,	$\phi+\varepsilon(x+\frac{1}{2}\varepsilon t)$	$)$
$\Phi^{7}_{\varepsilon}(t,x,\phi)$	$=($	$e^{\varepsilon}t$	,	x	,	$e^{-\varepsilon}\phi$	$)$
$\Phi^{8}_{\varepsilon}(t,x,\phi)$	$=($	$t+\frac{\varepsilon x^2}{1-2\varepsilon\phi}$	,	$\frac{x}{1-2\varepsilon\phi}$	,	$\frac{\phi}{1-2\varepsilon\phi}$	$)$
$\Phi^{9}_{\varepsilon}(t,x,\phi)$	$=($	$\frac{t}{1-2\varepsilon t}$	,	$\frac{x}{1-2\varepsilon t}$	,	$\phi+\frac{\varepsilon x^2}{1-2\varepsilon t}$	$)$
$\Phi^{10}_{\varepsilon}(t,x,\phi)$	$=($	$\frac{t}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}$	,	$\frac{x(1-\varepsilon x)+2t\phi\varepsilon}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}$	,	$\frac{\phi}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}$	$)$

(20)

Using these, we can generate non-trivial solutions from other known solutions, however trivial these may be. For instance, from $\phi = c$, which indeed is a solution, we obtain

$$\Phi^{i}_{\varepsilon}(t,\mathbf{x},c) = (\tilde{t},\tilde{\mathbf{x}},\tilde{\phi}) \tag{21}$$

The resulting graph $\tilde{\phi} = \tilde{\phi}(t,\mathbf{x},c) = f(\tilde{t},\tilde{\mathbf{x}})$ is another solution of the original equation. In the cases where we can project-out the ϕ-part of the group action, both t and $\mathbf{x}$ can be uniquely represented as functions $t = t_{(\tilde{t},\tilde{\mathbf{x}})}$, $\mathbf{x} = \mathbf{x}_{(\tilde{\mathbf{t}},\tilde{\mathbf{x}})}$ and then the new solution can immediately be calculated as

$$\tilde{\phi} = \tilde{\phi}(t_{(\tilde{t},\tilde{\mathbf{x}})}, \mathbf{x}_{(\tilde{\mathbf{t}},\tilde{\mathbf{x}})}) = f(\tilde{t},\tilde{\mathbf{x}}) \tag{22}$$

Now neither Φ^{1}, Φ^{2}, ϕ^{3}, Φ^{4}, Φ^{5} nor Φ^{7} will change the nature of the $\phi = c$ solution. However $\Phi^{3}_{-c}\Phi^{6}_{\kappa}$ will generate the solution

$$\tilde{\phi} = \kappa\tilde{x} - \frac{1}{2}\kappa^{2}\tilde{t} \tag{23}$$

Indeed, from Φ^{6}_{κ} we gather that

$$\begin{cases} \tilde{t} = t \\ \tilde{x} = x + \kappa t \\ \tilde{\phi} = c + \kappa(x + \frac{1}{2}\kappa t) \end{cases} \tag{24}$$

from which we get

$$\begin{cases} t = \tilde{t} \\ x = \tilde{x} - \kappa\tilde{t} \\ \tilde{\phi} = c + \kappa(\tilde{x} - \frac{1}{2}\kappa\tilde{t}) \end{cases} \tag{25}$$

Thus $\phi = \kappa x - \omega t$ with $\omega = \frac{1}{2}\kappa^2$ is a solution. Likewise, we can generate with $\Phi^3_{-c}\Phi^9_\sigma$ a new solution

$$\tilde{\phi} = \frac{\sigma \tilde{x}^2}{1 + 2\sigma \tilde{t}} \tag{26}$$

Finally, Φ^{10}_σ generates solutions similar to these plus an interesting new solution from $\phi = \kappa x - \frac{1}{2}\kappa^2 t$. In order to get it we have to invert the system of algebraic equations $(\tilde{t}, \tilde{x}, \tilde{\phi}) = \Phi^{10}_\sigma(t, x, \kappa x - \frac{1}{2}\kappa^2 t)$ and obtain

$$\begin{cases} t = \dfrac{\tilde{t}(1 + 2\sigma\kappa\tilde{t})}{(1 + \sigma(\kappa\tilde{t} + \tilde{x}))^2} \\ x = \dfrac{\tilde{x} + (\kappa^2\tilde{t}^2 + \tilde{x}^2)\sigma}{(1 + \sigma(\kappa\tilde{t} + \tilde{x}))^2} \\ \tilde{\phi} = \dfrac{\kappa\tilde{x} - \frac{1}{2}\kappa^2\tilde{t} + \sigma\kappa\tilde{x}^2}{1 + 2\sigma\kappa\tilde{t}} \end{cases} \tag{27}$$

3 Symmetries of the Continuity Equation

Now we have to analyze the symmetries of the second equation in (10) to get the full symmetry algebra of the nonlinear equation (9). Since it is a second order equation, we'll have to use second prolongation theory on the function

$$G(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)}) = \phi_{xx} + 2\alpha_x\phi_x + 2\alpha_t = 0 \tag{28}$$

where $\phi^{(2)} = \{\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{tx}, \phi_{xx}\}$ and similarly for $\alpha^{(2)}$, and thus must substitute (9) and (10) by

$$\mathbf{X} = X^t(t, \mathbf{x}, \phi, \alpha)\partial_t + X^x(t, \mathbf{x}, \phi, \alpha)\partial_x + Z(t, \mathbf{x}, \phi, \alpha)\partial_\phi + W(t, \mathbf{x}, \phi, \alpha)\partial_\alpha \tag{29}$$

with first prolongation

$$\begin{aligned} \mathbf{X}^{(1)} = \mathbf{X} &+ Z^t(t, \mathbf{x}, \phi^{(1)}, \alpha^{(1)})\partial_{\phi_t} + Z^x(t, \mathbf{x}, \phi^{(1)}, \alpha^{(1)})\partial_{\phi_x} + \\ &+ W^t(t, \mathbf{x}, \phi^{(1)}, \alpha^{(1)})\partial_{\alpha_t} + W^x(t, \mathbf{x}, \phi^{(1)}, \alpha^{(1)})\partial_{\alpha_x} \end{aligned} \tag{30}$$

and second prolongation

$$\begin{aligned} \mathbf{X}^{(2)} = \mathbf{X}^{(1)} &+ Z^{tt}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\phi_{tt}} + Z^{tx}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\phi_{tx}} + \\ &+ Z^{xx}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\phi_{xx}} + W^{tt}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\alpha_{tt}} + \\ &+ W^{tx}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\alpha_{tx}} + W^{xx}(t, \mathbf{x}, \phi^{(2)}, \alpha^{(2)})\partial_{\alpha_{xx}} \end{aligned} \tag{31}$$

Although these expressions quickly get out of hand, a suitable symbolic manipulation program can make computations automatic. We therefore obtain after some programming that

$$\mathbf{X}^{(2)}G = Z^{xx} + 2W^{t} + 2Z^{x}\alpha_{x} + 2W^{x}\phi_{x} = 0 \tag{32}$$

In the resulting expressions we must substitute $\phi_t = -\frac{1}{2}\phi_x^2$ and $\alpha_t = -\frac{1}{2}\phi_{xx} - \phi_x\alpha_x$ and then we obtain the additional equations to (17)

$$\begin{cases} X^t_\alpha &= 0 \\ X^x_\alpha &= 0 \\ Z_\alpha &= 0 \\ W_\alpha &= 0 \end{cases} \qquad \begin{cases} W_t &= -\frac{1}{2}X^x_{tx} \\ W_x &= \frac{1}{2}X^x_{xx} - X^x_{t\phi} \\ W_\phi &= -\frac{1}{2}X^x_{x\phi} \end{cases} \tag{33}$$

Integration of these equations is now straightforward, and the alteration on the Lie algebra of symmetries, apart from the addition of a generator of scale transformations $\mathbf{X}_0$, is mainly on the last three fields which should now read:

$$\begin{cases} \mathbf{X}_0 &= \partial_\alpha \\ \mathbf{X}_8 &= x^2\partial_t + 2\phi x\partial_x + 2\phi^2\partial_\phi - \phi\partial_\alpha \\ \mathbf{X}_9 &= 2t^2\partial_t + 2tx\partial_x + x^2\partial_\phi - t\partial_\alpha \\ \mathbf{X}_{10} &= 2tx\partial_t + (x^2 + 2t\phi)\partial_x + 2x\phi\partial_\phi - x\partial_\alpha \end{cases} \tag{34}$$

The corresponding flows are:

$$\begin{array}{|l|}\hline \Phi^0_\varepsilon = (\ t\ ,\ x\ ,\ \phi\ ,\ \alpha+\varepsilon\ ) \\ \hline \Phi^8_\varepsilon = (\ t+\frac{\varepsilon x^2}{1-2\varepsilon\phi}\ ,\ \frac{x}{1-2\varepsilon\phi}\ ,\ \frac{\phi}{1-2\varepsilon\phi}\ ,\ \alpha+\log(\sqrt{1-2\varepsilon\phi})\ ) \\ \hline \Phi^9_\varepsilon = (\ \frac{t}{1-2\varepsilon t}\ ,\ \frac{x}{1-2\varepsilon t}\ ,\ \phi+\frac{\varepsilon x^2}{1-2\varepsilon t}\ ,\ \alpha+\log(\sqrt{1-2\varepsilon t})\ ) \\ \hline \Phi^{10}_\varepsilon = (\ \frac{t}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}, \frac{x(1-\varepsilon x)+2t\phi\varepsilon}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}, \frac{\phi}{(1-\varepsilon x)^2-2t\phi\varepsilon^2}, \alpha+\log(\sqrt{(1-\varepsilon x)^2-2t\phi\varepsilon^2})) \\ \hline \end{array} \tag{35}$$

In summary, we have the full 11-dimensional Lie algebra of symmetries

$$
\begin{array}{lllll}
X_0 = & & & & \partial_\alpha \\
X_1 = & \partial_t & & & \\
X_2 = & & \partial_x & & \\
X_3 = & & & \partial_\phi & \\
X_4 = & 2t\,\partial_t & +x\,\partial_x & & \\
X_5 = & x\,\partial_t & +\phi\,\partial_x & & \\
X_6 = & & t\,\partial_x & +x\,\partial_\phi & \\
X_7 = & t\,\partial_t & & -\phi\,\partial_\phi & \\
X_8 = & x^2\,\partial_t & +2\,x\,\phi\,\partial_x & +2\,\phi^2\,\partial_\phi & -\phi\,\partial_\alpha \\
X_9 = & 2\,t^2\,\partial_t & +2\,x\,t\,\partial_x & +x^2\,\partial_\phi & -t\,\partial_\alpha \\
X_{10} = & 2\,x\,t\,\partial_t & +\left(2\,t\,\phi + x^2\right)\partial_x & +2\,x\,\phi\,\partial_\phi & -x\,\partial_\alpha
\end{array}
\tag{36}
$$

We can now look for the solution generated by the new $\Phi_\sigma^{10}-$symmetries starting from the plane-wave solution ($\phi = \kappa x - \omega t$ with $\omega = \frac{1}{2}\kappa^2$) . This will be (27) and additionally

$$\tilde{\alpha} = \alpha + Log(\frac{\sqrt{1+2\sigma\kappa\tilde{t}}}{1+\sigma(\kappa\tilde{t}+\tilde{x})}) \tag{37}$$

where α is any initial solution for the equation

$$-\alpha_t = \kappa\alpha_x \tag{38}$$

resulting from substitution of $\phi = \kappa x - \omega t$ in the second equation (10). This must therefore be of the form $\alpha = f(x - \kappa t)$ for any arbitrary function $f(x)$ specifying the initial amplitude of the wave. Expressing this in the transformed coordinate system $\{\tilde{t},\ \ \tilde{x}\}$, we get

$$\begin{cases} \tilde{\phi} & = \dfrac{\kappa\tilde{x} - \frac{1}{2}\kappa^2\tilde{t} + \sigma\kappa\tilde{x}^2}{1+2\sigma\kappa\tilde{t}} \\ \tilde{\alpha} & = f(\frac{\tilde{x}-\kappa\tilde{t}}{1+\sigma(\kappa\tilde{t}+\tilde{x})}) + Log(\frac{\sqrt{1+2\sigma\kappa\tilde{t}}}{1+\sigma(\kappa\tilde{t}+\tilde{x})}) \end{cases} \tag{39}$$

A possible solution $\Psi(t, \mathbf{x})$ to the nonlinear Schrödinger equation (9) describing a *free* particle in the classical sense (i.e. no classical potential term V

exists) can well be

$$\Psi(t,\mathbf{x}) = \frac{\sqrt{1+2\sigma\kappa t}}{1+\sigma(\kappa t+x)} F\left(\frac{x-\kappa t}{1+\sigma(\kappa t+x)}\right) e^{i\frac{\kappa x-\frac{1}{2}\kappa^2 t+\sigma\kappa x^2}{1+2\sigma\kappa t}} \tag{40}$$

$[\mathbf{X}_i,\mathbf{X}_j]$	$\mathbf{X}_0$	$\mathbf{X}_1$	$\mathbf{X}_2$	$\mathbf{X}_3$	$\mathbf{X}_4$	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_0$	0	$\mathbf{X}_1$	$\mathbf{X}_2$	$\mathbf{X}_3$	$\mathbf{X}_4$	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_1$	0	0	$\mathbf{X}_2$	$\mathbf{X}_3$	$\mathbf{X}_4$	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_2$	0	0	0	$\mathbf{X}_3$	$\mathbf{X}_4$	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_3$	0	0	0	0	$\mathbf{X}_4$	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_4$	0	$-2\mathbf{X}_1$	$-\mathbf{X}_2$	0	0	$\mathbf{X}_5$	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_5$	0	0	$-\mathbf{X}_1$	$-\mathbf{X}_2$	$\mathbf{X}_5$	0	$\mathbf{X}_6$	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_6$	0	$-\mathbf{X}_2$	$-\mathbf{X}_3$	0	$-\mathbf{X}_6$	$\mathbf{X}_7$	0	$\mathbf{X}_7$	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_7$	0	$-\mathbf{X}_1$	0	$\mathbf{X}_3$	0	$-\mathbf{X}_5$	$\mathbf{X}_6$	0	$\mathbf{X}_8$	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_8$	0	0	$-2\mathbf{X}_5$	$\mathbf{X}_0-2\mathbf{X}_4+4\mathbf{X}_7$	0	0	$-\mathbf{X}_{10}$	$\mathbf{X}_8$	0	$\mathbf{X}_9$	$\mathbf{X}_{10}$
$\mathbf{X}_9$	0	$\mathbf{X}_0-2\mathbf{X}_4$	$-2\mathbf{X}_6$	0	$-2\mathbf{X}_9$	$-\mathbf{X}_{10}$	0	$-\mathbf{X}_9$	0	0	$\mathbf{X}_{10}$
$\mathbf{X}_{10}$	0	$-2\mathbf{X}_5$	$\mathbf{X}_0-2\mathbf{X}_4+2\mathbf{X}_7$	$-2\mathbf{X}_6$	$-\mathbf{X}_{10}$	$-\mathbf{X}_8$	$-\mathbf{X}_9$	0	0	0	0

LIE ALGEBRA OF SYMMETRY GENERATORS FOR NONLINEAR SCHRÖDINGER EQUATION

Notice that for $\sigma = 0$ we recover the plane wave solution if $F(s) = const.$ or a wave of fixed shape but propagating with velocity $v = k$ and frequency $\omega = \frac{1}{2}k^2$. This reminds us that the waves here described cannot simply be summed to obtain a non-dispersive wave-packet.

A close examination of expression (40) convinces us that most such solutions (especially those that are bounded) will experience dispersion as they propagate. Can this be understood as a reflection of the existence of boundary conditions that affect the propagation of the particle? Louis de Broglie did interpret the existence of the quantum potential $\frac{\nabla^2|\Psi|}{|\Psi|}$ as a consequence of obstacles in the propagation of the particle, not so much as potential barriers (which might perhaps be appropriately described as classical potentials $V(x)$) but as boundary descriptions.

4 STUDY OF 2+1-DIMENSIONAL CASE

Another interesting development is the study of 2+1-dimensional case, which should be a suitable generalization of the 1+1-dimensional case already computed. This may provide a better insight on what is at stake when we wish to consider, for instance, the scattering of two particles, or the influence of boundary conditions, or obstacles to propagation...

First of all, the dynamical equations to consider are

$$\begin{cases} \phi_x^2 + \phi_y^2 & = -2\,\phi_t \\ \phi_{xx} + \phi_{yy} & = -2\,\left(\alpha_t + \phi_x\,\alpha_x + \phi_y\,\alpha_y\right) \end{cases} \tag{41}$$

From these we get the replacements

$$\begin{cases} \phi_t & = -\frac{1}{2}\left({\phi_x}^2 + {\phi_y}^2\right) \\ \alpha_t & = -\frac{1}{2}\left(\phi_{xx} + \phi_{yy}\right) - \phi_x\,\alpha_x - \phi_y\,\alpha_y \end{cases} \tag{42}$$

that we will be using in the prolongation formulas. The equations defining the Lie algebra of symmetries are now

$$\begin{cases} \partial_\phi X^t & = 0 \\ \partial_\alpha X^t & = 0 \\ \partial_x X^t & = \partial_\phi X^x \\ \partial_y X^t & = \partial_\phi X^y \\ \partial_t X^t & = 2\,\partial_x X^x - \partial_\phi Z \end{cases} \qquad \begin{cases} \partial^2_{\phi\phi} X^x & = 0 \\ \partial_\alpha X^x & = 0 \\ \partial_x X^x & = \partial_y X^y \\ \partial_y X^x & = -\partial_x X^y \\ \partial_t X^x & = \partial_x Z \end{cases} \qquad \begin{cases} \partial^2_{\phi\phi} X^y & = 0 \\ \partial_\alpha X^y & = 0 \\ \partial_t X^y & = \partial_y Z \end{cases}$$

$$\begin{cases} \partial_\phi W & = -\partial^2_{\phi x} X^x \\ \partial_\alpha W & = 0 \\ \partial_x W & = -\partial^2_{\phi x} Z \\ \partial_y W & = -\partial^2_{\phi y} Z \\ \partial_t W & = -\partial^2_{tx} X^x \end{cases} \qquad \begin{cases} \partial_\alpha Z & = 0 \\ \partial_t Z & = 0 \end{cases} \qquad \begin{cases} \partial^2_{tt} X^x & = 0 \\ \partial^2_{tt} X^y & = 0 \\ \partial^2_{tt} X^t & = 2\,\partial^2_{tx} X^x \\ \partial^2_{tx} W & = 0 \\ \partial^2_{ty} W & = 0 \\ \partial^2_{tt} W & = 0 \end{cases} \tag{43}$$

The general solution to these equations is then

$$\begin{cases} X^t = a_1 + 2\,t\,a_6 + x\,a_7 + y\,a_8 + t\,a_{12} + \left(x^2+y^2\right)a_{13} + 2\,t^2\,a_{14} + 2\,t\,x\,a_{15} + 2\,t\,y\,a_{16} \\ X^x = a_2 + x\,a_6 + \phi\,a_7 - y\,a_9 + t\,a_{10} + 2\,x\,\phi\,a_{13} + 2\,t\,x\,a_{14} + \left(x^2 - y^2 + 2\,t\,\phi\right)a_{15} + 2\,x\,y\,a_{16} \\ X^y = a_3 + y\,a_6 + \phi\,a_8 + x\,a_9 + t\,a_{11} + 2\,y\,\phi\,a_{13} + 2\,t\,y\,a_{14} + 2\,x\,y\,a_{15} + \left(-x^2 + y^2 + 2\,t\,\phi\right)a_{16} \\ Z = a_4 + x\,a_{10} + y\,a_{11} - \phi\,a_{12} + 2\,\phi^2\,a_{13} + \left(x^2+y^2\right)a_{14} + 2\,x\,\phi\,a_{15} + 2\,y\,\phi\,a_{16} \\ W = a_5 - 2\,\phi\,a_{13} - 2\,t\,a_{14} - 2\,x\,a_{15} - 2\,y\,a_{16} \end{cases} \tag{44}$$

and the corresponding Lie algebra generators can be written as

$$
\begin{array}{lccccc}
\mathbf{X}_1 = & \partial_t & & & & \\
\mathbf{X}_2 = & & \partial_x & & & \\
\mathbf{X}_3 = & & & \partial_y & & \\
\mathbf{X}_4 = & & & & \partial_\phi & \\
\mathbf{X}_5 = & & & & & \partial_\alpha \\
\mathbf{X}_6 = & 2t\,\partial_t & +x\,\partial_x & +y\,\partial_y & & \\
\mathbf{X}_7 = & x\,\partial_t & +\phi\,\partial_x & & & \\
\mathbf{X}_8 = & y\,\partial_t & & +\phi\,\partial_y & & \\
\mathbf{X}_9 = & & -y\,\partial_x & +x\,\partial_y & & \\
\mathbf{X}_{10} = & & t\,\partial_x & & +x\,\partial_\phi & \\
\mathbf{X}_{11} = & & & t\,\partial_y & +y\,\partial_\phi & \\
\mathbf{X}_{12} = & t\,\partial_t & & & -\phi\,\partial_\phi & \\
\mathbf{X}_{13} = & (x^2+y^2)\,\partial_t & +2x\phi\,\partial_x & +2y\phi\,\partial_y & +2\phi^2\,\partial_\phi & -2\phi\,\partial_\alpha \\
\mathbf{X}_{14} = & 2t^2\,\partial_t & +2tx\,\partial_x & +2ty\,\partial_y & +(x^2+y^2)\,\partial_\phi & -2t\,\partial_\alpha \\
\mathbf{X}_{15} = & 2tx\,\partial_t & +(x^2-y^2+2t\phi)\,\partial_x & +2xy\,\partial_y & +2x\phi\,\partial_\phi & -2x\,\partial_\alpha \\
\mathbf{X}_{16} = & 2ty\,\partial_t & +2xy\,\partial_x & +(y^2-x^2+2t\phi)\,\partial_y & +2y\phi\,\partial_\phi & -2y\,\partial_\alpha
\end{array}
\tag{45}
$$

Notice the following particular combination of the generators, where $\mathbf{k}$ and $\mathbf{v}$ are constant:

$$
\begin{cases}
\mathbf{X}_6 = 2t\partial_t + \mathbf{x}\cdot\partial_{\mathbf{x}} \\
\mathbf{X}_{7,8} = (\mathbf{k}\cdot\mathbf{x})\partial_t + \phi\,\mathbf{k}\cdot\partial_{\mathbf{x}} \\
\mathbf{X}_9 = \mathbf{x}\wedge\partial_{\mathbf{x}} \\
\mathbf{X}_{10,11} = t\,\mathbf{v}\cdot\partial_{\mathbf{x}} + (\mathbf{v}\cdot\mathbf{x})\partial_\phi \\
\mathbf{X}_{13} = |\mathbf{x}|^2\partial_t + 2\,\phi\,\mathbf{x}\cdot\partial_{\mathbf{x}} + 2\,\phi^2\partial_\phi - 2\,\phi\,\partial_\alpha \\
\mathbf{X}_{14} = 2\,t^2\partial_t + 2\,t\,\mathbf{x}\cdot\partial_{\mathbf{x}} + |\mathbf{x}|^2\partial_\phi - 2\,t\,\partial_\alpha \\
\mathbf{X}_{15,16} = 2\,t\,(\mathbf{k}\cdot\mathbf{x})\partial_t + (2\,(\mathbf{k}\cdot\mathbf{x})\mathbf{x} + (2\,t\,\phi - |\mathbf{x}|^2)\mathbf{k})\cdot\partial_{\mathbf{x}} + \\
\qquad\qquad +2\,\phi\,(\mathbf{k}\cdot\mathbf{x})\partial_\phi - 2\,(\mathbf{k}\cdot\mathbf{x})\partial_\alpha
\end{cases}
\tag{46}
$$

The first 5 fields $\mathbf{X}_1, \ldots, \mathbf{X}_5$ are the generators of space-time translations, phase shift and scaling, revealing the corresponding basic invariances of the equations.

The field $\mathbf{X_6}$ generates scale changes $\{t,\ \mathbf{x}\} \to \{\lambda^2 t,\ \lambda\,\mathbf{x}\}$ in the independent coordinates that also leave the equations unchanged.

Similarly $\mathbf{X_{12}}$ generates time and phase scale changes $\{t, \phi\} \to \{\lambda t, \lambda^{-1}\phi\}$.

Obviously $\mathbf{X_9}$ generates rotations in space.

The field combination $k_1\mathbf{X_7} + k_2\mathbf{X_8} = (\mathbf{k}\cdot\mathbf{x})\partial_t + \phi\mathbf{k}\cdot\partial_{\mathbf{x}}$ generates $\phi-$dependent translations in space-time $\{t, \mathbf{x}\} \to \{t + \varepsilon\mathbf{k}\cdot\mathbf{x} + \frac{1}{2}|\varepsilon\mathbf{k}|^2\phi,\ \mathbf{x} + \varepsilon\mathbf{k}\,\phi\}$.

The field combination $v_1\mathbf{X_{10}} + v_2\mathbf{X_{11}} = t\mathbf{v}\cdot\partial_{\mathbf{x}} + (\mathbf{v}\cdot\mathbf{x})\partial_\phi$ generate a sort of *boost* in $\{\mathbf{x}, \phi\}$-space $\{\mathbf{x}, \phi\} \to \{\mathbf{x} + \varepsilon\mathbf{v}t,\ \phi + \varepsilon\mathbf{v}\cdot\mathbf{x} - \frac{1}{2}|\varepsilon\mathbf{v}|^2\mathbf{t}\}$.

Incidentally, when applied to the static solution $\Psi_o = f(\mathbf{x_o})$ (with $f > 0$ arbitrary), the latter transformations generate solutions of the type

$$\Psi(\tilde{t}, \tilde{\mathbf{x}}) = f(\mathbf{v}\cdot\tilde{\mathbf{x}} - |\mathbf{v}|^2\tilde{\mathbf{t}})\ \mathbf{e}^{\mathbf{i}(\mathbf{v}\cdot\tilde{\mathbf{x}} - \frac{1}{2}|\mathbf{v}|^2\tilde{\mathbf{t}})} \tag{47}$$

The flows of $\mathbf{X_{13}}$ or $\mathbf{X_{14}}$ are easily computable although we see no immediate geometrical meaning to them, but to remark that again here there is a sort of duality between phase ϕ and time t as symmetry transformation generators.

The flow of $\mathbf{X}_{15}$ is computable as follows:

$$\begin{gathered} \frac{d\,t}{d\,\varepsilon} = 2\,t\,x \quad ; \quad \frac{d\,x}{d\,\varepsilon} = 2\,\phi\,t + x^2 - y^2 \\ \frac{d\,y}{d\,\varepsilon} = 2\,x\,y \quad ; \quad \frac{d\,\phi}{d\,\varepsilon} = 2\,\phi\,x \quad ; \quad \frac{d\,\alpha}{d\,\varepsilon} = -2\,x \end{gathered} \tag{48}$$

Set $\tau = \int_0^\varepsilon x(\varepsilon')\,d\varepsilon'$, i.e. $d\tau = x(\varepsilon)d\varepsilon$. Then we obtain the following implicit equations:

$$\begin{cases} t(\varepsilon) = t_0 e^{2\tau(\varepsilon)} \\ y(\varepsilon) = y_0 e^{2\tau(\varepsilon)} \\ \phi(\varepsilon) = \phi_0 e^{2\tau(\varepsilon)} \\ \alpha(\varepsilon) = \alpha_0 - 2\tau(\varepsilon) \end{cases} \qquad \text{and} \qquad \frac{d\,x^2}{d\,\tau} = 2x^2 + 2e^{4\tau}(2t_0\phi_0 - y_0^2) \tag{49}$$

Integration of the last equation yields

$$x^2(\tau) = e^{2\tau}\left(x_0^2 + (2t_0\phi_0 - y_0^2)(e^{2\tau} - 1)\right) \tag{50}$$

From this we can infer that

$$\begin{aligned} \varepsilon = \pm\int_0^\tau \frac{e^{-2\tau'}\,d\tau'}{\sqrt{2t_0\phi_0 - y_0^2 + (x_0^2 + y_0^2 - 2t_0\phi_0)e^{-2\tau'}}} = \\ = \pm\int_1^{e^{-2\tau}} \frac{d\,\xi}{\sqrt{2t_0\phi_0 - y_0^2 + (x_0^2 + y_0^2 - 2t_0\phi_0)\xi}} = \\ = \pm\frac{\sqrt{2t_0\phi_0 - y_0^2 + (x_0^2 + y_0^2 - 2t_0\phi_0)e^{-2\tau}} - x_0}{x_0^2 + y_0^2\ - 2t_0\phi_0} \end{aligned} \tag{51}$$

which we can solve to obtain

$$e^{-2\tau(\varepsilon)} = (1 \pm \varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0) \tag{52}$$

The only consistent solution is

$$\begin{cases} t(\varepsilon) &= \dfrac{t_0}{(1-\varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0)} \\ x(\varepsilon) &= \dfrac{x_0(1-\varepsilon x_0) - \varepsilon(y_0^2 - 2t_0\phi_0)}{(1-\varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0)} \\ y(\varepsilon) &= \dfrac{y_0}{(1-\varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0)} \\ \phi(\varepsilon) &= \dfrac{\phi_0}{(1-\varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0)} \\ \alpha(\varepsilon) &= \alpha_0 + Log((1-\varepsilon x_0)^2 + \varepsilon^2(y_0^2 - 2t_0\phi_0)) \end{cases} \tag{53}$$

Given the fact that fields $\mathbf{X_{15}}$ and $\mathbf{X_{16}}$ commute, we can view the one-parameter groups generated by arbitrary $\mathbf{X} = k_1\mathbf{X_{15}} + k_2\mathbf{X_{16}}$ as the composition of the separate flows of $k_1\mathbf{X_{15}}$ and $k_2\mathbf{X_{16}}$. Also notice that $\mathbf{X_{16}}$ is similar to $\mathbf{X_{15}}$ with the exchange of x and y, so without further ado we can write down the flow of $\mathbf{X}$ as

$$\Phi_\varepsilon^{k_1\mathbf{X_{15}}+k_2\mathbf{X_{16}}} = \begin{cases} \tilde{t} = \dfrac{t_o}{|\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2|\mathbf{x}_o|^{-2} - 2\varepsilon^2 t_o\phi_o} \\ \tilde{x} = \dfrac{\mathbf{x}_o - \varepsilon\mathbf{k}(|\mathbf{x}_o|^2 - 2t_o\phi_o)}{|\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2|\mathbf{x}_o|^{-2} - 2\varepsilon^2 t_o\phi_o} \\ \tilde{\phi} = \dfrac{|\mathbf{x}_o|^2\phi_o}{|\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2|\mathbf{x}_o|^{-2} - 2\varepsilon^2 t_o\phi_o} \\ \tilde{\alpha} = \alpha_o + Log(|\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2|\mathbf{x}_o|^{-2} - 2\varepsilon^2 t_o\phi_o) \end{cases} \tag{54}$$

Let us see the effect of these transformations in some simple cases. We start with the case $\phi_o = const.$.

In order to easily obtain the inversion formulas we shall reformulate these as

$$\begin{cases} |\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2 |\mathbf{x}_o|^{-2} = \left(\frac{t_o}{\tilde{t}}\right)(1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o) \\ \mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2 = \left(\frac{t_o}{\tilde{t}}\right)(\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o) \\ \tilde{\phi} = \left(\frac{\tilde{t}}{t_o}\right)\phi_o \\ \tilde{\alpha} = \alpha_o - Log\left(\frac{\tilde{t}}{t_o}\right) \end{cases} \tag{55}$$

Squaring the second equation and substituting in the first for $|\mathbf{x}_o - \varepsilon\mathbf{k}|\mathbf{x}_o|^2|^2 = |\mathbf{x_o}|^2(1 - 2\varepsilon\mathbf{k}\cdot\mathbf{x}_o + \varepsilon^2|\mathbf{k}|^2|\mathbf{x}_o|^2)$ we get

$$|\mathbf{x_o}|^2 = \left(\frac{t_o}{\tilde{t}}\right)\frac{|\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o|^2}{1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o} \tag{56}$$

Substituting this back in the second equation we obtain

$$\mathbf{x_o} = \left(\frac{t_o}{\tilde{t}}\right)(\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o + \varepsilon\mathbf{k}\ \frac{|\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o|^2}{1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o}) \tag{57}$$

Again, making the inner product of the second equation with $\varepsilon\mathbf{k}$ and using the first equation we get

$$(1 - \varepsilon\mathbf{k}\cdot\mathbf{x_o}) = \left(\frac{t_o}{\tilde{t}}\right)(1 + \varepsilon\mathbf{k}\cdot\tilde{\mathbf{x}}) \tag{58}$$

These last two equations yield

$$\left(\frac{\tilde{t}}{t_o}\right) = (1 + \varepsilon\mathbf{k}\cdot\tilde{\mathbf{x}}) + \varepsilon\mathbf{k}\cdot\tilde{\mathbf{x}} - 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o + |\varepsilon\mathbf{k}|^2\ \frac{|\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o|^2}{1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o} =$$
$$= \frac{|\tilde{\mathbf{x}} + |\tilde{\mathbf{x}}|^2\varepsilon\mathbf{k}|^2}{|\tilde{\mathbf{x}}|^2(1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o)} \tag{59}$$

Using the designation $\sigma(\tilde{t}, \tilde{\mathbf{x}})$ for the function

$$\sigma_{\varepsilon\mathbf{k}}(\tilde{t}, \tilde{\mathbf{x}}) = \frac{|\tilde{\mathbf{x}} + |\tilde{\mathbf{x}}|^2\varepsilon\mathbf{k}|^{\mathbf{2}}}{|\tilde{\mathbf{x}}|^2(1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o)} \tag{60}$$

then

$$\tilde{\Psi}(\tilde{t}, \tilde{\mathbf{x}}) = \sigma_{\varepsilon\mathbf{k}}^{-1}(\tilde{t}, \tilde{\mathbf{x}})\ f\left(\sigma_{\varepsilon\mathbf{k}}^{-1}(\tilde{t}, \tilde{\mathbf{x}})(\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}\ \phi_o + \varepsilon\mathbf{k}\ \frac{|\tilde{\mathbf{x}} - 2\varepsilon\mathbf{k}\ \tilde{t}|^2}{1 + 2|\varepsilon\mathbf{k}|^2\ \tilde{t}\ \phi_o})\right) \times$$
$$\times\ e^{i\sigma_{\varepsilon\mathbf{k}}(\tilde{t}, \tilde{\mathbf{x}})} \tag{61}$$

Particularly, when $\phi_o = 0$, i.e. when $\Psi_o(t_o, \mathbf{x_o}) = \mathbf{f(x)} > \mathbf{0}$ we get

$$\tilde{\Psi}(\tilde{t}, \tilde{\mathbf{x}}) = \frac{|\tilde{\mathbf{x}}|^2}{|\tilde{\mathbf{x}} + |\tilde{\mathbf{x}}|^2 \varepsilon \mathbf{k}|^{\mathbf{2}}} \, f\left(|\tilde{\mathbf{x}}|^2 \, \frac{\tilde{\mathbf{x}} + |\tilde{\mathbf{x}}|^2 \varepsilon \mathbf{k}}{|\tilde{\mathbf{x}} + |\tilde{\mathbf{x}}|^2 \varepsilon \mathbf{k}|^2}\right) \tag{62}$$

Let us now consider the case $\Psi_o = f(\mathbf{v} \cdot (\mathbf{x}_o - \mathbf{v}t_o)) e^{i(\mathbf{v} \cdot \mathbf{x}_o - \frac{1}{2}|\mathbf{v}|^{\mathbf{2}} \mathbf{t_o})}$. It proves easier to work with the moving coordinates $\mathbf{z} = \mathbf{x} - \mathbf{v}t$, and the inversion formulas should read

$$\begin{cases} |\mathbf{z}_o - \varepsilon \mathbf{k}|^2 |\mathbf{z}_o|^{-2} = \left(\frac{t_o}{\tilde{t}}\right)(1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\, \tilde{t}) \\ \mathbf{z}_o - \varepsilon \mathbf{k}\, |\mathbf{z}_o|^2 = \left(\frac{t_o}{\tilde{t}}\right) \tilde{\mathbf{z}} \\ \tilde{\phi} = \left(\frac{\tilde{t}}{t_o}\right) \phi_o \\ \tilde{\alpha} = \alpha_o - Log\left(\frac{\tilde{t}}{t_o}\right) \end{cases} \tag{63}$$

After some of the same manipulations as we did before, we get

$$|\mathbf{z_o}|^2 = \left(\frac{t_o}{\tilde{t}}\right) \frac{|\tilde{\mathbf{z}}|^2}{1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\, \tilde{t}} \tag{64}$$

After completing the square in

$$\varepsilon \mathbf{k} \cdot \mathbf{z}_o - |\varepsilon \mathbf{k}|^2 \, |\mathbf{z}_o|^2 = (1 - \varepsilon \mathbf{k} \cdot \mathbf{z}_o) - |\mathbf{z}_o - \varepsilon \mathbf{k}|^2 |\mathbf{z}_o|^{-2} = \left(\frac{t_o}{\tilde{t}}\right) \varepsilon\, \mathbf{k} \cdot \tilde{\mathbf{z}} \tag{65}$$

we get, solving for $(\frac{\tilde{t}}{t_o})$

$$\begin{cases} \left(\frac{\tilde{t}}{t_o}\right) = 1 + 2\varepsilon \mathbf{k} \cdot (\tilde{\mathbf{z}} + \mathbf{v}\tilde{t}) + \frac{|\varepsilon \mathbf{k}|^{\mathbf{2}} |\tilde{\mathbf{z}}|^{\mathbf{2}}}{1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\tilde{t}} \\ \\ \mathbf{z}_o = \left(\frac{t_o}{\tilde{t}}\right)(\tilde{\mathbf{z}} + \frac{|\tilde{\mathbf{z}}|^2}{1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\tilde{t}} \varepsilon \mathbf{k}) \end{cases} \tag{66}$$

We then shall obtain

$$\tilde{\phi}(\tilde{t}, \tilde{\mathbf{x}}) = \mathbf{v} \cdot \tilde{\mathbf{x}} - \tfrac{1}{2}|\mathbf{v}|^2 \tilde{t} + \frac{|\tilde{\mathbf{x}} - \mathbf{v}\tilde{t}|^2}{1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\tilde{t}} \varepsilon \mathbf{k} \cdot \mathbf{v} \tag{67}$$

and $\tilde{\alpha} = \alpha_o(\mathbf{v} \cdot \mathbf{z}_o) - Log(\frac{\tilde{t}}{t_o})$ thus, setting

$$\sigma_{\varepsilon \mathbf{k}, \mathbf{v}}(\tilde{t}, \tilde{\mathbf{x}}) = 1 + 2\varepsilon \mathbf{k} \cdot \tilde{\mathbf{x}} + \frac{|\varepsilon \mathbf{k}|^{\mathbf{2}} |\tilde{\mathbf{x}} - \mathbf{v}\tilde{\mathbf{t}}|^{\mathbf{2}}}{1 + 2\varepsilon \mathbf{k} \cdot \mathbf{v}\tilde{t}} \tag{68}$$

we have

$$\tilde{\Psi}(\tilde{t},\tilde{\mathbf{x}}) = \sigma^{-1}_{\varepsilon\mathbf{k},\mathbf{v}}\ f\left(\sigma^{-1}_{\varepsilon\mathbf{k},\mathbf{v}}\ \frac{\mathbf{v}\cdot(\tilde{\mathbf{x}}-\mathbf{v}\tilde{t}+(|\tilde{\mathbf{x}}|^2-|\mathbf{v}\tilde{t}|^2)\varepsilon\mathbf{k})}{1+2\varepsilon\mathbf{k}\cdot\mathbf{v}\tilde{t}}\right)\times \\ \times e^{i\frac{\mathbf{v}\cdot(\tilde{\mathbf{x}}-\frac{1}{2}\mathbf{v}\tilde{t}-|\tilde{\mathbf{x}}|^2\varepsilon\mathbf{k})}{1+2\varepsilon\mathbf{k}\cdot\mathbf{v}\tilde{t}}} \tag{69}$$

5 Symmetries of the Free Linear Schrödinger Equation

As mentioned above, it might prove instructive to do these calculations for the simple case of the *linear* free Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{70}$$

and try to get some perspective of what kind of group operation might be involved there that could account for the *superposition principle* of solutions to that equation. In the absence of quantum and classical potentials, and keeping with the notation (2), the equations to consider are now

$$\begin{cases} 2\,\phi_t + {\phi_x}^2 - {\alpha_x}^2 - \alpha_{xx} & = 0 \\ \phi_{xx} + 2\left(\alpha_t + \phi_x\,\alpha_x\right) & = 0 \end{cases} \tag{71}$$

In the ensuing second-prolonged equations, we will perform always the substitutions

$$\begin{cases} \phi_t & = \frac{1}{2}\left({\alpha_x}^2 + \alpha_{xx} - {\phi_x}^2\right) \\ \alpha_t & = -\frac{1}{2}\phi_{xx} - \phi_x\,\alpha_x \end{cases} \tag{72}$$

Recall that the generators of symmetries for this second order system must be the second prolongations $\mathbf{X}^{(\mathbf{2})}$ of the tangent vector fields to the manifold $\mathcal{S} = \{\phi(t,\mathbf{x}),\alpha(\mathbf{t},\mathbf{x})\}$ which is a solution to these equations, where

$$\mathbf{X}_{(t,\mathbf{x},\phi,\alpha)} = X^t\,\partial_t + X^{\mathbf{x}}\cdot\partial_{\mathbf{x}} + Z^1\,\partial_\phi + Z^2\,\partial_\alpha \tag{73}$$

and

$$\mathbf{X}^{(2)}{}_{(t,\mathbf{x},\phi^{(\mathbf{2})},\alpha^{(\mathbf{2})})} = \mathbf{X} + \sum_J\left(Z^{1J}\,\partial_{\phi_J} + Z^{2J}\,\partial_{\alpha_J}\right) \tag{74}$$

with $J \in \{t, x, tt, tx, xx\}$, $\phi^{(2)} = \{\phi_t, \phi_x, \phi_{tt}, \phi_{tx}, \phi_{xx}\}$ (likewise for $\alpha^{(2)}$) and

$$\begin{cases} Z^{1J'\nu} &= D_\nu Z^{1J'} - \sum\limits_{\sigma\in\{t,\mathbf{x}\}} D_\nu X^\sigma \, \partial_\sigma \phi_{J'} \\ Z^{2J'\nu} &= D_\nu Z^{2J'} - \sum\limits_{\sigma\in\{t,\mathbf{x}\}} D_\nu X^\sigma \, \partial_\sigma \alpha_{J'} \end{cases} \tag{75}$$

Performing the usual symbolic manipulations we arrive at the defining PDE's for the Lie algebra generators

$$\begin{cases} X^t_t &= 2\,X^x_x \\ X^t_x &= 0 \\ X^t_\phi &= 0 \\ X^t_\alpha &= 0 \end{cases} \quad \begin{cases} X^x_\phi &= 0 \\ X^x_\alpha &= 0 \end{cases} \quad \begin{cases} Z^1_x &= X^x_t + Z^2_{x\phi} \\ Z^1_t &= \frac{1}{2} Z^2_{xx} \\ Z^1_\phi &= Z^2_\alpha \\ Z^1_\alpha &= -Z^2_\phi \end{cases} \quad \begin{cases} Z^2_t &= -\frac{1}{2} Z^1_{xx} \\ Z^2_x &= \frac{1}{2} X^x_{xx} - Z^2_{x\alpha} \\ Z^2_\phi &= -Z^2_{\phi\alpha} \\ Z^2_\alpha &= Z^2_{\phi\phi} \end{cases} \tag{76}$$

These lead to the following statements

$$\begin{cases} X^x_{xx} &= 0 \\ Z^1_{xx} &= -2\,Z^2_t \\ Z^2_{xx} &= 2\,Z^1_t \\ Z^1_\phi &= Z^2_\alpha \end{cases} \quad \begin{cases} \partial_t \left(Z^1 + Z^1_\alpha\right) &= 0 \\ \partial_x \left(Z^1 + Z^1_\alpha\right) &= X^x_t \\ \partial_\phi \left(Z^1 + Z^1_\alpha\right) &= 0 \\ \partial_\alpha \left(Z^1 + Z^1_\alpha\right) &= 0 \end{cases} \quad \begin{cases} \partial_t \left(Z^2 + Z^2_\alpha\right) &= -\frac{1}{2} X^x_{tx} \\ \partial_x \left(Z^2 + Z^2_\alpha\right) &= 0 \\ \partial_\phi \left(Z^2 + Z^2_\alpha\right) &= 0 \\ \partial_\alpha \left(Z^2 + Z^2_\alpha\right) &= 0 \end{cases} \tag{77}$$

with general solution

$$\begin{cases} X^t &= a_6\,t^2 + 2a_4\,t + a_0 \\ X^x &= a_6\,x\,t + a_5\,t + a_4\,x + a_1 \\ Z^1 &= \left(-a_7\,\sin(\phi) + a_8\,\cos(\phi)\right) e^{-\alpha} + \frac{1}{2}\,a_6\,x^2 + a_5\,x + a_2 \\ Z^2 &= \left(a_7\,\cos(\phi) + a_8\,\sin(\phi)\right) e^{-\alpha} - \frac{1}{2}\,a_6\,t + a_3 \end{cases} \tag{78}$$

This leads to the Lie algebra generators

$$\begin{cases} \mathbf{X}_0 = \partial_t \\ \mathbf{X}_1 = \partial_x \\ \mathbf{X}_2 = \partial_\phi \\ \mathbf{X}_3 = \partial_\alpha \\ \mathbf{X}_4 = 2t\,\partial_t + x\partial_x \\ \mathbf{X}_5 = t\,\partial_x + x\,\partial_\phi \\ \mathbf{X}_6 = t^2\,\partial_t + x\,t\,\partial_x + \frac{1}{2}\,x^2\,\partial_\phi - \frac{1}{2}\,t\,\partial_\alpha \\ \mathbf{X}_7 = \beta(t,x)\,e^{-\alpha}\left(-\sin(\phi)\,\partial_\phi + \cos(\phi)\,\partial_\alpha\right) \\ \mathbf{X}_8 = \gamma(t,x)\,e^{-\alpha}\left(\cos(\phi)\,\partial_\phi + \sin(\phi)\,\partial_\alpha\right) \end{cases} \tag{79}$$

where $\beta(t,x)$ and $\gamma(t,x)$ are both solutions of

$$\left(\partial_{xx} + 2i\,\partial_t\right)\left(\partial_{xx} - 2i\,\partial_t\right)\Theta = 0 \tag{55}$$

verifying the conditions

$$\begin{cases} \partial_{xx}\beta &= 2\,\partial_t\gamma \\ \partial_{xx}\gamma &= -2\,\partial_t\beta \end{cases} \qquad i.e. \qquad \left(\partial_{xx} - 2i\,\partial_t\right)(\gamma + i\beta) = 0 \tag{80}$$

The fields $\mathbf{X}_7$ and $\mathbf{X}_8$ are thus the generators of an infinite-dimensional sub-algebra responsible for the superposition principle as shown in [1].

The fields $\mathbf{X}_0,\ \mathbf{X}_1,\ \mathbf{X}_2,\ \mathbf{X}_3$ again generate translations, phase and scale changes.

The field $\mathbf{X}_4 = 2\,t\,\partial_t + x\,\partial_x$ generates scale transformations in configuration space $\{t,\ x\} \to \{e^{2\varepsilon}t,\ e^{\varepsilon}x\}$

$\mathbf{X}_5 = t\,\partial_x + x\,\partial_\phi$ is the generator for *Galilean* transformations which affect the phase also $\{t, x, \phi\} \to \{t, x + \varepsilon t, \phi + \varepsilon x + \frac{\varepsilon^2}{2}t\}$. This means new solutions

$$\Psi_\varepsilon(t,x) = e^{i(\varepsilon x - \frac{\varepsilon^2}{2}t)}\,\Psi_o(t, x - \varepsilon t) \tag{81}$$

are obtainable from old ones $\Psi_o(t_o, x_o)$. E.g., with $\phi_o(t,x) = const.$, then α_o must solve

$$\begin{cases} \alpha_t = 0 \\ \alpha_x^2 + \alpha_{xx} = 0 \end{cases} \tag{82}$$

thus generally $\alpha_o = Log(ax + b)$, with arbitrary constants a, b. Then $\mathbf{X}_5$ generates new solutions of the form

$$\Psi(t,x) = (a(x - \varepsilon\,t) + b)\,e^{i(\varepsilon x - \frac{\varepsilon^2}{2}t)} \tag{83}$$

$\mathbf{X}_6 = t^2\,\partial_t + x\,t\,\partial_x + \frac{1}{2}\,x^2\,\partial_\phi\ - \frac{1}{2}\,t\,\partial_\alpha$ generates the transformation

$$\begin{cases} t(\varepsilon) = \dfrac{t_o}{1 - \varepsilon t_o} \\ x(\varepsilon) = \dfrac{x_o}{1 - \varepsilon t_o} \\ \phi(\varepsilon) = \phi_o + \dfrac{\varepsilon x_o^2}{2(1 - \varepsilon t)} \\ \alpha(\varepsilon) = \alpha_o + \frac{1}{2}Log(1 - \varepsilon t_o) \end{cases} \tag{84}$$

The inverse transformation in position space is

$$\begin{cases} t_o = \dfrac{t}{1 + \varepsilon t} \\ x_o = \dfrac{x}{1 + \varepsilon t} \end{cases} \tag{85}$$

from which we get new solutions with "Gaussian phase"

$$\Psi_\varepsilon(t, x) = \frac{1}{\sqrt{1 + \varepsilon t}}\, e^{i \frac{\varepsilon x^2}{2(1+\varepsilon t)}}\, \Psi_o\left(\frac{t}{1 + \varepsilon t},\ \frac{x}{1 + \varepsilon t}\right) \tag{86}$$

We notice that under the very restrictive conditions for the existence of nonlinear superposition rules in the case of nonlinear partial differential equations (see e.g. [9]) such as Lie-Bäcklund transformations, the absence of an infinite-dimensional symmetry sub-algebra in our case is a good indicator that no such rule may exist.

6 Conclusion

Through this work we have been able to show via the study of the symmetry algebras of free equations that there is an intrinsic difference between approaches to field theories that produce the linear Schrödinger equation or its non-linear type that displays the Quantum Potential term. To obtain the first one we are forced to extend the Hamilton-Jacobi equation with an additional non-local term, therefore compromising its connection to a variational principle for the path mechanics (which, nonetheless, is used with apparent success anyway in many applications in quantum mechanics). The second approach does not change the classical equations, and therefore their interpretation and connection to variational principles is preserved, but this comes at a cost of nonlinearity, which is very inconvenient for physics. The intrinsic difference is manifest through the distinct nature of the equation's symmetry algebra in both cases, and we were able to show that multiplying the quantum potential by a constant different than unity leads to the a theory mathematically equivalent to the linear one in the free case.

Using the symmetry generators of the nonlinear equation we show how to generate solutions from given solutions, e.g. constants or other standard (linear phase) solutions. We thus obtain Gaussian and more complex phase solutions which could be of physical interest.

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INVESTIGATING THE INFINITY SLOPE IN A NONLINEAR APPROACH

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Summary: The aim of this article is to present some mathematical approaches to a nonlinear dynamics derived from two fundamental classical principles: a Variational Principle (Hamilton-Jacobi equation) and a Balance Principle (Continuity equation). The global dynamics is described by the conjunction of these two equations and we will apply it in a specific and idealized problem: the Infinity Slope. In the first section, we make a general presentation of the nonlinearity, a brief algebraic reference and state our motivations. The second section presents the formal aspects behind our specific nonlinear dynamics. Finally, in the third section we consider the central scheme of our article: we present our problem, the Similarity Methods and Characteristics Equation. With the application of these methods in some particular and idealized cases, we show a way to obtain exact solutions of the nonlinear dynamics.

Keywords: Nonlinearity, Variational Principles, Partial Differential Equation, Hamilton-Jacobi Equation, Continuity Equation, Similarity Methods, Characteristics Equation, Nonlinear Dynamics.

1. NONLINEARITY: MOTIVATION AND PRELIMINARIES

We think that the nonlinear character in an equation can be explored so as to enable a richer range of solutions. Nonlinear solutions can represent situations more general and express non-trivial descriptions for a specific problem. In fact, an equation of the type $y_L = a_0 + a_1 x$ becomes a particular case of more general equations. We can see this algebraic character in the $\mathbb{R}^2$ space. If we consider the linear case y_L and, for example, the two parabolas for $a_0, a_1, \pm a_2 \in \mathbb{R}$:

$$y_{NL} = a_0 + a_1 x + a_2 x^2 \tag{1}$$

then, the general expression (1) includes the more simple equation $y_L = a_0 + a_1 x$ for $a_2 \to 0$. The Taylor series of an arbitrary function $f(x)$ is another clear example of this conception.

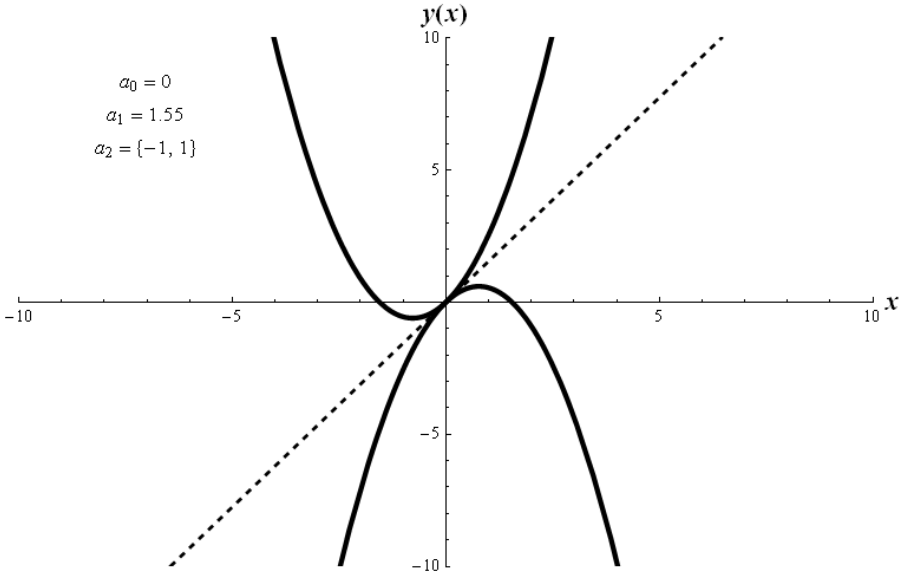


Fig. 1

Thus, a nonlinear situation as the one in equation (1) can incorporate a lot of new characteristics that the linear case doesn't. An example of this is the non null curvature in a geometrical point of view. Since nineteen century the first works of Lobatchevski, János Bolyai, Gauss and Riemann showed that the Euclidean Geometry is a special case when the curvature is zero. Beyond this result we have, for example, the cases for a positive curvature (spherical surface) or a negative curvature (hyperbolic surface). In this sense, the linearity should not be taken as the rule. The linearity should be taken as an exception.

Putting apart this brief historical note, we can ask some questions about the expressions y_L and y_{NL}. For example, we can ask: What is the solution for the problems: $y_L = 0$ and $y_{NL} = 0$? If we take $a_1 \neq 0$, the linear solution is trivially given by the single value:

$$x = -\frac{a_0}{a_1} \tag{2}$$

But in the nonlinear case we can have more than a single value:

$$x = -\frac{a_1}{2a_2} \pm \frac{\sqrt{a_1^2 - 4a_0a_2}}{2a_2} \tag{3}$$

We see that nonlinear equation put in front of us more options of results. In this sense, the nonlinear equation is richer. Eventually, nonlinear equation can contain the linear case. If we take the Taylor expansion, around $a_2 = 0$, in second terms of the solution (3) we obtain:

$$x = -\frac{a_1}{2a_2} \pm \left[\frac{a_1}{2a_2} - \frac{a_0}{a_1} - \frac{a_0^2}{a_1^3}a_2 - \frac{2a_0^3}{a_1^5}a_2^2 + O[a_2^3]\right] \tag{4}$$

Now, for particular limit $a_2 \to 0$, we have:

$$x \cong -\frac{a_1}{2a_2} \pm \left[\frac{a_1}{2a_2} - \frac{a_0}{a_1}\right] \tag{5}$$

And, finally, the two cases:

$$\begin{aligned} x_1 &= -\frac{a_0}{a_1} \\ x_2 &= -\frac{a_1}{a_2} + \frac{a_0}{a_1} \end{aligned} \tag{6}$$

We see that the solution x_1 is compatible with the linear case (2), i. e., the nonlinear equation contains the linear solution as a particular case. But we also have, the singularity solution given by x_2. This solution contains the information of the limit $a_2 \to 0$, because in the original nonlinear equation we had two solutions for each parabola. Then, x_2 informs if the convergence is by left or right, i. e., $a_2 \to 0^{\mp}$. Depending of this convergence, we have the second root of equation (1) in $x_2 \to \pm\infty$.

On the other hand, we can also compare (2) with (3) and reflect it in a physical point of view: If we have two kinds of mathematical models (one linear and the other nonlinear) to describe a particular phenomenological behaviour of a physical system, how many kinds of solution are lost if we don't explore the nonlinear description? Maybe one of those nonlinear solutions would have an important significance in the interpretation of a physical system, but we don't know because we don't have a chance to study

it. Is it worth it to pay that price just for the easiness granted by the linear models? We need to encourage ourselves to research the nonlinear via. These are our initial and general motivations.

Indeed, if we want to attack a nonlinear problem, we will find that it is technically a very hard way. The introductory algebraic example in the relations (1), (2) and (3) is very easy. In a more interesting physical problem we work with Partial Differential Equations (EDP). The search for exact solutions of a nonlinear differential equation becomes a difficult task because this kind of problem lacks analytical general methods of application. The difficulties also become progressive with the increase of dimensionality of the problem: 1D, 2D or 3D. For these reasons, many researchers prefer to continue in linear paradigm.

Nevertheless, it is possible by some work and insight to get interesting results. As an example, we set in the next sections some exact nonlinear solutions derived from the system of fundamentals equations. One of the methods used to obtain some of these solutions was the Similarity and Characteristics Methods. In this paper, we consider an ideal problem in order to apply those methodologies. Before that, we show, in the next section, the fundamental framework on the nonlinear dynamic of our problem.

2. ON A NONLINEAR DYNAMIC

We consider a Lagrangian of type $\mathcal{L}(q_i, \dot{q}_i, t) = T(q_i, \dot{q}_i) - V(q_i, t)$, where q_i and $\dot{q}_i$ are the coordinates and the time derivatives of the points of a space of configurations. Using Hamilton's principle on the integral that defines the action $S = \int_{t_0}^{t} \mathcal{L} dt$ and considering a particle with mass m, $T = m\dot{q}_i{}^2/2$, we are led to the extreme condition $\delta S = 0$ on all possible paths of integration between the points $P_0 = P|_{t0}$ and $P = P|_t$. Consequently, we obtain a dynamics described by the following equation:

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + \mathrm{V}\right] \tag{7}$$

The equation (7) is a natural consequence and a general result of the application of variational principles. From the generality of equation (7) we can derive well know particular results. For instance, in terms of conventional symbolism $\vec{p} = \nabla S$ and $E = -\frac{\partial S}{\partial t}$ the equation of Hamilton-Jacobi (7) takes the form $E = T + V$. We can thus find fundamental features

characterizing the Classical Mechanics. If we differentiate (7) with respect to a classical trajectory (applying the gradient ∇), we get:

$$\frac{\partial \vec{p}}{\partial t} = -\vec{p}.\frac{\nabla \vec{p}}{m} - \nabla \mathrm{V} \implies \frac{\partial \vec{p}}{\partial t} + \vec{p}.\frac{\nabla \vec{p}}{m} = -\nabla \mathrm{V} \tag{8}$$

But the total derivative of the moment $\vec{p}$ can be written as: $d\vec{p} = \frac{\partial \vec{p}}{\partial t} dt + \nabla \vec{p}.\, d\vec{q}$. Therefore, by substituting this result in (8) and using the definition $\vec{p} = m\frac{d\vec{q}}{dt}$, we obtain the Newton's second law:

$$\frac{d\vec{p}}{dt} = -\nabla \mathrm{V} = \vec{\mathrm{F}} \tag{9}$$

However, we are interested in observing another more general result that can be obtained from the equation (7). To that end, we consider the Continuity Equation. Thus, we want to work with the system:

$$\begin{cases} E = T + V \\ \dfrac{\partial \rho}{\partial t} = -\nabla J \end{cases} \tag{10}$$

If we use again the relations $\rho = R^2$ and $J = \rho v = R^2 \frac{\nabla S}{m}$, following a series of algebraic manipulations (see [2] for this steps), we obtain:

$$-\frac{\hbar^2}{2m}\nabla^2 \Theta + \left(\frac{\hbar^2}{2m}\frac{\nabla^2 \mathrm{R}}{\mathrm{R}} + V\right)\Theta = \mathrm{i}\hbar\frac{\partial \Theta}{\partial t} \tag{11}$$

where $\Theta(\vec{r}, t) = R(\vec{r}, t)e^{\frac{i}{\hbar}S(\vec{r},t)}$. If we compare the equation (11) with the Schrödinger Equation, we observe that the equation (11) represents a nonlinear dynamics due to the term $\frac{\hbar^2}{2m}\frac{\nabla^2 \mathrm{R}}{\mathrm{R}}$ defined as the "quantum potential". The equation (11) is a natural consequence of Hamilton's Principle in conjunction with the Continuity Equation. Now, if, for convenience, we use the scale transformations which was suggest by Rica da Silva and Croca [3]:

$$\Theta(\vec{r},t) = R(\vec{r},t)e^{\frac{i}{\hbar}S(\vec{r},t)} \;\longrightarrow\; e^{\alpha+i\phi}$$
$$\left(\frac{mc^2}{\hbar}\mathrm{t}, \frac{mc}{\hbar}\vec{\mathrm{r}}, \frac{1}{mc^2}\mathrm{V}\right) \;\longrightarrow\; (t, \vec{\mathrm{r}}, \mathrm{V}) \tag{12}$$

the system of equations (10) can be rewritten as:

$$\begin{cases} \frac{1}{2}(\nabla\phi)^2 + \frac{\partial\phi}{\partial t} + \mathrm{V} = 0 \;\; \text{(Hamilton-Jacobi)} \\ \frac{\partial\alpha}{\partial t} + \nabla\alpha\nabla\phi + \frac{1}{2}\nabla^2\phi = 0 \;\; \text{(Continuity)} \end{cases} \tag{13}$$

And the quantum potential can be expressed as:

$$\frac{\hbar^2}{2m}\frac{\nabla^2\mathrm{R}}{\mathrm{R}} = \frac{1}{2}(\nabla^2\alpha + (\nabla\alpha)^2) \tag{14}$$

The system (13), and thus the nonlinear equation (11), synthesizes two concepts. The first: the equation of Hamilton-Jacobi reflects a dynamic derived from the application of the Extreme's Principle to the action ϕ (phase), of which the following is an illustration.

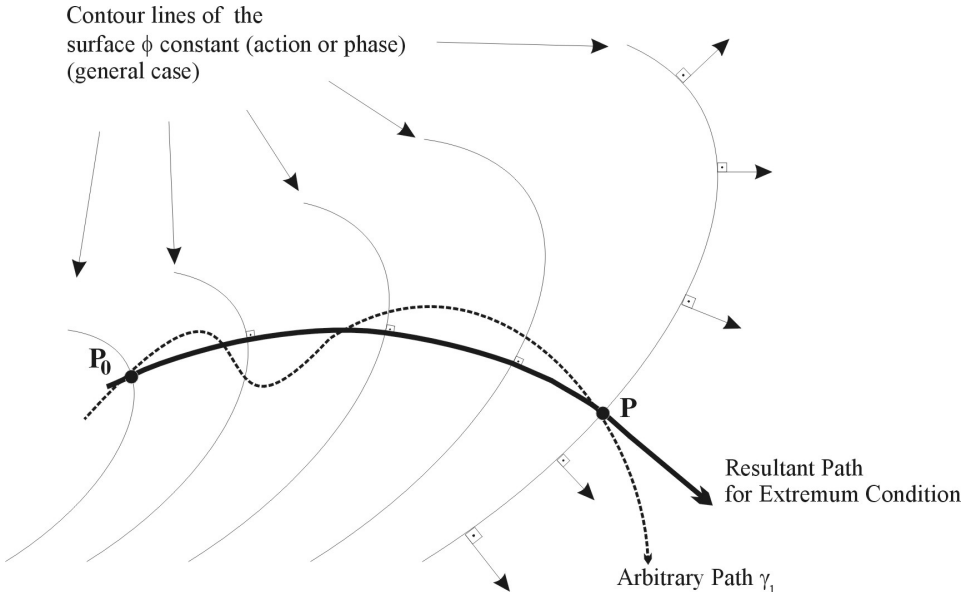
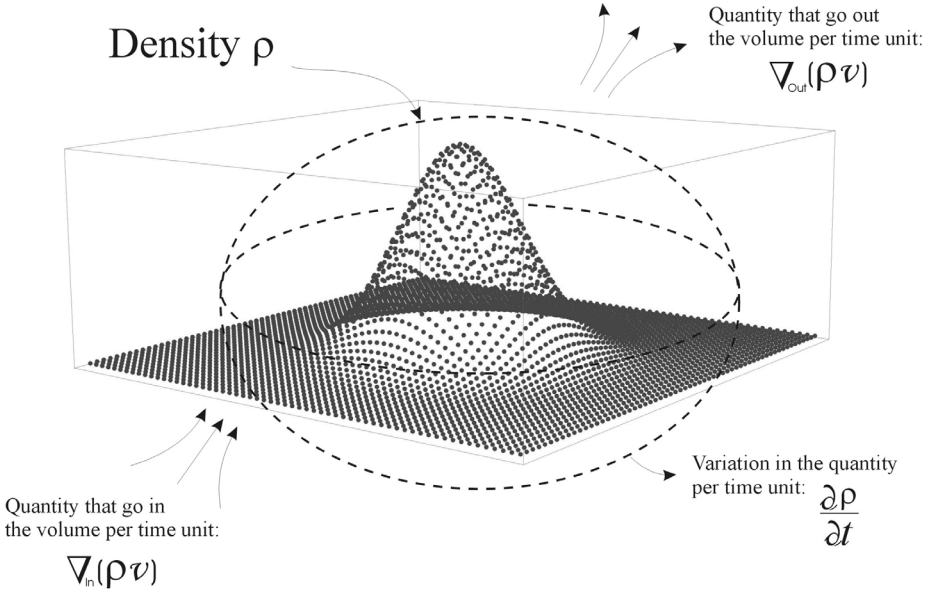


Fig. 2

Generally, the shape of the surface ϕ is arbitrary and the condition $\phi = cte$ allows us to visualize the contours lines of ϕ, connecting two points P_0 and P in the space. In general, any arbitrary paths $\gamma_1$connect P_0 and P. However, according to the Hamilton Principle, the "good path" must be the one that derives from the condition that makes null the variation of the action. The result of this condition is the first equation of the system (13). The notion of trajectory is inherent in this equation and, consequently, the notions of "localized nature" are closely related.

However, the Continuity Equation in the system (13) expresses a balance condition. Previously, we defined the relationship $\rho = R^2$ and $J = \rho v = R^2 \frac{\nabla \phi}{m}$. This mean that we assume some property of the physical system characterized by a density ρ and a current J. We don't discuss here this physical property (mass, charge, etc.). The important point to stress is the fact that during the evolution of the system, according to the trajectory resulting from the extreme condition shown in the previous figure, this physical property is conserved (the balance notion). The figure below provides an intuitive notion on the meaning contained in the Continuity Equation.



Resultant Balance Equation:

$$\nabla_{In}(\rho v) - \nabla_{Out}(\rho v) = -\nabla \mathbf{J} = \frac{\partial \rho}{\partial t}$$

Fig. 3

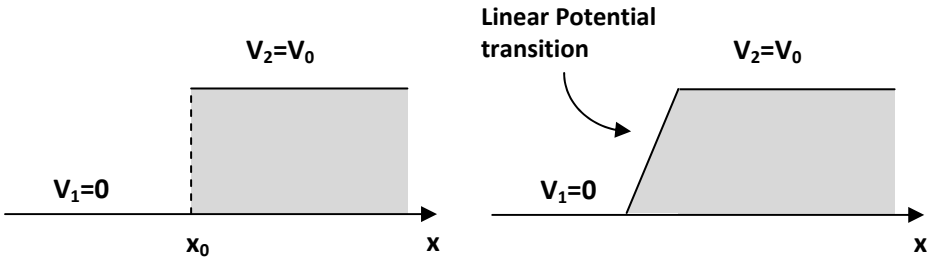
With this last scheme, it is also clear that what lies behind the Continuity Equation is an essence of "extended nature".

Therefore, we think that the combination of both equations of system (13) includes, in a sense, the integration of two kinds of nature: localized and extended. Physically, this interpretation leads us to imagine an object with a certain extension, where a particular property is described by the density function ρ. If we identify each point of this extended object with the coordinates $\vec{q}$, the evolution of these points will be given while respecting, at the same time, the principle of extreme (Hamilton's Principle) and a balance condition type (Continuity Equation). Overall, the evolution of the particle follows a nonlinear process characterized by a current density $J = \rho v$ and described by the nonlinear global equation (11).

Technically, the strategy of resolution of the equation (11) considers the system of equations (13). First, we must solve the Hamilton-Jacobi equation because the phase solution, $\phi(\vec{r}, t)$, will incorporate the potential effects $V(\vec{r}, t)$. The next step is to use the phase solution in Continuity equation to determine the amplitude solution $\alpha(\vec{r}, t)$. We observe in this last case that the amplitude solution depends implicitly on the potential $V(\vec{r}, t)$, because it is incorporated in the phase solution.

3. THE INFINITE SLOPE IN NONLINEAR APPROACH

Consider an ideal situation. We are in a very large and perfect slope. We cannot see the bottom or the upper part of the slope. The slope is like a huge mountain. We are interested in this kind of problem because it will allow us, in future analyses, to elaborate a problem composed of more than one potential conjunction. In fact, a common description, it is natural to begin by describing a potential barrier (Figure 4a) where we have two regions with potentials constants and a jump in x_0. However, in Figure 4b we adjust this idea to a slightly more soft transition: a linear transition between V_1 and V_2. For this reason, we'll only study the linear potential in the nonlinear dynamic exposed in the last section.



(a) (b)

Fig. 4

Let's return to both equations (13). We'll only consider problem 1+1 (i. e., one spatial dimension and the temporal parameter). Our problem deals with the linear potential given by $V(x) = \gamma x + \beta$ (where $\gamma, \beta \in \mathbb{R}$ are constants). Thus, the system to be considered must be simplified in the following way:

$$\begin{cases} \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{\partial \phi}{\partial t} + \gamma x + \beta = 0 \quad \text{(Hamilton-Jacobi)} \\ \frac{\partial \alpha}{\partial t} + \left(\frac{\partial \alpha}{\partial x}\right)\left(\frac{\partial \phi}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right) = 0 \quad \text{(Continuity)} \end{cases} \tag{15}$$

In accordance with our strategy, we must begin by solving the Hamilton-Jacobi equation. The first step is to consider the following basic transformation:

$$V(x) = \gamma x + \beta = \gamma\left(x + \frac{\beta}{\gamma}\right) = \gamma X = V(X) \tag{16}$$

It is trivial to verify that in the new variable we have $\frac{\partial \phi(x,t)}{\partial x} = \frac{\partial \Phi(X,t)}{\partial X}$, $\frac{\partial \phi(x,t)}{\partial t} = \frac{\partial \Phi(X,t)}{\partial t}$. Now, the Hamilton-Jacobi equation has a little more simplified form:

$$\frac{1}{2}\left(\frac{\partial \Phi}{\partial X}\right)^2 + \frac{\partial \Phi}{\partial t} + \gamma X = 0 \tag{17}$$

In this point, we apply the Similarity Method. The fundamental idea is to look for a transformation that maintains invariant the form of the equation (17). We consider the one-parameter family of stretching transformations:

$$\bar{X} = \lambda^a X \quad ; \quad \bar{t} = \lambda^b t \quad ; \quad \bar{\Phi} = \lambda^c \Phi \tag{18}$$

With this transformation, we observe that:

$$\frac{\partial \bar{\Phi}}{\partial \bar{t}} = \lambda^{c-b} \left(\frac{\partial \Phi}{\partial t}\right) \quad ; \quad \frac{\partial \bar{\Phi}}{\partial \bar{X}} = \lambda^{c-a} \left(\frac{\partial \Phi}{\partial X}\right) \tag{19}$$

Now, let's rewrite the equation (17) to investigate what are the invariance conditions. In other words, let's consider:

$$\frac{1}{2}\left(\frac{\partial \bar{\Phi}}{\partial \bar{X}}\right)^2 + \frac{\partial \bar{\Phi}}{\partial \bar{t}} + \gamma \bar{X} = 0 \tag{20}$$

After the substitution of transformation (18) and (19) we obtain:

$$\frac{1}{2}\lambda^{2(c-a)} \left(\frac{\partial \Phi}{\partial X}\right)^2 + \lambda^{c-b} \left(\frac{\partial \Phi}{\partial t}\right) + \gamma \lambda^a X \ = 0 \tag{21}$$

Comparing the equations (17) and (21), we conclude that to guarantee the similarity between both equations, we have to impose:

$$2(c-a) = c - b = a \tag{22}$$

In other words, the conditions in the transformation are:

$$\bar{X} = \lambda^{\frac{2c}{3}} X \quad ; \quad \bar{t} = \lambda^{\frac{c}{3}} t \quad ; \quad \bar{\Phi} = \lambda^c \Phi \tag{23}$$

Now, we'll employ a fundamental theorem of the Similarity Method and, for the current purposes of this paper, we'll state this theorem using the specific equations (transformations) mentioned above.

<u>Theorem</u>: If a Partial Differential Equation, as the equation (17), is invariant for a transformation of the kind (18), then the transformation:

$$\Phi(\mathrm{X}, \mathrm{t}) = \mathrm{t}^{\mathrm{c/b}}\, \Gamma(\mathrm{z}) \quad ; \quad \mathrm{z} = \frac{\mathrm{X}}{\mathrm{t}^{\mathrm{a/b}}} \tag{24}$$

reduce the equation (17) to a Ordinary Differential Equation in $\Gamma(z)$. This ODE will has the form: $\mathrm{g}\big(\mathrm{z}, \Gamma(\mathrm{z}), \Gamma'(\mathrm{z})\big) = 0$.

Appling the condition (22) to this theorem, we obtain explicitly:

$$\begin{gathered} \Phi(\mathrm{X},\mathrm{t}) = t^3\,\Gamma(z) \quad ; \quad z = \frac{X}{t^2} \\ \frac{\partial \Phi}{\partial X} = t\Gamma'(z) \quad ; \quad \frac{\partial \Phi}{\partial t} = 3t^2\Gamma(z) - 2X\Gamma'(z) \end{gathered} \tag{25}$$

When we substitute this transformation in the original equation (17), we reduce it to a Nonlinear Ordinary Differential Equation given by:

$$2\gamma \mathrm{z} + 6\Gamma(z) - 4z\Gamma'(z) + \Gamma'(z)^2 = 0 \tag{26}$$

We want to explore this equation in order to obtain nonlinear solutions depending of X and t. To this effect, we'll write the solutions in a quadratic polynomial form and, then, see if this form can give us exact and nonlinear solutions. Thus, we propose:

$$\Gamma(z) = \mathrm{a}_0 + \mathrm{a}_1\mathrm{z} + \mathrm{a}_2\mathrm{z}^2 \tag{27}$$

In fact, when we investigate solutions of this form in equation (26), we obtain two possible set of valid coefficients conditions:

$$\begin{gathered} \mathrm{a}_0 \to -\gamma^2/6 \ , \ \mathrm{a}_1 \to -\gamma \ , \ \mathrm{a}_2 \to 0 \\ \text{or} \\ \mathrm{a}_0 \to -\gamma^2/24 \ , \ \mathrm{a}_1 \to -\gamma/2 \ , \ \mathrm{a}_2 \to 1/2 \end{gathered} \tag{28}$$

Explicitly, we have:

$$\begin{gathered} \Phi^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t}) = t^3\left[-\frac{\gamma^2}{6} - \gamma\left(\frac{X}{t^2}\right)\right] = -\frac{\gamma\, t(\gamma \mathrm{t}^2 + 6X)}{6} \\ \text{and} \\ \Phi^{\mathrm{NL}}_{\mathrm{type2}}(\mathrm{X},\mathrm{t}) = t^3\left(-\frac{\gamma^2}{24} - \frac{\gamma}{2}\left(\frac{X}{t^2}\right) + \frac{1}{2}\left(\frac{X}{t^2}\right)^2\right) = \frac{-\gamma^2 t^4 - 12\gamma t^2 X + 12X^2}{24t} \end{gathered} \tag{29}$$

Remember that, in accordance to relation (16), we have $X = (\mathrm{x} + \beta/\gamma)$. Our next step is to solve the continuity equation that, in variable X, maintains the same form:

$$\frac{\partial \alpha}{\partial t} + \left(\frac{\partial \alpha}{\partial X}\right)\left(\frac{\partial \phi}{\partial X}\right) + \frac{1}{2}\left(\frac{\partial^2 \phi}{\partial X^2}\right) = 0 \text{ (Continuity Equation)} \tag{30}$$

The solution of amplitude $\alpha(X,t)$ depend of each analytical form of nonlinear phase of type 1 or 2 in solutions (29). We will treat each one separately.

3.1. NONLINEAR SOLUTION TYPE 1

When we substitute the first type of nonlinear solutions for phase (29) in continuity equation (30), we have:

$$\frac{\partial \alpha}{\partial t} = \gamma\, t\, \frac{\partial \alpha}{\partial X} \tag{31}$$

If we adopt a new notation $\vec{q} = \{q_1, q_2\} = \{t, X\}$ and $\vec{p} = \{p_1, p_2\} = \left\{\frac{\partial \alpha}{\partial t}, \frac{\partial \alpha}{\partial X}\right\}$ we have a correspondent algebraic equation:

$$\mathcal{F}(\vec{q}, \alpha, \vec{p}) = p_1 - \gamma q_1 p_2 = 0 \tag{32}$$

Now we can calculate the characteristic equations. In general form, we have:

$$\begin{gathered} \dot{q}_i = \partial_{p_i} \mathcal{F} \\ \dot{\alpha} = \vec{p} \cdot \nabla_{\vec{p}} \mathcal{F} \\ \dot{p}_i = -\left(\partial_{q_i} \mathcal{F} + (\partial_\alpha \mathcal{F}) p_i\right) \end{gathered} \tag{33}$$

where we put $\partial_{w_i} = \frac{\partial}{\partial w_i}$ and the total derivation ($\dot{q}_i$, $\dot{\alpha}$ and $\dot{p}_i$) is effected in relation to the parameter τ. After applying (33) in algebraic equation (32) we obtain:

$$\frac{dq_1}{1} = \frac{dq_2}{-\gamma q_1} = \frac{d\alpha}{0} = \frac{dp_1}{\gamma p_2} = \frac{dp_2}{0} \tag{34}$$

The integration of first three equations (34) is trivial. It gives us:

$$\begin{gathered} q_1(\tau) = \tau + c_0 \\ q_2(\tau) = -\frac{\gamma \tau^2}{2} - \gamma c_0 \tau + c_1 \\ \alpha(\tau) = c_2 \end{gathered} \tag{35}$$

In these equations, c_0, c_1 and c_2 are integration's constants. Particularly, our solution α depend of constant c_2. For this raison, we can write c_2 in terms of others constants c_0 and c_1. When we eliminate the parameter τ in first two equations (35) and put $c_2 = \Lambda(c_0, c_1)$, we obtain, in old variables $\{t, X\}$, the general form solution:

$$\alpha^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t}) = \Lambda\left(\frac{2\mathrm{X} + \gamma\mathrm{t}^2}{2}\right) \tag{36}$$

We note that $\Lambda(w)$ is a arbitrary function in coupled variable $w = \frac{2\mathrm{X}+\gamma\mathrm{t}^2}{2}$. In other words, the solution (36) gives us infinities of particular solutions. In short, we have the nonlinear solution type 1 given by (29) and (36), i. e.:

$$\begin{aligned} \Phi^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t}) &= -\frac{\gamma\, t(6X + \gamma\mathrm{t}^2)}{6} \\ \alpha^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t}) &= \Lambda\left(\frac{2\mathrm{X} + \gamma\mathrm{t}^2}{2}\right) \end{aligned} \tag{37}$$

But we want to find at least one interesting solution to see its global scattering behaviour. We propose to write the general solution in amplitude $R(\vec{r}, t) = e^{\alpha(\vec{r},t)}$ as a Gaussian distribution. For this, we need a quadratic function to $\alpha^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t})$. If we return to the original coordinate $\mathrm{X} = \left(\mathrm{x} + \frac{\beta}{\gamma}\right)$, we have:

$$\alpha^{\mathrm{NL}}_{\mathrm{type1}}(\mathrm{X},\mathrm{t}) = -\left[\left(\mathrm{x} + \frac{\beta}{\gamma}\right) + \frac{\gamma\mathrm{t}^2}{2}\right]^2 \tag{38}$$

We can to rearrange the argument in expression (38) and put it in terms of the average result $\langle \mathrm{x}^{\mathrm{cl}} \rangle$, i. e.:

$$\langle \mathrm{x}^{\mathrm{cl}} \rangle := x(t) = -\gamma\frac{t^2}{2} + v_0 t + x_0 \tag{39}$$

Thus, the amplitude function $R(\vec{r}, t) = e^{\alpha(\vec{r},t)}$ can be read in form:

$$R^{NL}_{type1}(\mathrm{x},\mathrm{t}) = \sqrt{\frac{2}{\pi}}\mathrm{e}^{-\left[\mathrm{x}-\langle \mathrm{x}^{\mathrm{cl}}\rangle+\mathrm{v}_0\mathrm{t}\right]^2} \tag{40}$$

In expression (38) we put an additional normalization constant. The amplitude in (40) is related with a density $\rho = R^2$. Thus, the expression (40) give us an expectation value of position x related with a temporal translation in classical result, i. e.:

$$\langle x\rangle = \int_{-\infty}^{+\infty} \mathrm{x}\rho(\mathrm{x},\mathrm{t})\mathrm{dx} = \langle \mathrm{x}^{\mathrm{cl}}\rangle - \mathrm{v}_0\mathrm{t} \tag{41}$$

A plot-3D of amplitude function (40) is given bellow:

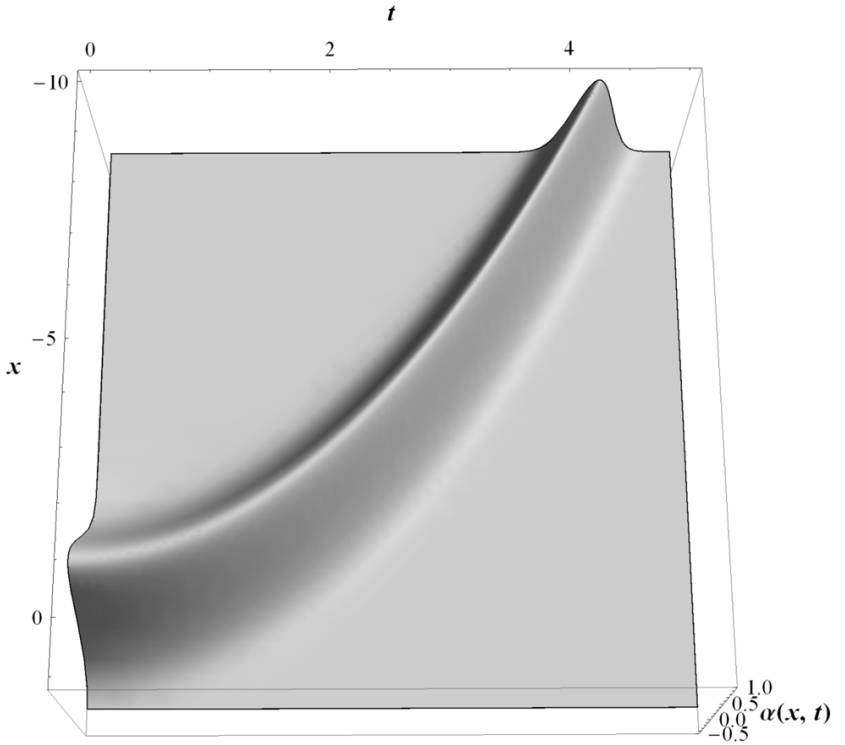


Fig. 5

If we use the scale transformation (12) to the phase, the complete solution is given by:

$$\Theta_1(x,t) = e^{\alpha^{NL}_{type1} + i\Phi^{NL}_{type1}} = \sqrt{\frac{2}{\pi}}\, e^{-\left[\left(x+\frac{\beta}{\gamma}\right)+\frac{\gamma t^2}{2}\right]^2} . e^{-i\frac{\gamma\, t\left(6\left(x+\frac{\beta}{\gamma}\right)+\gamma t^2\right)}{6}} \qquad (42)$$

If we define $\vec{p} = \nabla\Phi$ as a momentum, the global structure moves with a decreasing tax given by:

$$p^{NL}_{type1} = -\gamma\, t \qquad (43)$$

After $t = 0$, the structure goes with a negative momentum to direction $x \to -\infty$. The next figure shows us the Real part of expression (42) and we can note the global Gaussian envelope. Its velocity change depending if it is near or far of $t = 0$.

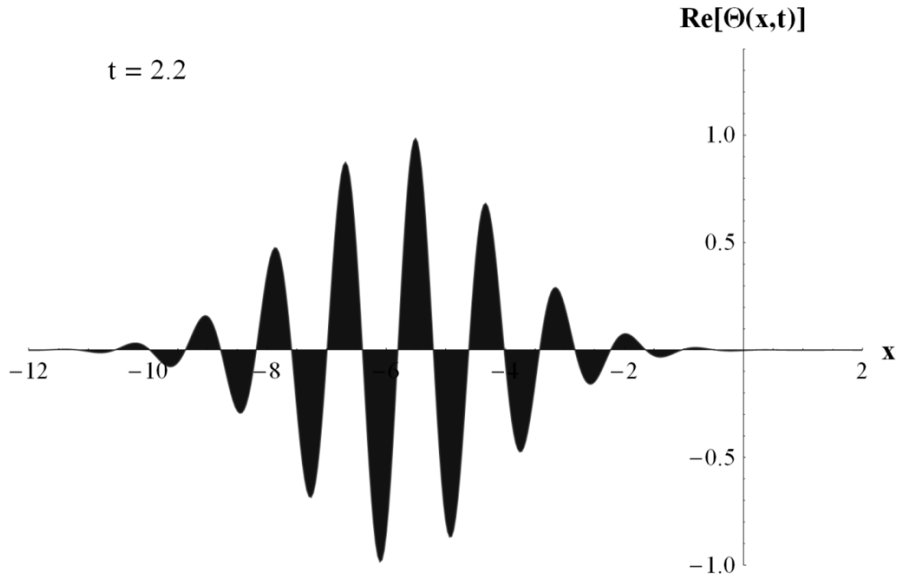


Fig. 6

3.2. NONLINEAR SOLUTION TYPE 2

Now, we consider the second type of nonlinear solutions for phase (29). Putting it into continuity equation (30), we have:

$$\frac{1+2t\partial_t\alpha(X,t)+(-kt^2+2X)\partial_X\alpha(X,t)}{2t}=0 \tag{44}$$

We can to adopt the same way that was used in steps (32) and (33). Thus we have:

$$\frac{dq_1}{2q_1}=\frac{dq_2}{-\gamma {q_1}^2+2q_2}=\frac{d\alpha}{-1}=\frac{dp_1}{2\gamma p_2 q_1-2p_1}=\frac{dp_2}{-2p_2} \tag{45}$$

After integration of first three equations (45), we have:

$$\begin{aligned} q_1(\tau) &= Ae^{2\tau} \\ q_2(\tau) &= -\frac{1}{2}\gamma A^2 e^{4\tau}+Be^{2\tau} \\ \alpha(\tau) &= -\tau+C \end{aligned} \tag{46}$$

We can associate the first two integration constants (A and B) and obtain a particular relation to the originals variables $\{q_1,q_2\}=\{t,X\}$ given by:

$$\frac{B}{A}=\frac{2X+\gamma t^2}{2t} \tag{47}$$

With this expression, we can look for a particular form to $\alpha(\tau)$. We can suggest a format that depends on an arbitrary function $\Sigma(\mathrm{B/A})$:

$$\alpha_{\mathrm{type2}}^{\mathrm{NL}}(\mathrm{X,t})=-\frac{1}{2}\mathrm{Log}\left[\pm\left(\frac{2X+\gamma t^2}{2t}\right)+\sqrt{-2\gamma X+\left(\frac{2X+\gamma t^2}{2t}\right)^2}\right]+\Sigma\left(\frac{2X+\gamma t^2}{2t}\right) \tag{48}$$

Thus, our nonlinear solution type 2 is given by these amplitude solutions (48) and the phase solution (29), i. e. $\Phi_{\mathrm{type2}}^{\mathrm{NL}}(\mathrm{X,t})=(-\gamma^2t^4-12\gamma t^2X+12X^2)/(24t)$. Again, we have infinity possible mathematical solutions. So, to obtain a Gaussian behaviour, we will consider the particular quadratic case in which:

$$\Sigma\left(\frac{2X+\gamma t^2}{2t}\right) = -\left(\frac{2X+\gamma t^2}{2t}\right)^2 \tag{49}$$

where we remember the original coordinate $\mathrm{X} = \left(\mathrm{x} + \frac{\beta}{\gamma}\right)$. With this proposal and with the phase solution $\Phi^{\mathrm{NL}}_{\mathrm{type2}}(\mathrm{X},\mathrm{t})$, the global composition give us:

$$\Theta_2(x,t) = e^{-\frac{1}{2}\mathrm{Log}\left[\pm\left(\frac{2X+\gamma t^2}{2t}\right)+\sqrt{-2\gamma X+\left(\frac{2X+\gamma t^2}{2t}\right)^2}\right]-\left(\frac{2X+\gamma t^2}{2t}\right)^2} \times e^{-i\frac{\left(-\gamma^2 t^4-12\gamma t^2 X+12X^2\right)}{24t}} \tag{50}$$

In contrast with the solution type 1, the present solution produces a little more complicated momentum with time and space dependence:

$$\mathrm{p}^{\mathrm{NL}}_{\mathrm{type2}}(X,t) = \frac{24X-12\gamma t^2}{24t} \tag{51}$$

Here, we have not the single linear proportionality classical relation (43). We also note that there is a singularity in $t = 0$. The motion of particle depends, in space and time, of the potential presence. We can think that the density $\rho = R^2$ suffers, progressively, a transformation in its form. A plot-3D of amplitude function (48) with the quadratic function (49) and the potential $\mathrm{V(x)} = \gamma\mathrm{x} + \beta$ is given bellow:

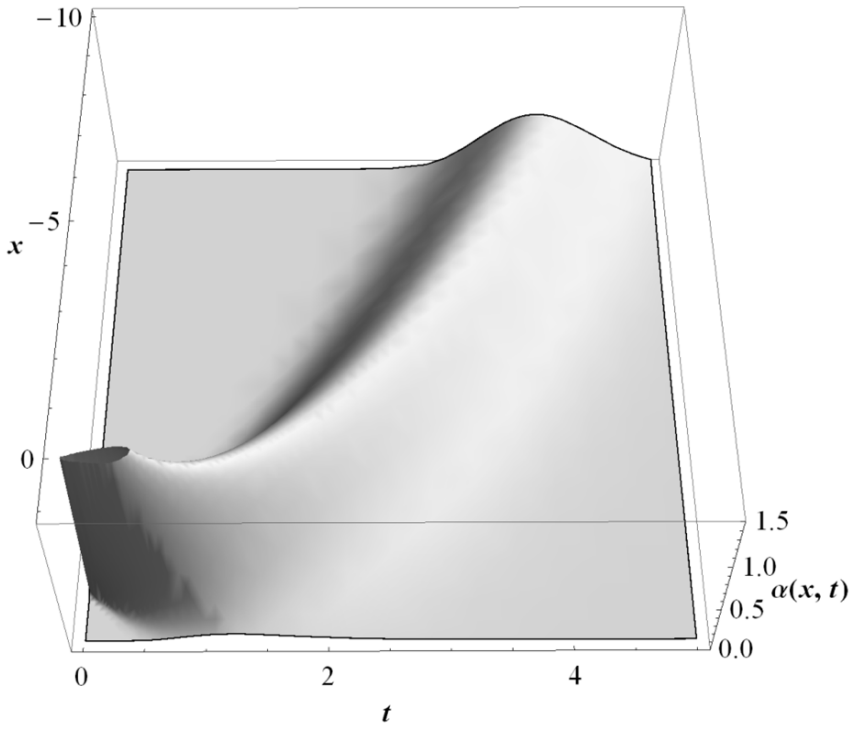


Fig. 7

In the next figure we have the Real part of expression (50). We can observe the behaviour of structure for the time $t = 2.7$. We note that the global envelop tends to modify when is near of $t = 0$. Here, the effects of potential $V(x)$ are described in a more interesting way. The central structure of this "particle" is modified. This response is a more explicit nonlinear effect. When the particle comes in very large distance of $t = 0$, its central structure expands. Thus, we can think that, in our theoretical model, the particle suffers a kind of "dilatation" due to the presence of potential.

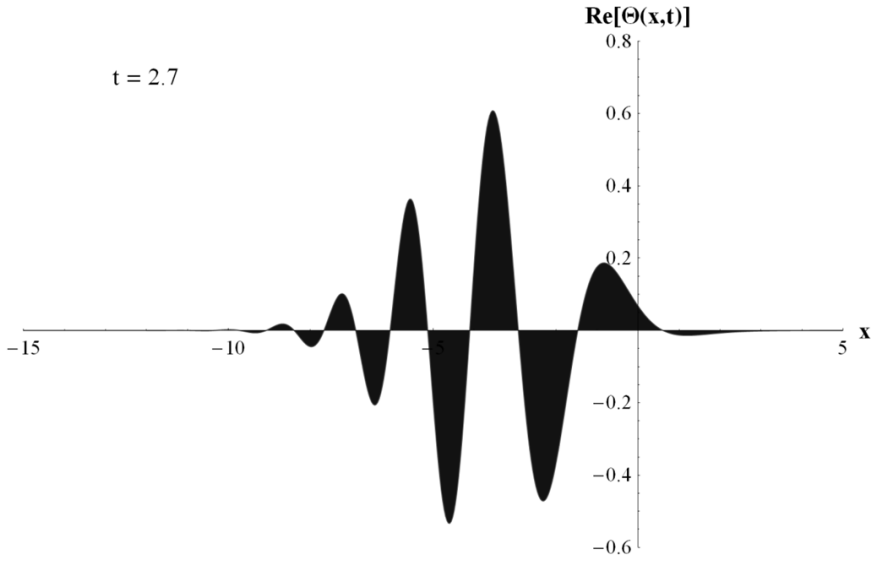


Fig. 8

3.3. NONLINEAR EQUATION IN SEPARATION VARIABLE APPROACH

In general, in the technical schema behind the Orthodox Quantum Mechanics approach, we work with a stationary field. The Schrödinger equation can have solutions of type $\Psi_k(\vec{r},t) = a_k(\vec{r})e^{-\frac{i}{\hbar}E_k t}$. This takes us to an eigenvalue problem. In mathematical point of view, this linearization is a very great simplification. We note that in these situations we have constant amplitude $a_k(\vec{r})$ in time (stationary states).

Inspired by this scheme, we take again the general case given by the nonlinear equation (11) or (13) where the solutions can be written in an arbitrary manner as $\Theta(\vec{r},t) = e^{\alpha(\vec{r},t)+i\phi(\vec{r},t)}$. From these considerations we will assume some particularizations. The first hypothesis states that the amplitude is independent of time, i. e., $e^{\alpha(\vec{r})}$ (stationary states). To the second hypothesis, we assume that for phase solution: $\phi(\vec{r},t) = \varphi(\vec{r}) - Et$.

This means that we are restricted to a particular subset of solutions contained in the much broader set of global nonlinear dynamics. These particular subsets are, indeed, a local expression of a more general and global nonlinear dynamics. In turn, these subsets of solutions make a partial

correspondence with the solutions of an essentially linear dynamics described by the Schrödinger equation. In this last case, the linearity is extrapolated for the whole set of solutions physically interesting, being a global feature in its own universe.

Then, when we make the two hypotheses above, we obtain the follows solutions for phase and amplitude:

$$\begin{aligned}\phi_{SV1}(x,t) &= \varphi(x) - Et = \pm\frac{2\sqrt{2}[\mathrm{E}-(\gamma x+\beta)]^{3/2}}{3\gamma} - Et + c_0 \\ \alpha_{SV1}(x) &= -\frac{1}{4}\mathrm{Log}[4\mathrm{E}-4(\gamma x+\beta)] + c_1\end{aligned} \tag{52}$$

where c_0 and c_1 are constants. A fast inspection in phase (52) shows us that the momentum is given by:

$$p_{SV1} = \partial_x\phi_{SV1}(x,t) = \pm\sqrt{2}\sqrt{\mathrm{E}-(\gamma x+\beta)} \tag{53}$$

This result is compatible with the classical conservation relation $E = E_c + V(x)$, where E_c is the kinetics energy. The hypothesis in amplitude assumed above can be, smoothly, modified in another way. We can assume the general dependence in time: $\alpha(x,t)$. Then, the phase solution $\phi_{SV2}(x,t)$ is the same in expression (52), but for the amplitude we obtain:

$$\alpha_{SV2}(x) = -\frac{1}{4}\mathrm{Log}[4\mathrm{E}-4(\gamma x+\beta)] + \Omega\left[\frac{\gamma t \pm \sqrt{2}\sqrt{\mathrm{E}-(\gamma x+\beta)}}{\gamma}\right] \tag{54}$$

In the amplitude solution (54) $\Omega\left[\frac{\gamma t+\sqrt{2}\sqrt{\mathrm{E}-(\gamma x+\beta)}}{\gamma}\right]$ is an arbitrary function of the argument.

4. CONCLUSIONS

This paper aimed to discuss and present some methods to obtain exact results on the nonlinear dynamics described by equation (11). The key point we want to highlight in this analysis is the fact that it is possible to develop an exact nonlinear dynamics based on two fundamental classical equations: the Hamilton-Jacobi Equation and the Continuity Equation. Particularly, it is preserved the classical variational principle (the principle of Hamilton) without the need to change its foundation.

We emphasize the richness provided by the exploration of nonlinear set of solutions from equation (11). In this sense, the non-linear solutions can

help us to develop more sophisticated models, thus providing more and richer descriptions about a particular problem.

We also emphasize our interest in future studies of nonlinear solutions in an alternative way. Since the analytical difficulties in this kind of problems are clear, we have interested in alternatives tools that enable the exploration of new results in the nonlinear dynamics described by equation (11). For instance, we can pay attention to the numerical calculation, specifically in what regards the Finite Element Method (FEM). Our intention is to use this method in order to compare the possible results with the analytical solutions that were presented in this work. This first step will serve to confirm the good convergence of numerical solutions. Thus, we will have an auxiliary and powerful research tool to find solutions in situations where more complex analytical methods have more dramatic problems.

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SOME FUNDAMENTALS OF PARTICLE PHYSICS IN THE LIGHT OF HYPERPHYSIS

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Summary: Some key concepts of particle physics like mass, charge, forces and fundamental interactions are here seen not as basic but as mere useful derived concepts. It will be shown that these derived concepts can be understood in a unitary way in light of the New global physics, the Hyperphysis, rooted into the organizing principle of eurhythmy.

Keywords: Hyperphysis, particle physics, principle of eurhythmy, complex particle, basic physical interactions, concepts of mass and charge.

1. INTRODUCTION

In the present work we intent to show that some basic concepts of particle physics like mass, charge, fundamental forces, fundamental interactions and so can all be understood in a unitary way as particular cases, or better saying as branches of the New global causal and nonlinear approach to better comprehend Nature, the Hyperphysis[1].

Hyperphysis, being developed in its fundamental by one of us[2], is mainly based upon the concept of complex particle and on the organizational principle of eurhythmy[3]. In this approach any particle is understood as a very complex entity. This complex entity being a more or less stable organization of the basic chaotic physical medium, the subquantum medium, is compose of an extended part, the wave, and inside there is a well-localized and in general indivisible structure relatively very small compared to its wave. The wave is named theta wave and the small localized structure the acron. Mathematically we could write

$$\boldsymbol{\phi} = \phi(\theta, \xi) \tag{1.1}$$

or, assuming the simplest linear approach, as de Broglie did[4],

$$\boldsymbol{\phi} = \theta + \xi \ , \tag{1.2}$$

where ξ stands for the acron and θ, naturally for the theta wave.

In previous works, following de Broglie, this very small high energetic region of the complex particle was called, singularity or even corpuscle. Still due to the confusion with the concept of mathematical singularity and from the fact that this region of the particle has an inner very complex structure it is now named by the Greek word acron[5]. This word comes from the Greek άκρον meaning the higher pike like in acropolis, the higher city.

The following drawing, Fig.1.1, tries, roughly, to picture the complex particle.

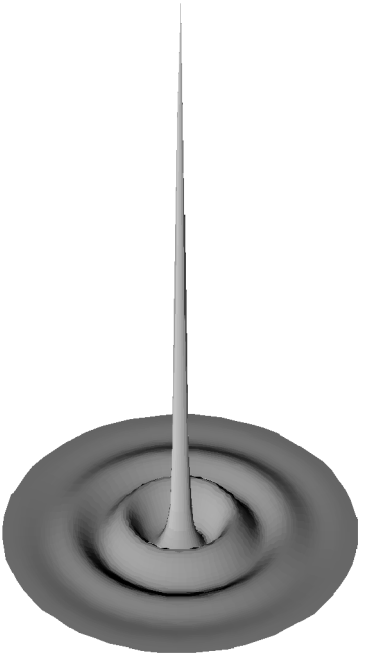

Fig.1.1 – Graphic sketch of a complex particle.

Now the acron moves in a random way inside the theta wave field according to the organizing principle of eurhythmy[6]. This principle states that the acron inside the theta wave field follows a stochastic path that in

average leads it to the regions were the intensity of the theta wave field is greater. The relative energy of the acron is much greater than the one of the theta wave, $E_\xi/E_\theta \gg 1$, therefore in the most common detection processes what really is measured is the acron. Only in very special conditions and by indirect methods we can access to the theta wave.

2. MOTION OF THE COMPLEX PARTICLE

When dealing with the complex particle we need to consider two kinds of motions. The global motion that is, the velocity of the extended theta wave v_θ and the motion of the acron v_ξ. The motion of the acron is always relative to the surrounding theta wave field. The acron moves in a chaotic way in the theta wave field with an instantaneous huge velocity its natural velocity $v_\xi = v_N \ggg c$. This natural velocity would be the velocity the acron would acquire when the probability of going in one direction would approach one. Still the observed velocity, the average velocity, the one that is observed may be zero or may, in very particular cases, like in tunneling conditions, approach the natural velocity $0 \leq \bar{v}_\xi \leq v_N$. In any case this average velocity of the acron, relative to the surrounding theta wave field for each medium and for particular physical conditions like temperature and so, has a maximal allowed average velocity such that,
$0 \leq \bar{v}_\xi = v_M \leq v_N$ being independent of the emitting source.

2.1. MOTION OF THE COMPLEX PARTICLE IN OTHER THETA WAVE FIELDS

Suppose now that the a complex particle, a photon for instance, enters another different theta wave field, a gravitic field or other, as shown in Fig.2.1.

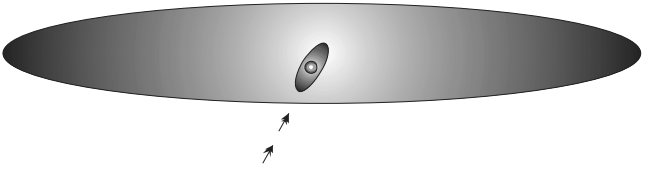

Fig.2.1 – The particle, a small theta wave with an acron enters a large theta wave field

We know that the motion of the acron is always relative to surrounding theta wave field. So, in these conditions it is clear that before entering the large theta wave θ_L only sees its own theta field θ_0. Now what happens when the particle enters the large theta wave field θ_L? Two extreme situations may occur:

1 – The relative intensity of the entering theta wave is much greater than the one of the extended theta wave field

$$|\theta_0|^2/|\theta_L|^2 \gg 1. \tag{2.1}$$

In this situation the acron ignores completely the extended theta wave field θ_L and sees only its initial own theta wave θ_0. In the case of a photon entering a gravitic theta wave field this same very phenomenon may be interpreted saying that the photon is massless. Meaning that in this situation the photon is not subject to gravitic interaction. The same conclusion could be drawn if the same photon enters an electromagnetic field. Either in this case we would be lead to say that the photon is a chargeless particle in the sense that it does not respond, interacts that is, depends on the electromagnetic field.

2 – The relative intensity of the entering theta wave is much smaller than the wave field

$$|\theta_0|^2/|\theta_L|^2 \ll 1. \tag{2.2}$$

In this case the acron, for all practical purposes, sees only the large theta wave field θ_L. Therefore its motion is now not relative to its initial theta wave field θ_0 but to the large extended theta wave field θ_L. Keeping in mind the same case of the photon entering a gravitic field, but now a very intense gravitic field, verifying condition (2.2), we would be lead to say that the acron, the part of the photon we can directly detect, is now sensitive to the gravitic field. In this conditions, as experimental evidence tell us, it will be deflected by the very intense gravitic field, meaning that the photon has, in this case, mass.

For the intermediary cases, between these two extreme situations, the motion of the acron would depend on the composite action of the global field being a composition of the two fields $\theta = \theta(\theta_0, \theta_L)$.

3. COALESCENCE AND ANTI-COALESCENCE

Two well separated complex particles, such that their respective theta waves do not superpose do not interact, see Fig. 3.1.

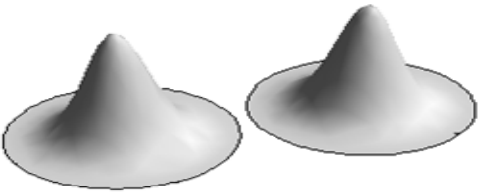

Fig.3.1. Two well separated particles do not interact

Still when they approach, see Fig.3.2 the respective theta waves may overlap and then the acrons

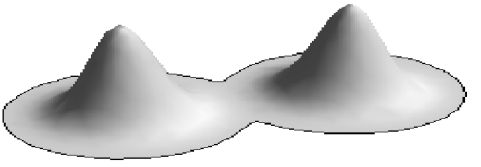

Fig.3.2. The two theta waves overlap

start being receptive the joint action of the two theta wave fields. In this situation the nonlinear interaction between the particles may be such that after a certain initial complex process the phase of the theta waves stablize[7] maintaining a relative phase difference constant. When this phase difference is equal it can be shown[8] that the intensity of the connecting region starts increasing till the waves coalescence into a single wave as shown in Fig. 3.3.

Fig.3.2. The two theta waves coalesce into a single wave

In this situation the acrons tend to move to the central region according to the principle of eurhythmy. In more common language we would say that the particles, the acrons attract themselves. Everything can be described saying that a pushing force attracts the particles. It is relatively easy to show[2] that relatively far from the origin, due to the spherical radial symmetry of the theta wave field, this pushing force can be described by the inverse square law of their distance $F \propto 1/r^2$.

For certain kinds of complex particles, fermions for instance, the nonlinear complex stabilizing process is such that the constant phase difference between the two theta waves assume the value of π. That is, the waves are then in phase opposition and in the region of overlapping it gives a wave of less or even null intensity. In this conditions and following the principle of eurhythmy the acrons are lead to move apart of each other, so we have anti-coalescence. Everything happens as if a kind of repulsive force leads the acrons to move apart. Also in this situation it can be shown that this average tendency, this repulsive force, has the same $F \propto 1/r^2$ variation.

In these conditions we can see that what is commonly called force, gravitic, electromagnetic and so, are nothing more than useful concepts at a certain level of description of the physical reality for the average motion of the acron in a composite theta wave field. The concept of mass or charge being a function of the number of interacting acrons depends basically, as we have seen, on the relative intensity of the overlapping theta waves. Meaning that in the regions where the theta wave field intensity is greater the number of acrons is also greater.

4. CONCLUSION

In the light of the global unifying nonlinear physics, the hyperphysis, the concepts of mass, charge, gravitic interaction, electromagnetic interaction and so are nothing more than derived concepts and not, as have

been till now asserted, fundamental concepts. Naturally they are very useful at certain levels of description of the physical reality. Therefore such concepts must be used with some caution and above all do not pretend that they have a general validity at the diverse scales of description of reality.

In this new more general approach to understand physical reality many somehow incomprehensible things like for instance the strange properties of the neutrinos that are said to be devoid of charge and of mass and furthermore very difficult to detect are now properly understood. If the neutrino has no mass and no charge this simply means that the relative intensity of its own theta wave is much greater than the electromagnetic and gravitic fields we deal with. In this case the acron of the neutrino does not see the relative feeble electric and gravitic fields and consequently is insensitive to them. In the common language this natural situation, that has nothing strange in it, is described by saying that the neutrino is a chargeless and massless particle.

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A PHYSICAL THEORY ON THE COMPLEX AND THE NONLINEAR. A CARTOGRAPHIC PERSPECTIVE

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Summary: in this work, using Lakato's methodology of research programmes, we propose a brief analysis of a Nonlinear Causal Theory. Our main purpose is to put forward a provisional cartography of that theory and, at the same time, to highlight some critical essential topics of the Orthodox Quantum Theory.

Keywords: Nonlinearity, Nonlinear Master Equation, Wavelet Analysis, Research Programme's Methodology.

1. INTRODUCTION

Lakato's methodology offers an organized and structured view of scientific theories. It is for this reason that we use it as an auxiliary tool in the present work. This organization is extremely helpful in the elucidation of the basic hypotheses that support the research program and that, according to Lakatos, form its *irreducible core*. The historical analysis of a research program shows that the scientific community develops a theory in several ways. Such developments lead theory's followers to constantly formulate auxiliary hypotheses, which form a sort of *protecting belt* of the theory's irreducible core. Throughout this kind of progressive game of attack and defence, an intellectual battle is put forward. Science expands, the program is questioned anew, and old problems resurface with new outlines. A researcher, working in a particular program of research, assumes the consequences of the theory's *negative heuristics*, proposes adjustments and gives contextual answers. Scientific expansion is also accomplished through the advances given by the theory's *positive heuristics*. For instance: new mathematical formalisms are developed and applied, and the institutions promote the construction of experimental techniques and increasingly sophisticated apparatus.

In this work, the methodology of scientific research programmes is applied to a part of a theoretical plan inspired by the work of physicist Louis de Broglie. Recently, this program has been taken over by the physicist J. R. Croca and has gained new nuances and contours: this research programme will be referred as «Nonlinear Causal Theory». We'll not consider it in its entirety. We'll simply focus some of its particular topics in order to develop our analysis.

Besides providing a critical analysis of the Orthodox Quantum Theory, we believe that establishing a structural mapping can also help to point out some fundamental differences between that theory and the nonlinear causal research programme.

2. INTEGRATION VS. COMPLEMENTARITY: THE PARTICLE-WAVE DUALITY REVISITED

We'll restrict ourselves to the field of quantum phenomena. Initially, we'll assume a basic statement formulated upon the interpretation of experimental data and extracted from textbooks on Orthodox Quantum Theory. The statement says:

> *Young's double-slit experiment […] led us to the following conclusions: both wave and particle aspects of light are needed to explain the observed phenomena; but they seem to be mutually exclusive, in the sense that 'it is impossible to determine through which slit each photon has passed without destroying, by this very operation, the interference pattern'. The wave and particle aspects are sometimes said to be 'complementary'.*[1]

According to Lakatos, the irreducible core of a research programme should be taken as irrefutable. Therefore, to disagree with a program's irreducible core can be interpreted as a refusal of its entire theoretical corpus.

Orthodox Quantum Theory offers an answer that originates a fundamental dilemma associated with the classical perspective. A satisfactory quantum phenomena description implies that the wave and the corpuscle properties must be *necessarily* admitted for the *same* entities. However, these properties must be admitted as "complementary".[2] This

[1] Coen-Tannoudji, C., Diu, B., Laloë, F. (1977). *Quantum Mechanics – Vol. I*, p. 50.
[2] One of the first essays on the topic was: N. Bohr, in *The quantum postulate and the recent development of atomic theory*, Atomic Theory and the Description of Nature. Nature, **121**, 580 (1928).

means that a phenomenon can be wavelike "or" corpuscular, but not both at the same time. Thus, the statement tries to reconcile two fundamental classical heritages.

On the one hand, the Euclidean inheritance, attached to the following definition: *a point is that which has no parts, or that which has no size*.[3] The corpuscle image referred in the studies of Newtonian Mechanics is derived from this idealization of the mathematical point. Ideas like indivisibility and exact position are associated with the point (considered as an element of space).

The other classical inheritance found its highest moment with the Electromagnetism of Maxwell. In this theory the background image is the notion of wave, to which are associated ideas such as divisibility and extension.

We can summarize these physical notions with two ontological and disjoint aspects: the discrete and the continuous.

The Orthodox Quantum Theory preserves both aspects in a complementary manner. Complementarity was proposed by Niels Bohr and, here, we consider, in short, the wave/corpuscle version of complementarity. In the research programme of the Orthodox Quantum Theory the Complementarity Principle is crucial, which is why we consider it as one of the constituent elements of the irreducible core of that program.

On the other hand, the denial of the irreducible core formed by the dilemma of complementarity would put us out of the research programme. Consequently, to think that we would still be in that theoretical domain wouldn't make sense and so we could ask: How could we formulate another scientific research program that may be adequate do the description of the quantum world and, at the same time, deny the core of complementarity?

We'll follow the proposal of the Nonlinear Causal Theory[4] that has 5 basic assumptions. We could examine these assumptions, find connections and developments among them; however, that will not be our task here. We want to develop a small, and provisional, structural map based on the basic assumptions of that program.

To start with, we'll invoke the second assumption:

> *2. Second assumption: There is a basic physical natural chaotic medium named the subquantum medium. All physical processes occur in this natural chaotic medium.*

[3]Book 1 of Euclid's *Elements* contains this definition of point. See the link: http://www.mat.uc.pt/~jaimecs/euclid/1parte.html (23/06/09).

[4] See the article HYPERPHYSIS – The Unification of Physics in the present volume.

It is our belief that *the basic physical natural chaotic medium named the subquantum* can be interpreted as a kind of "unifying support" where all physical processes occur. Therefore, in this case, we no longer have an ontological discrete/continuous dichotomy. The background role is performed by the subquantum medium and, at this level, its essence, discrete or continuous, is not questioned. Likewise, it is not relevant to consider the discrete/continuous dilemma, and the notions of space and time are not basic. These notions (space and time) are closely related with the notions of discrete and continuous and will only emerge later, as tools, for example, which help in the further understanding and description of some physical phenomena, on an operating level.

When we add the fourth assumption to the second, we get closer to the corpuscle/wave impasse:

> *4. Fourth assumption: In general the complex particles, stable organizations of the subquantum medium, are composed of an extended region, the so called theta wave,* ***and*** *inside it there is a kind of a very small localized structure, the acron.*

The duality particle/wave is explicit in this assumption. The Nonlinear Causal Theory assumes the wave and corpuscle behaviours, not in a complementary way, but in an "integrated" one. That is, these characteristics, wave and particle, are not considered as simultaneously inconsistent (this is the classical point of view). On the other hand, they are not considered as complementary (this is the view of Orthodox Quantum Theory). Thus, we have here a divergence with complementarity. We assume a new perspective where, for instance, for the same physical phenomenon, we can have a wavelike "and" a corpuscular description. This perspective is different from the previous programme, which ultimately considers a phenomenon as wavelike "or" corpuscular.

The adjective we use is related with the verb *to be integrated* and can mean *to integrate an element in a set, forming a coherent whole; to be incorporated*. Etymologically, the word comes from the Latin *integro* and means: to start, to renew, to restore. In all senses, the description of the new program will be eventually, and radically, opposed to the previous programme, creatively *renewing* the theoretical domain.

We notice that the fourth assumption only safeguards the fact *that in general the complex particles, stable organizations of the subquantum medium* integrate both characters (theta wave and acron). This does not exclude the possibility of other types of phenomena. That is, we may have eventually an experimental situation where the wavelike character has dominance or where the corpuscular nature has relevance. Consider what the

third assumption states: *What are called physical entities that is, the particles, fields and so on, are more or less stable local organizations of the basic chaotic subquantum medium*. Namely, we can have theta waves independently of the presence of an acron, but an acron needs the theta waves as waves-guide for his movement. This last statement is connected to the principle of eurhythmy, which appears by the fifth assumption of the research programme. Let us imagine that an extension of this principle, at a level of mathematical applicability, constitutes a sort of a bridge with the stochastic processes.

> *5. Fifth assumption: The principle of eurhythmy. This organizing principle states that the acron inside the theta wave field follows a stochastic path that in average leads it to the regions were the intensity of the theta wave field is greater.*

In the case of the Orthodox Quantum Theory, we are unable to propose an image of a quantum entity. However, in the case of the Nonlinear Causal Theory, we can follow the fourth assumption and propose some illustrative sketches. We can show how some characteristics of the wave and of the corpuscle are *integrated* into a single image. The wave remains in an extensive region (called *theta* wave), while the corpuscular aspect is represented by a sort of peak (called *acron*) limited to a small region, thus corresponding to the location characteristics. This *integrated* image is the sketch of a *quantum entity*, that is, a stable organization of the subquantum medium. Somehow, the image aims to correspond to a portion of an *objective reality*. The next figure gives a visual idea of this model through a one-dimensional graphic of the real part of Morlet's wavelet.

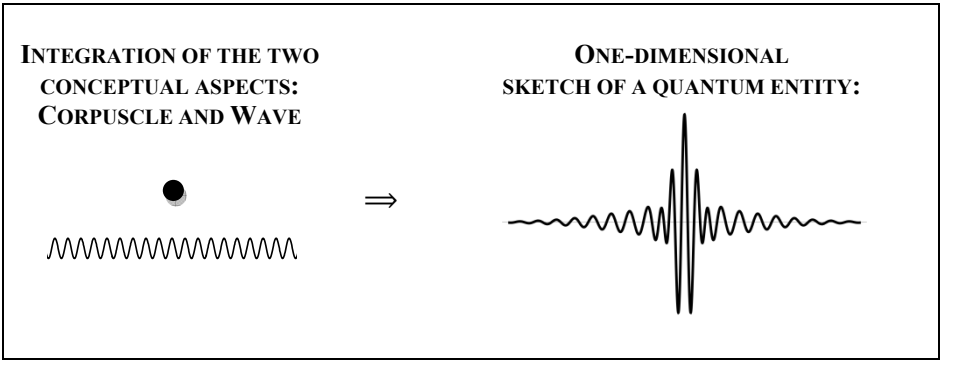

Fig. 1

Figure 1 (to the left) offers two classical images that describe the physical phenomena. The point (a non-dimensional abstract entity) and the wave (normally, classical physics uses waves of the sinusoidal type, infinite in temporal and spatial domains).

In the case of Orthodox Quantum Theory, an image that sums up both concepts makes no sense. However, that is not the case with the Nonlinear Causal Theory. On the right side of Figure 1, we have a one-dimensional sketch of a concentrated energy region (the *acron*) in permanent interaction with an extended region (the *theta* wave). Therefore, we have an *integrated* picture of what would be a quantum entity. It is clear that the sketch in Figure 1, to the right, is a rough draft of reality. It's only a model among others. Still, although simple, it suggests a whole new way to describe the complexity of nature and it does not refuse objectivity. In this causal scheme, the *particle-wave integration* appears as a consequence of the fourth assumption (that defines the complex particles). Nevertheless, such *integration* is not required for all physical phenomena. Generally speaking, according to the Nonlinear Causal Program, we may have three types of physical phenomena: wavelike, corpuscular, or both simultaneously.

At the end of this section we propose a provisional version of the Nonlinear Causal Program descriptive map.

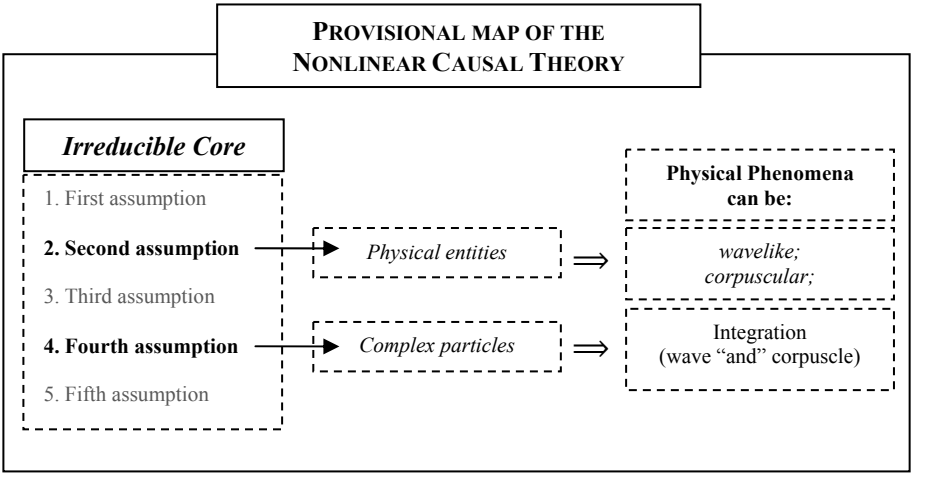

Fig. 2

3. The Linear Exception and the Nonlinearity Saved

Generally speaking, modern physics advanced under various mottos of universalistic nature. We'll identify one of those mottos as *the motto of simplicity* and we'll try to show some of the developments that such a motto might have.

Physicists often defend the idea that a physical "law" should be as simple as possible. "The simpler the better" must be followed strictly. Here is a practical example: imagine two equations that, in general, can be used to describe the same phenomenological domain and that provide, *apparently*, very close results; if both are put on trial, why not choose the equation with the simpler structure and the faster solution? In this case, the strategy is to assume additional hypotheses to "simplify" the complex equation and thus to reduce it to a simple equation. However, there is clearly a problem in this approach. An equation of physics is much more than a simple mathematical equation. Behind an equation of physics, besides epistemological sense, there may be a subtle connection with the metaphysical. Although providing under certain conditions reasonable and practical results, there is a serious risk in constantly taking the path of simplicity: it can lead us to an extrapolated and paradoxical interpretation of the world. What's more, the choice for simplicity may be based in hasty cuts, careless omissions and in the use of incompatible premises, when associated to the fundamental principles.

We believe that the *simplicity motto* is related with another concept: *symmetry*. It is usual to think that the world is perfectly symmetrical. It is usual to say that we must work with symmetric equations or symmetrical laws. However, we think that this attitude minimizes the descriptive richness that we may have of the cosmos. Our everyday observations show that the most basic things – for instance, the right and the left side of a face – are asymmetric. We know that *symmetry* may contribute to obtain simpler equations. Nonetheless, if we want to refine our understanding and our ability to act upon the world, we must work with tools that better represent the world. There is no reason to assume, as a general rule, that the cosmos is perfectly symmetrical. It would be an artificial world, a world of mirrors and, surely, a boring world.

There is another point strongly related with simplicity and symmetry: *linearity*. We'll begin the analysis of this concept with a short reference to the mathematical formalism of the Orthodox Quantum Theory. This theory postulates that the state of a physical system is defined by a wave function Ψ. The wave function Ψ must fulfil the famous non-relativistic Schrödinger's Equation. A fundamental mathematical property of this equation is the linearity:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{3.1}$$

We may interpret the Principle of Linear Superposition as: what is valid for one entity is also valid for many entities, regardless of how many entities are involved. This is a huge simplification and it reduces the description of the physical phenomena complexity. For what reason we'll we use the same process to describe three boxes: the first with 1 entity, the second with 10 entities and the third with 1000 entities? Or, to put it in another way, how can we think that 1000 entities will have a simply addictive phenomenological behaviour equal to the behaviour described by 1 entity? In the Orthodox Quantum Theory, the principle of linear superposition (the linearity) assumes a universal status. This implicit assumption appears when the theory states that Schrödinger's Equation and the wave function Ψ describe *thoroughly* a quantum system.

Regarding these considerations, let us discuss the mathematical formalism of the Nonlinear Causal Theory.

The first point to highlight gives us a contrast with the Principle of Linear Superposition. Every situation, phenomenon, and experiment must be worthy of a differentiated attention. The description of a system with $N > 1$ entities shouldn't be considered as simply addictive, based on a linear superposition. What is equal to one is not always equal to many.

The interaction between two or more physical entities may bring non-trivial news: new phenomena may *emerge*. The emergence of such phenomena reflects the enormous complexity of nature; this emergence shows that the Principle of Linear Superposition has limits of applicability; through it, the Principle of Linear Superposition loses its universal status.

Similarly, every function or equation has its limitations. They only serve to draw a rough sketch of what actually happens in the physical world. In the best of chances, an equation, or function, will be a temporary tool that describes a small portion of the universe in a given temporal and spatial scale.

It is too imprudent to state the validity of any physical-mathematical law concerning the whole cosmos. To do it is to commit a rough mistake of inferential extrapolation.

Eventually, we may think that in other galaxies, or in other regions of our Milky Way, distant from our Solar System, other phenomena, different from the ones we observe on Earth, are occurring. The variety of phenomena that we observe in our home shows us the multiple configurations of stability that the subquantum medium may assume, locally, during a given period of time and under certain conditions. Perhaps other phenomena, inaccessible to our local experience, may occur in the farthest regions of the cosmos.

In this context, let us discuss some topics that are in contrast with the Schrödinger Equation (3.1).

First, we believe that the concept of *universal* makes no sense here. We must acknowledge that constants as the mass m or Planck's constant $\hbar$, for instance, have a local validity and are susceptible to variations, depending on the situation in question. For the Nonlinear Causal Program, in the particular case of mass, we can think it as a function of the number of *acrons* that constitute the total corpuscle, i.e., $m = m(\xi)$[5]. So, we can have a very complex situation where m is not a constant; m may endure significant variations.

Let's consider the Provisional Map of the Nonlinear Causal Theory outlined in the last section. In that map, the Fourth assumption leads to the *integrated* notion of the wave/particle duality. Inspired by this idea, let us consider the formal conjunction of two dynamics: one referring to the extensive aspect and the other to the located aspect. The two classical equations that contain, intuitively, such aspects are: the Equation of Continuity and the Hamilton-Jacobi Equation. With these considerations, we obtain the following Nonlinear Master Equation[6]:

$$-\frac{\hbar^2}{2m}\nabla^2\Theta + \frac{\hbar^2}{2m}\frac{\nabla^2(\Theta\,\Theta^*)^{1/2}}{(\Theta\,\Theta^*)^{1/2}}\Theta + V\Theta = i\hbar\frac{\partial\Theta}{\partial t} \qquad (3.2)$$

Now let's highlight the fact that this equation has no supremacy or universality status. The expression (3.2) is not "The Equation", "The Theory". What we have here is "a possible theoretical way" that takes a few fundamental principles: the Balance Principle (Continuity Equation) and the Variational Principle (Hamilton-Jacobi Equation). On the contrary, the Schrödinger Equation is presented as a fundamental postulate in textbooks on the Orthodox Quantum Theory. In that perspective, the Schrödinger Equation has the status of a basic *assumption*, that is, it is one of the elements that form the *hard core* of that theory. In a different way, equation (3.2) has a more malleable status, it is a proposal that coherently fulfils two fundamental principles and aims to describe a portion of the complexity of nature, and it isn't a basic *assumption*. We defend the idea that the equation (3.2) is a consequence of the developments of at least 3 basic assumptions:

- Based on the fourth assumption, the equation (3.2) somehow reflects the dynamics of the "extensive" (the continuous, the wavelike) and of the "located" (the discrete, the corpuscular);

[5] See the article HYPERPHYSIS – The Unification of Physics in the present volume.
[6] A heuristic process to obtain this equation can be seen in Croca (2003).

- According to the fifth assumption, the equation (3.2) is also in close relation with the Hamilton-Jacobi Equation, because the principle of eurhythmy deals with the movement of the acron on the theta wave. The Hamilton-Jacobi Principle (a variational principle) is basically connected with the *principle of eurhythmy*;

- In what concerns the second assumption, the equation (3.2) tries to be a descriptive and partial response to the complexity of the *subquantum chaotic medium*. A linear tool cannot be the most suitable one to describe a phenomenological reality supported in a chaotic medium. Mathematically, this complexity can be translated by nonlinear equations, for instance, the equation (3.2). However, nonlinearity is not presented here as a constituent element of the theory's hard core; it is a reflection of the second assumption;

Granted these arguments, we consider that the equation (3.2) is a constituent element of the protecting belt (to use Lakatos' terminology) of the Nonlinear Causal Program.

Based on this equation, several steps can be taken and the research programme can dynamically develop its positive heuristic. Therefore, in this theory, the equation (3.2) has a different status from the one assumed by the Schrödinger Equation. We believe that Schrödinger's Equation has a place in the hard core of the Orthodox Quantum Program. The map below summarizes the ideas discussed in this section:

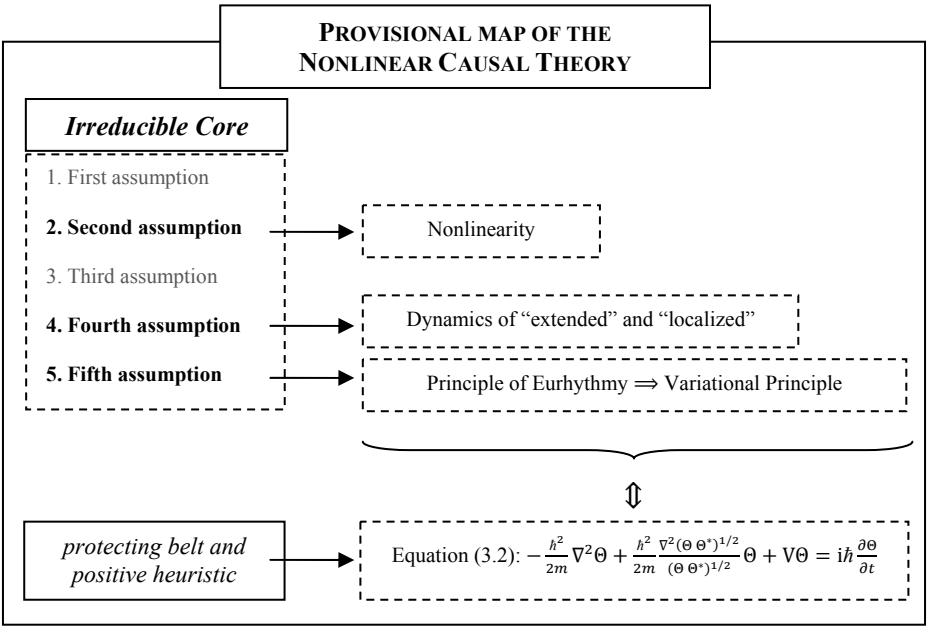

Fig. 3

3.1. SOME PRACTICAL CONSIDERATIONS ABOUT NONLINEARITY

The Master Equation (3.2) clearly indicates an entire research plan, involving techniques for the resolution of Nonlinear Partial Differential Equations. Due to its nonlinear character, the equation (3.2) is part of a class of EDP's with a high degree of technical difficulty. Particularly, the equation can provide a formal solution, called Morlet's wavelet, for a free particle. With this specific solution, we can build an analytical representation of the initial image of the quantum entity (Figure 1, right); this figure is formed by a located part and by an extended part.

In equation (3.2), nonlinearity is generated by the term $\nabla^2(\Theta\,\Theta^*)^{1/2}/(\Theta\,\Theta^*)^{1/2}$. This term emerges naturally from two fundamental principles: the Balance Principle (Continuity Equation) and a Variational Principle (Hamilton-Jacobi Equation). In other words, the nonlinearity of equation (3.2) is connected to some "root" principles. This is important because we often find scientific papers with alternative proposals that generate a forced nonlinearity. It is quite common to generate a nonlinear behaviour by adding *ad hoc* the wave function Ψ raised to the power of n, in the Schrödinger Equation. Although we believe this to be an alternative way of proceeding,

we ask nonetheless: where are the "root" formal principles of this dynamic? The nonlinear dynamics created along these lines seems a "patched" dynamics because it is often based in a linear equation in which a term is forced *ad hoc*. This type of nonlinear dynamics has a deficit of formal justifications and of basic principles. It is for this reason that we say that this kind of dynamic is a *pseudo nonlinear dynamics*.

Now let's discuss the dilemma linearity/nonlinearity from a formal point of view and through a particular case. We want to put nonlinearity in the place it has always occupied, that is to say, in the place of the rule and not in the place of the exception.

To that end, let us focus the functional of the phase $\phi(\vec{r},t)$. The behaviour of this functional is described by the Hamilton-Jacobi Equation. We'll give two examples of solutions for the one-dimensional case. We assume the potential constant $V(\vec{r}) = V_0$. Under these conditions, two formally valid solutions for to the equation (3.2) are:

$$\phi_L(x,t) = \pm\sqrt{2(\mathcal{E}-V_0)}\,x - \mathcal{E}t$$

and

$$\phi_{NL}(x,t) = \frac{x^2 + \mathrm{B}x + \frac{\mathrm{B}^2}{4}}{2t} - V_0 t \tag{3.3}$$

Where B is a parameter and $\mathcal{E}$ a constant that can be interpreted, classically, as the total energy of the particle. The next graphic corresponds to the phases (3.3) for the case $t = 1$, $B = 0$, $\mathcal{E} = 0$ e $V_0 = 1$:

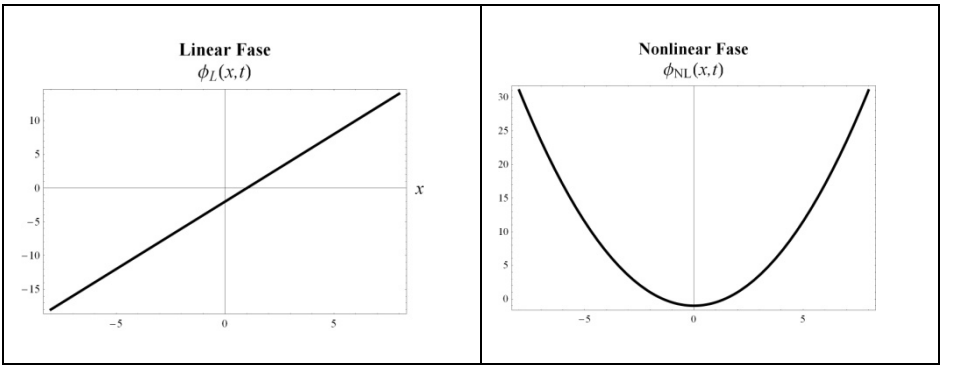

Fig. 4

On the other hand, we observe that the dependence of these phases appears in the final solution in the form of an imaginary exponential, i.e., $\Theta(\vec{r},t) = R(\vec{r},t)e^{i\phi(\vec{r},t)}$, where R is the amplitude. Therefore, we have, for the real parts $e^{i\phi(\vec{r},t)}$ the following corresponding graphics:

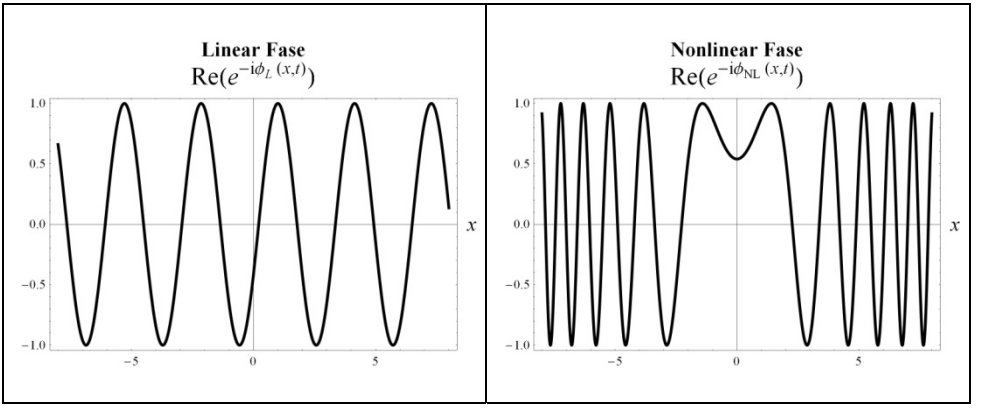

Fig. 5

We noticed that the behaviour of the linear phase is much simpler than the one presented by the nonlinear case. We observe, for a fixed time, the variation of the successive peaks in both cases. In the linear behaviour we have:

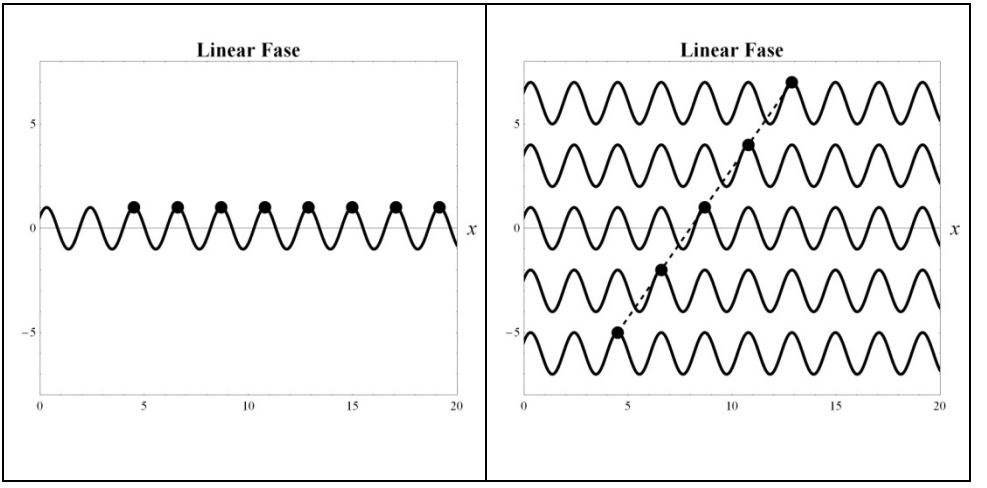

Fig. 6

In a different way, the behaviour of the nonlinear can be observed in the following graphics:

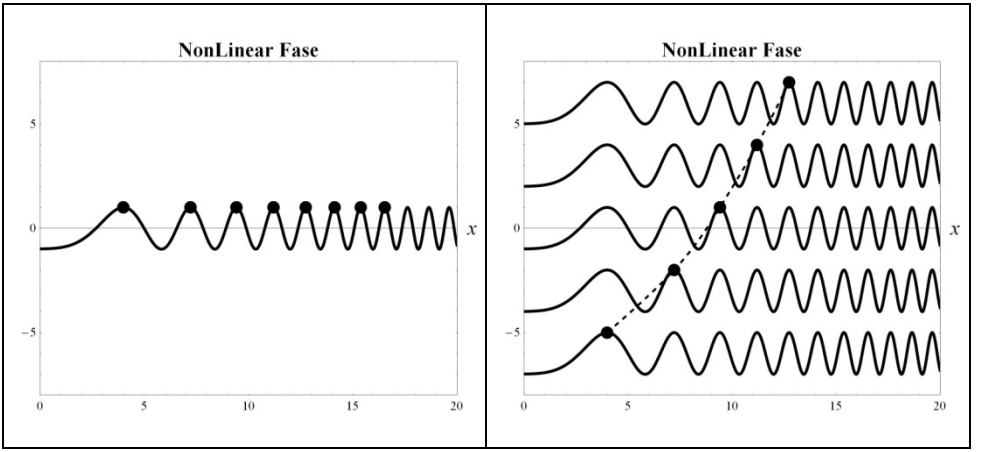

Fig. 7

Besides these basic observations on the mathematical structure of phases (3.3), we also can account for the relations that help us in an intuitive understanding of the physical problem. One of these relations associates the momentum of the particle and of the phase: $\vec{p} = \nabla\phi$. Using this relationship to solve the linear phase in (3.3) we get:

$$\vec{p}_L = \nabla\phi_L(x,t) = \nabla\left(\pm\sqrt{2(\mathcal{E}-V_0)}\,x - \mathcal{E}t\right) = \pm\sqrt{2(\mathcal{E}-V_0)}\,\hat{\imath} \qquad (3.4)$$

In this expression $\hat{\imath}$ is the unitary vector in the x direction. Here, to an intuitive explanation, the result (3.4), in a sense, can be interpreted as the momentum of a particle of mass $m = 1$. Now, if we use the relation $E_L = -\frac{\partial\phi_L}{\partial t}$, we get $E_L = \mathcal{E}$. However, for the solution of nonlinear phase (3.3), we get:

$$\vec{p}_{NL} = \tfrac{1}{2t}(2x + \mathrm{B})\hat{\imath} \quad \text{and} \quad E_{NL} = \tfrac{1}{2t^2}\left(x^2 + \mathrm{B}x + \tfrac{\mathrm{B}^2}{4}\right) + V_0 \qquad (3.5)$$

We note that the relations $\vec{p}_{NL} = \nabla\phi_{NL}$ and $E_{NL} = -\frac{\partial\phi_{NL}}{\partial t}$ are not constant. In this sense the results of (3.5) can be interpreted as a progressive dependence, on the momentum and energy of a particle, in space x at time t. Physically, this is a possible description of the effects of a constant potential, for instance. The potential should be felt by the particle in time and space

dependences. In relation (3.4) this does not occur. The linear structure in $\phi_L(x,t)$ is derived from the orthodox Theory's scheme, where we choose $\phi(\vec{r},t) = \varphi(\vec{r}) - Et$. This choice limits the valid solutions for the equation (3.2) only in a small part of the universe of possible solutions. In solution $\phi_{NL}(x,t)$ we see that x e t are coupled in a non-trivial relation.

After this discussion, we propose an intuitive and visual layout in order to understand a global relation between the nonlinear and linear solutions. In general, we have a nonlinear dynamics and in very particular cases, we investigate the solutions of this equation for stationary states. This means that we are restricted to a particular subset of solutions contained in the much broader set of global nonlinear dynamics (3.2). These particular subsets are, indeed, a local expression of a more general and global nonlinear dynamics (see left in the figure). In turn, these subsets of solutions make a partial correspondence with the essentially linear solutions. In this last case, the linearity cannot be extrapolated for the whole set of solutions physically interesting, being a global feature in its own universe in very particular situations.

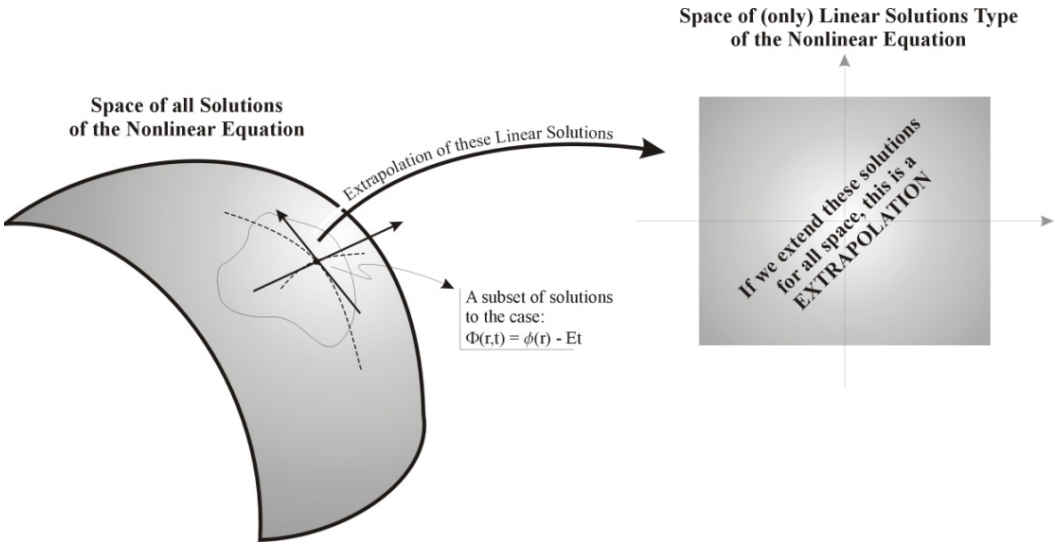
Space of (only) Linear Solutions Type
of the Nonlinear Equation
Space of all Solutions
of the Nonlinear Equation
Extrapolation of these Linear Solutions
A subset of solutions
to the case:
Φ(r,t) = φ(r) - Et
If we extend these solutions
for all space, this is a
EXTRAPOLATION

Fig. 8

3.2. NONLINEARITY, COMPLEXITY AND THE EXPERIMENTS: A BRIEF NOTE

Based in the conceptual scheme of Lakatos, we'll propose a provisional map of the Nonlinear Causal Research Programme. However, before doing so, we would like to add some notes on the preceding discussion. It is our belief that these final comments exemplify part of the program's positive heuristics (in Lakatos' terminology).

Imagine a source **S**, where quantum entities (photons or electrons, for instance) are emitted in the direction of a semi-mirrored surface **M**. The reflection and transmission coefficients, in **M**, are given by $R = T = 1/2$. Thus, the quantum entity can follow two possible paths. At the end of each of these possible paths, there are detectors $\mathbf{D_1}$ and $\mathbf{D_2}$. Now, remembering that the Orthodox Quantum Theory describes what happens in this simplified interferometer (see next figure). After the semi-mirror **M**, and before any measurement, the system is described by a total wave function composed by wave functions ψ_1 and ψ_2 (the wave function of path 1 and the wave function of path 2). At this point of the description, we can imagine, for instance, that the particle has a "potential existence", in both arms of the interferometer.

When the measurement is done, however, the theory uses the expression "collapse of the total wave function". For instance, if the $\mathbf{D_1}$ detector registers the arrival of the particle, then that means that the composite state $\psi_1 + \psi_2$ collapsed in state ψ_1. This does not mean that the particle was already in state ψ_1 before the measurement was made. Before the measurement by $\mathbf{D_1}$, we cannot despise the function ψ_2 in the global composition $\psi_1 + \psi_2$. Thus, the term "collapse" is required in the theory.

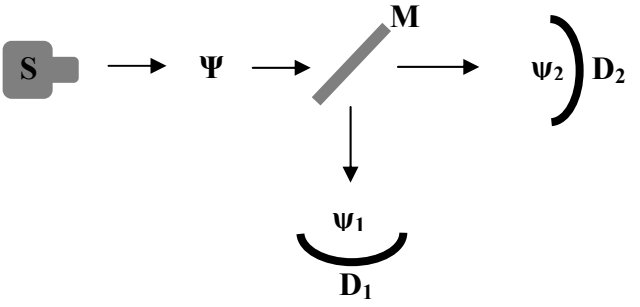

Fig. 9

Now, let's try to think otherwise. To the Nonlinear Causal Research Programme, the chaotic character of nature is implicit in this experiment. All physical processes occur in the basic physical natural chaotic medium named the subquantum medium (Second assumption). With the Fourth assumption, we can propose the follow symbolic figure to the complex particles ϕ (an extended region, called theta wave θ, and a very small localized structure, the acron ξ):

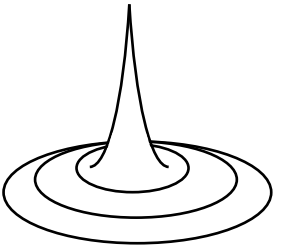

Fig. 10

Thus, when the particle ϕ arrives in mirror **M**, it enters in a very complex process of interactions. We suppose here that, after this process, the particle goes by the path 1, i. e., the acron ξ goes by path 1 with its theta wave θ_1. In this situation, is natural to image that the initial theta wave θ, the extended region of particle ϕ, was divided in two parts in mirror **M**: in that θ_1 and in another θ_2 that passes to the path 2. The next figure shows this brief description:

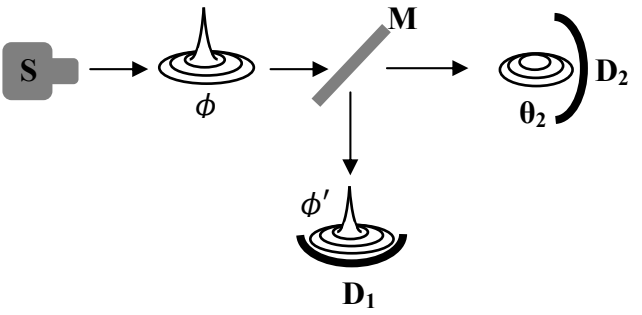

Fig. 11

In the configuration of the last figure, the detector $\mathbf{D_1}$ will make *click* because the acron ξ is present with its theta wave θ_1. But the detector $\mathbf{D_2}$ won't make a *visible click* because the amplitude of theta wave θ_2 is very

small. So, we have a different explanation from the one given by the Orthodox Program. Here, before a click in detector $\mathbf{D_1}$ the particle (acron ξ and theta wave θ_1) was still in path 1 *and* a theta wave θ_2 was in path 2. Eventually, we can have the contrary situation.

The description of this simple experiment is important because it suggests a description that does not use the word "collapse". From our causal and nonlinear point of view, the term "collapse" appears in the orthodox plan to conceal the complex and chaotic essence of nature through a linear theoretical domain.

On the other hand, the experiment suggests some procedures of the Nonlinear Causal Program. We incorporate these procedures in the program's positive heuristics; an example of which is given by the following figure of an experimental proposal: the generator of theta waves.

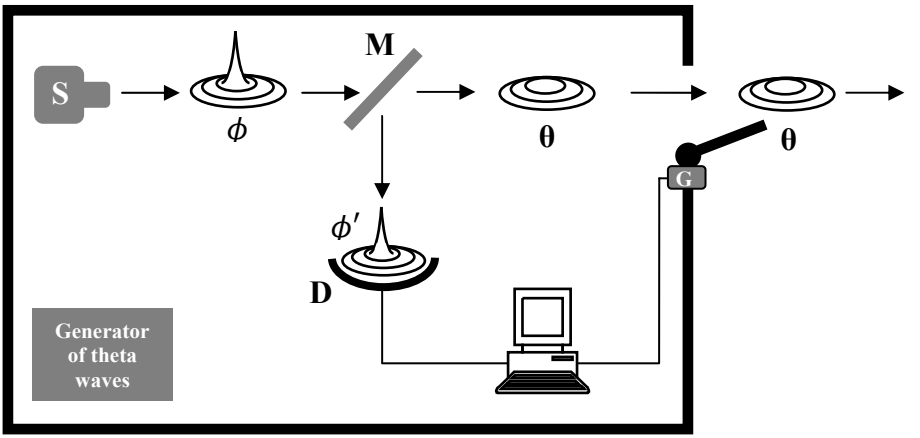

Fig. 12

The scheme has the same principles of the previous figures, with few differences: now, we don't have the detector $\mathbf{D_2}$. In this case $\mathbf{D_1}$ = **D**. Furthermore, we connect the detector **D** to a system with the following function: when a particle (acron ξ and theta wave θ_1) arrives to the detector **D**, it triggers a device to open gate **G** thus allowing the passing of the theta wave θ_2. In the instance of the opposite situation, when it only a theta wave (without the acron) arrives at detector **D**, the device will not trigger; the gate remains closed and the device does not send anything. This experimental apparatus is called theta waves generator.

In the case of the Orthodox Quantum Theory's Program, this device doesn't make sense because this theory imposes the term collapse as a

barrier that prevents us from suggesting the existence of some entity in another branch of the interferometer, after the measurement in **D**.

Therefore, the theta waves generator is a clear example of a device that opens a path that was inaccessible before. This stimulates the theoretical and experimental exploration. We can imagine such generators coupled with other devices. Let us imagine that we have a device "A" that produces interference patterns. If we couple a theta waves generator to the device "A" we can induce a change in the original interference patterns. This suggests an experiment to measure, indirectly, the theta waves. These waves have a very limited range (low power) and it is for this reason that it is so difficult to detect them directly through conventional detectors.

4. A (STILL) PROVISIONAL CARTOGRAPHIC PROPOSAL

We can propose now a provisional map inspired in some terms of Lakatos' methodology. We suggest as elements of the irreducible core the theory's 5 basic assumptions. However, let us remind that in this work we have limited ourselves to assumptions 2, 4 and 5. From each of these three assumptions, we discover developments at various levels: conceptual, interpretive, theoretical and experimental.

We note that the term nonlinearity, for instance, is not mentioned explicitly in this irreducible core. In the context of our discussion, the nonlinearity appears at an operational level, as a way to express the complexity of the basic physical natural chaotic medium named the subquantum medium (second assumption).

The operational level also suggests two other examples that follow the way open by the fifth assumption (which refers to the principle of eurhythmy). In the present context, the principle of eurhythmy proposes the foundations for the description of the stochastic processes that occur in the subquantum medium between the acron and the theta wave. On the other hand, in terms of classical physics, the principle of eurhythmy can also be connected with the variation principles (principle of the least action, principle of the least path).

In any case, the following map aims to sketch a provisional cartography of the discussion we've outlined.

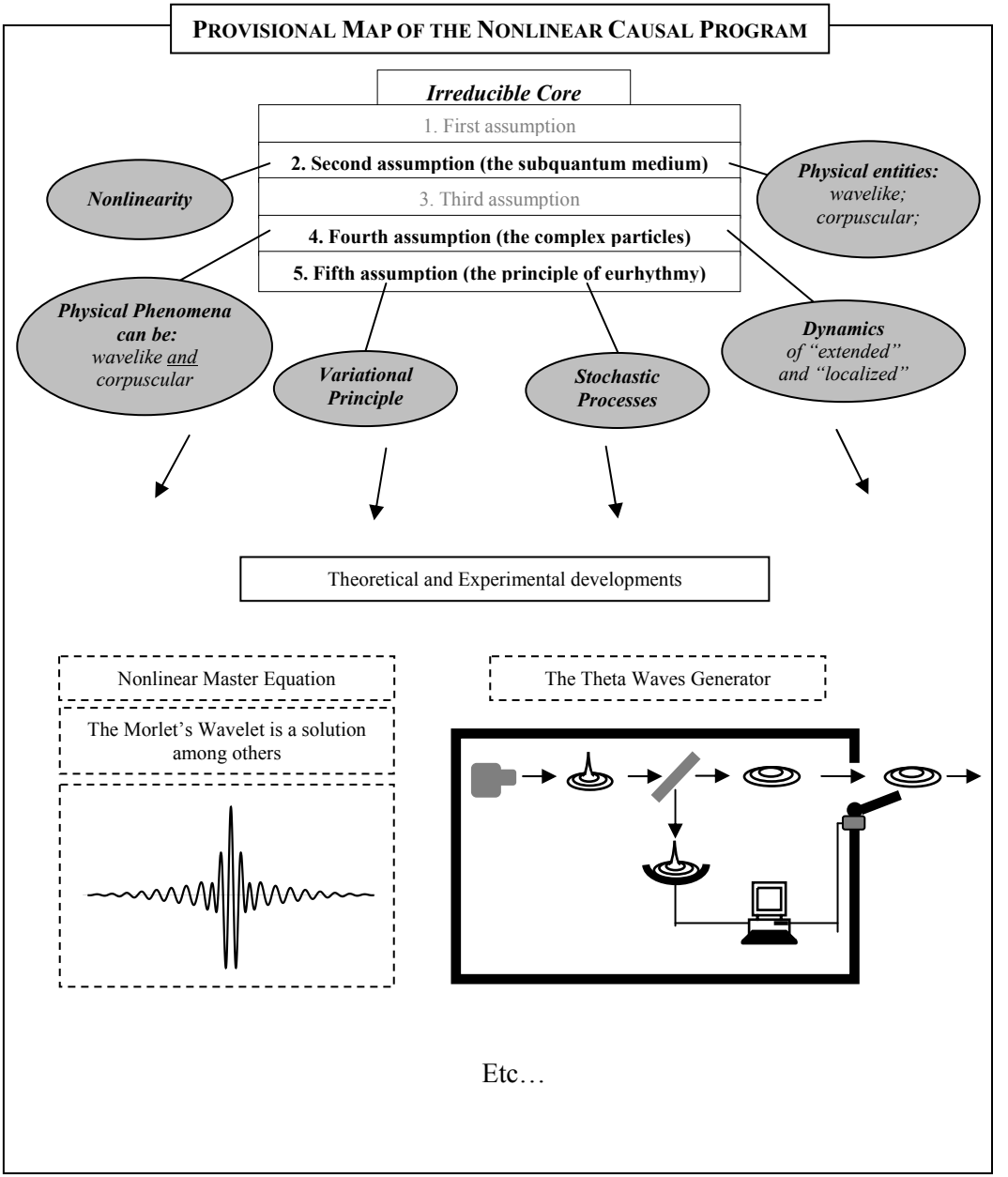
PROVISIONAL MAP OF THE NONLINEAR CAUSAL PROGRAM
Irreducible Core
1. First assumption
2. Second assumption (the subquantum medium)
3. Third assumption
4. Fourth assumption (the complex particles)
5. Fifth assumption (the principle of eurhythmy)
Nonlinearity
Physical entities: wavelike; corpuscular;
Physical Phenomena can be: wavelike and corpuscular
Variational Principle
Stochastic Processes
Dynamics of "extended" and "localized"
Theoretical and Experimental developments
Nonlinear Master Equation
The Morlet's Wavelet is a solution among others
The Theta Waves Generator
Etc…

Fig. 13

ACKNOWLEDGMENTS

I would like to thank the Project of the Foundation for Science and Technology (FCT) - Philosophical Problems of Quantum Physics (PTDC/FIL/66598/2006) and the Center of Philosophy of Science of the University of Lisbon, for the support given to the elaboration of this work.

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SOME PRINCIPLES OF PHILOSOPHY OF PHYSICAL NATURE[1]

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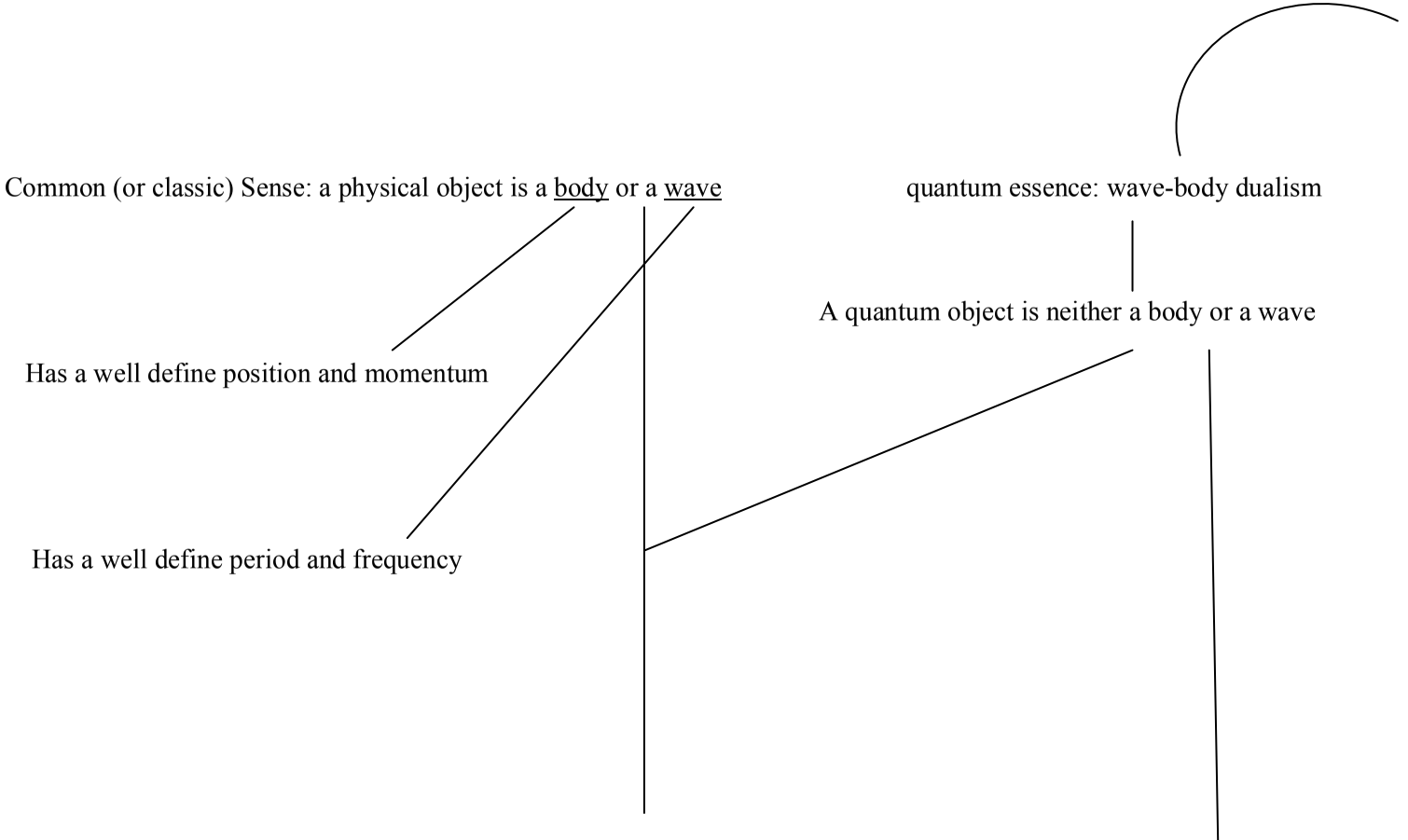
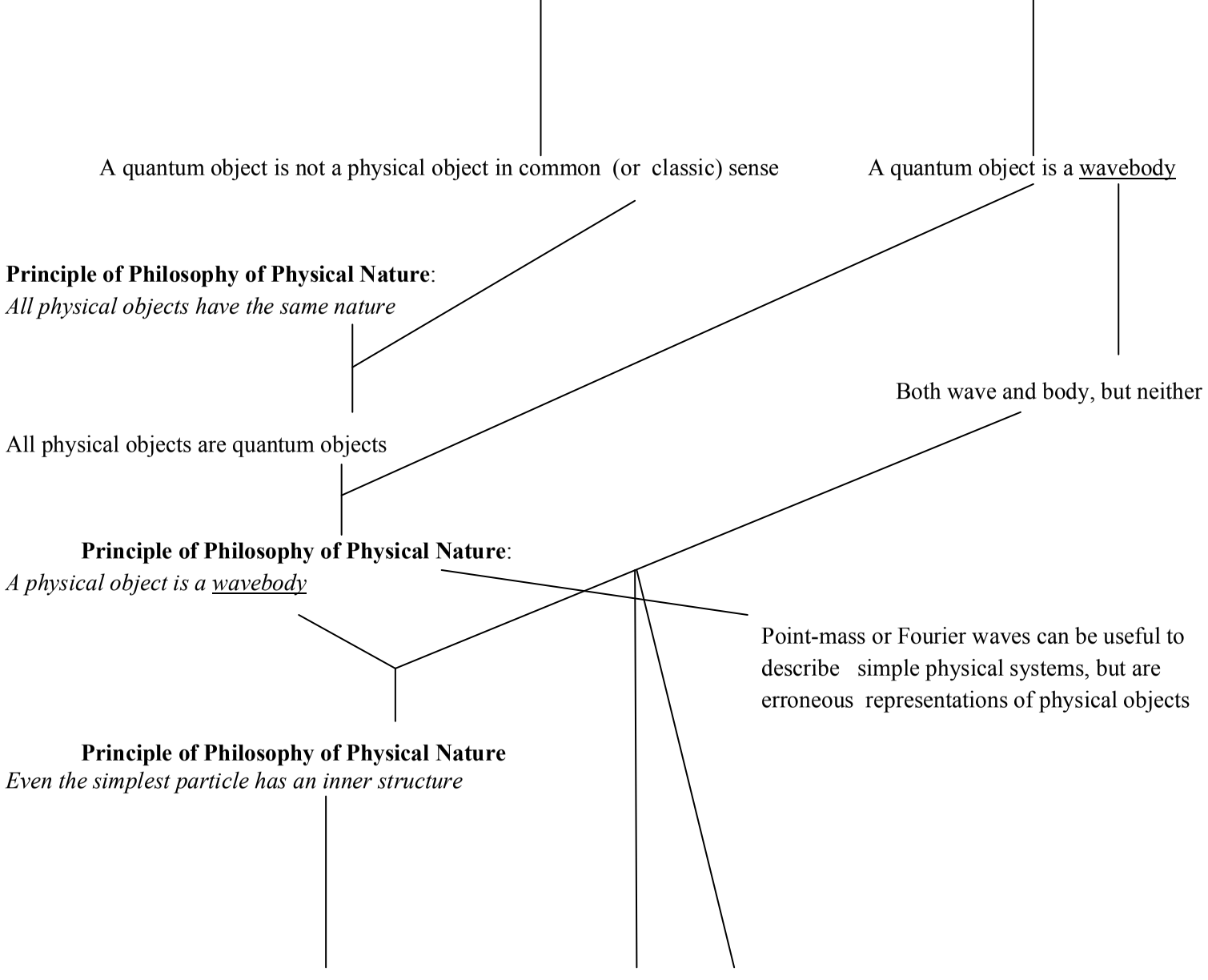
Summary: In this text, as his title indicates, I do not aim to show a complete or finish set of Philosophy of Physical Nature principles. I only want to propose some propositions that are, in my view, a priori to the Physics that is the subject in this book. To be more precise, my aim is only to suggest some propositions concerning the nature of physical objects. On the other hand, I don't intend to present those principles in a deductive way, but only in a schematic way. To a certain extent, there is not a center, a given order or, even, a well-defined hierarchy, and most of the details and explanations have been omitted in order to focus on the relations between the propositions.

Keywords: Philosophy of Nature, Ontology, Hyperphysics, wave-body dualism, diagrammatic reasoning.

[1] This work has been made with the support of the Project "What is Physical Theory?" (PTDC/FIL-104587/2008) and the Center for Philosophy of Sciences of University of Lisbon (CFCUL).


Common (or classic) Sense: a physical object is a body or a wave
Has a well define position and momentum
Has a well define period and frequency
quantum essence: wave-body dualism
A quantum object is neither a body or a wave


A quantum object is not a physical object in common (or classic) sense
A quantum object is a wavebody
Principle of Philosophy of Physical Nature:
All physical objects have the same nature
Both wave and body, but neither
All physical objects are quantum objects
Principle of Philosophy of Physical Nature:
A physical object is a wavebody
Point-mass or Fourier waves can be useful to describe simple physical systems, but are erroneous representations of physical objects
Principle of Philosophy of Physical Nature
Even the simplest particle has an inner structure

A physical object doesn't have a well define properties like, for example, position, momentum, period or frequency

The behavior of a physical object cannot be reduced to his external conditions

The behavior of a whole cannot be reduced to the behavior of none of his parts

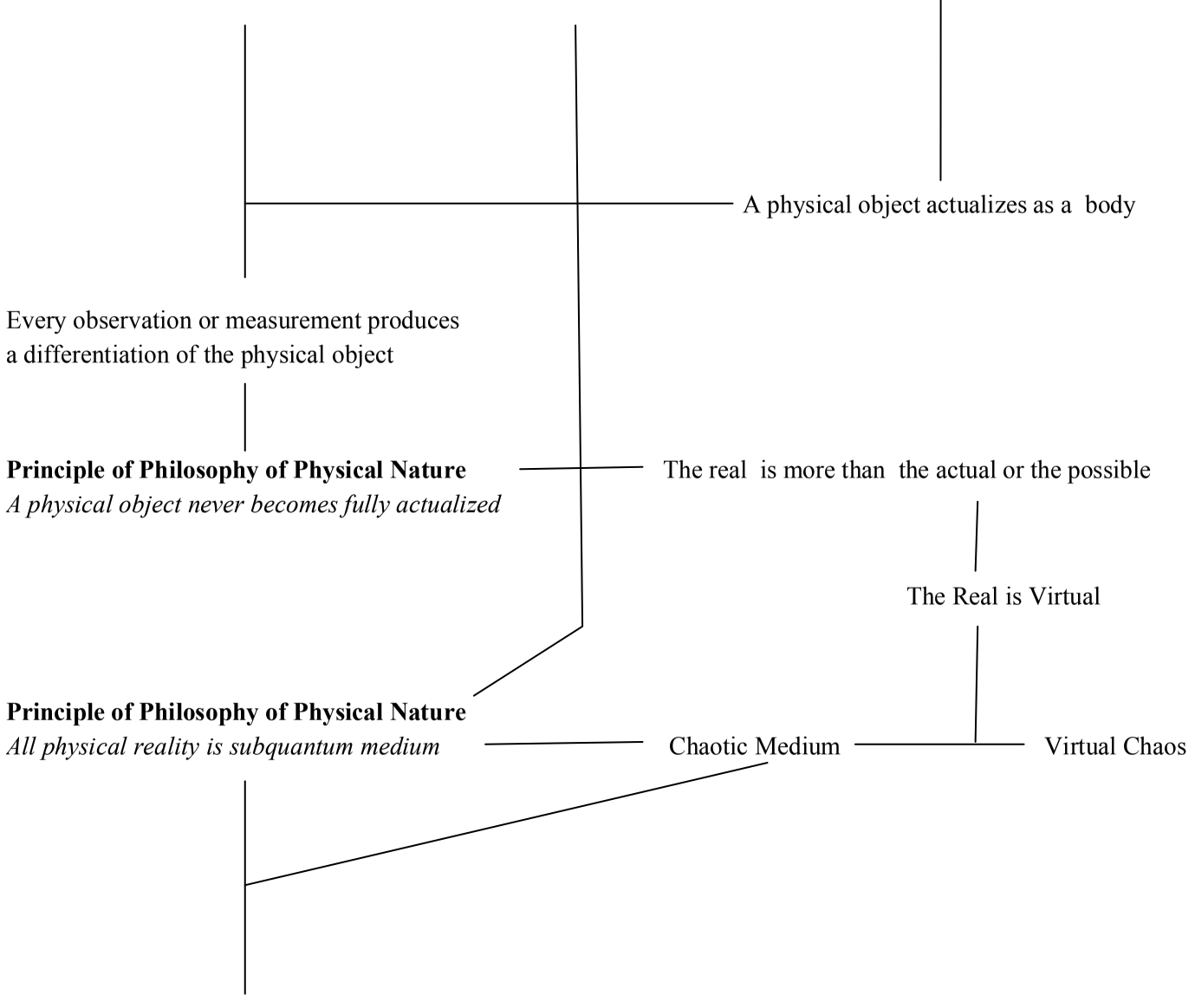
Principle of Philosophy of Physical Nature

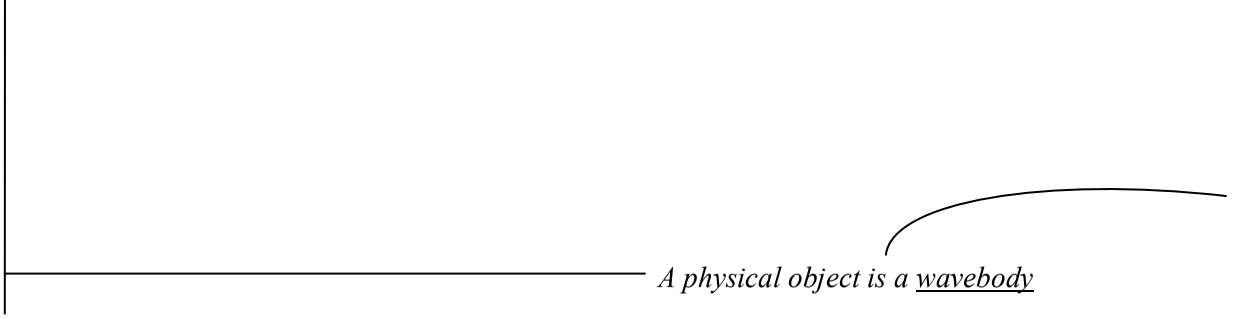
All physical objects have a non-linear behavior

Physical quantities are only statistical

Every measurement is a measurement of localization.
That is, we only observe body –type properties

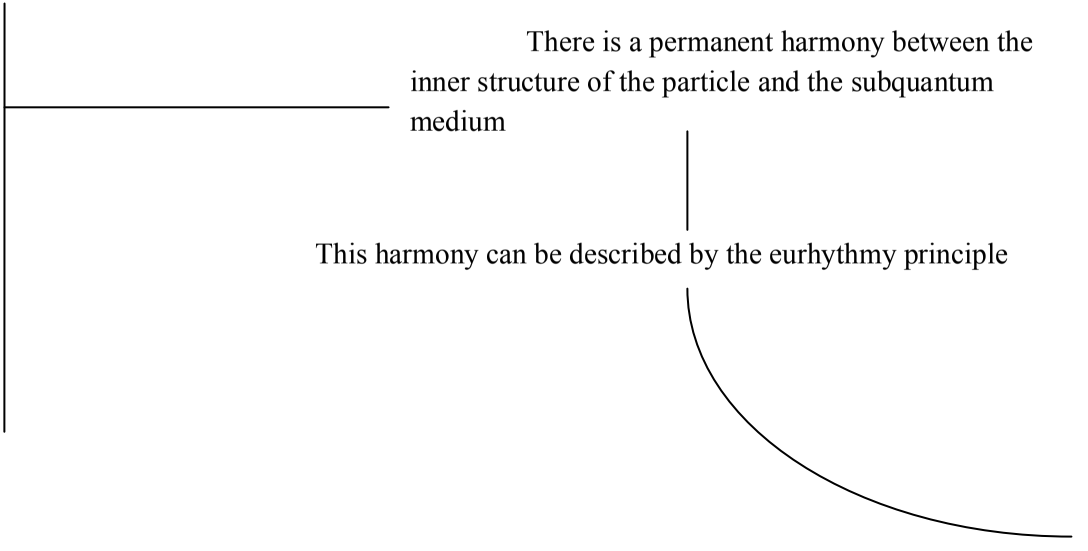
A physical object is a wavebody


A physical object actualizes as a body
Every observation or measurement produces
a differentiation of the physical object
Principle of Philosophy of Physical Nature
A physical object never becomes fully actualized
The real is more than the actual or the possible
The Real is Virtual
Principle of Philosophy of Physical Nature
All physical reality is subquantum medium
Chaotic Medium
Virtual Chaos



Principle of Philosophy of Physical Nature

A particle is a stable organized structure of the subquantum medium


There is a permanent harmony between the inner structure of the particle and the subquantum medium
This harmony can be described by the eurhythmy principle

THE CRISIS IN THEORETICAL PHYSICS SCIENCE, PHILOSOPHY AND METAPHYSICS

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Summary: This paper aims to analyze the crisis in 20th century physics. Despite the huge financial support for theoretical physics through the second half of the past century, no one developed new, effective, physical theories. Quantum Mechanics was the last theory to fulfill the six criteria we propose make a physical theory acceptable. A philosophical realist stance is assumed. There is a tautology between the theories developed in the past century and the dominant epistemological position, i.e. the *semantic view*. We defend that this impasse must be overcome by adopting a new vision of the world, i.e. a new ontology. Any ontology is always provisional but is necessary in order to emerge from the present crisis in physics. The scientific research program proposed by Louis de Broglie and the *principle of eurhythmy* proposed by Croca within this program may be a possible solution. The field of physics stresses the proposals of experiments, where the predicted results of the new quantum theory built by Louis de Broglie's scientific research program are different from the predicted results of the orthodox quantum theory. In the last section of this paper, we try to analyze the philosophical and scientific consequences of the new way of looking at the world proposed in this scientific research program. This new vision of the world would allow a glimpse into how the process of evolution has changed from the quantum level complex structures to the emergence of a complex high-level structure such as *Homo sapiens sapiens*. Evidently, this process would not be a teleological or deterministic one - it would be a process where adaptation and natural selection prevailed. Of course, our species is a mere evolutionary accident. The adaptation can be glimpsed through what we define as *weak teleology*. This *weak teleology* is a consequence of the tendency to persist, or self-preservation, which is a consequence of the *principle of eurhythmy*.

Keywords: principle of eurhythmy; principle of complementarity; reductionism; emergence; organism; complex structure; dualism; monism; θ wave; acron; inner structure; outer structure; extended structure; extended mind; Louis de Broglie scientific research program.

[1] Member of the Center of Philosophy of Sciences, Faculty of Sciences, University of Lisbon, Portugal.

1. INTRODUCTION

Why the title of this text? The answer is: the history of 20th century physics imposes it. The particle physicist and cosmologist Robert Oldershaw[2] wrote in 1988 that, throughout the second half of 20th century physics, physicists devoted their time to building unverifiable theories of the first order, and unverifiable theories of the second order. Clarifying the meaning of this statement, he defined these two types of "theories" as follows:

> **1. First order:** "Theories" that cannot be confronted with experiment; we cannot reach the energy level necessary to achieve it.
>
> **2. Second order:** "Theories" that depend on so many parameters (e.g. Standard Model – 19 parameters!) that we can manage them in order to get an agreement between the "theory" and the experimental data.

We know that scientists work peacefully until a crisis requires them to re-evaluate the foundations of their work. The existence of a profound present-day crisis in Physics is undeniable. Oldershaw is a significant example because he is a particle physicist and cosmologist, i.e. a physicist who works in mainstream physics.

This situation finds its roots in the humus created by the two main theories that emerged in the first thirty five years of that century, i.e. the Special Theory of Relativity/General Relativity and Quantum Physics. The inability to find a way out of the chaotic situation into which physics was plunged in the second half of the 20th century must be explained by an exhaustive analysis of these two theories, which is being done by many people. Here, we simply expose our own conclusions. One of the outcomes of this situation happened in epistemology when we consider, for example, Kuhn, Feyerabend and all those who defend an instrumentalist stance. Incommensurability between paradigms (Kuhn), anarchic scepticism (Feyerabend) and ontological scepticism (instrumentalism, Semantic View) are consequences of the chaos which theoretical physics experienced through the second half of 20th century.

It is urgent, not only to those who study the foundations of physics but to physicists as well, to establish a more accurate definition of a physical theory because this will allow them to separate sound physical theories from theoretical constructions that are mere projects of theories. If they achieve

[2] Robert L. Oldershaw, *American Journal of Physics*, Vol. 56 (12), December 1988.

this, it will be a basis for a better foundation to their work. Below is a description of the criteria that will enable us to decide if a theory is acceptable, or if it is no more than a project of theory, i.e. a mere conjecture.

Assuming from the very beginning a non-naive realistic stance, we can divide this ensemble of criteria into two smaller ensembles. The first group includes the theoretical criteria. The second group includes the empirical criteria.

First group (theoretical criteria):

1. Every physical theory must be based on an ontological postulate. Whether explicitly or implicitly, this must exist. Of course, this ontological postulate is always a provisional one. Between the world and what we think about it, there is only an analogy and never an identity. However, this ontological postulate is important because it enables us to get a positive heuristics deeply connected with the theory throughout its development, within a scientific research program. This postulate, as any other postulate, is not a dogma. In science, there is no place for dogma. This compromise is no more than a framework, a first but fundamental postulate. In some way, the cause of the problems that physics feel nowadays was the rejection of an ontological postulate, common to the two theories mentioned above. We must not forget the deep influence of Ernst Mach on the young Einstein, and the influence of Harald Høffding on Bohr. Recent works shed new light on this situation. (Croca[3], Selleri[4], Moreira[5], Faye[6])

2. The second theoretical criterion is the adoption of a definite number of postulates consistent with the underlying ontological postulate. It would be possible to insert the ontological postulate among these new postulates but, to stress its primordial role, it was decided to present it in isolation. Once again, all these postulates are not dogmas. Archimedes called them requests, and this is the name that will be used here.

[3] J. Croca, *Towards a Nonlinear Quantum Physics*, World Scientific, 2003.
[4] Franco Selleri, *Lezioni di relatività da Einstein all'etere di Lorentz,* Progedit, Bari, 2003.
[5] R. Moreira, *Contribuição para o estudo da génese do princípio de complementaridade*, PhD Thesis, 1993.
[6] J. Faye, *Niels Bohr: His Heritage and Legacy*, Kluwer Academic Publishers, Dordrecht, 1991.

3. The third theoretical criterion is the adoption of a quantitative description of phenomena, which physics must achieve after Galileo. In order to achieve this purpose, every acceptable theory should use a mathematical codification. However, it will be possible, in principle, to expose the whole of a theory without using a single mathematical formula, which will make that description immensely tedious, as tedious as writing, for example, Beethoven's 9th Symphony without using staffs, ledger lines, clefs, notes values, etc. However, mathematical codification is an unquestionable and precious reasoning auxiliary tool, but we must be aware that underlying this commitment with a quantitative description is a radical reductionism. Classical mechanics deals exclusively with local movement, and is not concerned with every change we detect in nature. Aristotle classified these changes into two main divisions: substantial and accidental. Within accidental changes, he considered three sub-types: qualitative, quantitative and local. Classical mechanics considered only the last one. The first blow to this reductionism came from electromagnetism. We will return to this question later.

Second group (empirical criteria):

4. The quantitative description of phenomena obtained by the theory must lie inside the empirical data errors. This is the most naive but unavoidable empirical criterion. For instance, Tycho's Geo-heliocentric system would, in principle, satisfy this criterion.

5. Every acceptable theory must be able to accommodate new and unsuspected phenomena. Consider the existence of Neptune (Newtonian mechanics), Poisson's spot (wave optics), electromagnetic waves (Maxwell's electromagnetism) and the tunnelling effect for massive particles (quantum mechanics). This is a much sounder criterion than the first one.

6. The last empirical criterion that an acceptable physical theory must fulfill is to be able to enlarge our ability to act in the world through the production of instruments inconceivable without that theory. An acceptable theory is not a simple instrument. We must conceive it not as a common tool but as (if we are allowed to use this metaphor) a machine tool that allows us to produce new tools, new instruments. This last empirical criterion is the ultimate evidence that leads us to admit some kind of harmony between the world and what we think about it (the theory). We do not dare say more than this. But we think

that it is this last criterion, being to some extent a pragmatic one, in combination with the first theoretical one, that allows us to foresee a very general positive heuristics that gives us the possibility to overcome the crisis in which theoretical physics has fallen. This last criterion is consistent with our vision of the world as it will be understood in the final section of this work.

In the following table, we analyze several examples of theories and projects of theory (ancient and present) to show which among them fulfill all these criteria. The question marks inserted in some of the answers represent the perplexity of the reader and not our own[7,8].

	Theoretical criteria			Empirical criteria		
	1	2	3	4	5	6
Mathematical Model of Deferent and Epicicles	Yes	Yes	Yes	Yes	No	No
Newtonian Mechanics	Yes	Yes	Yes	Yes	Yes	Yes
Thermodynamics	Yes	Yes	Yes	Yes	Yes	Yes
Maxwell's Electromagnetism	Yes	Yes	Yes	Yes	Yes	Yes
Special Theory of Relativity	No?	Yes	Yes	Yes	Yes	Yes
General Relativity	No?	Yes	Yes	Yes	Yes	Yes?
Quantum Mechanics	No?	Yes	Yes	Yes	Yes	Yes
Quantum Electrodynamics	No?	Yes	Yes	Yes	No	No
Standard Model (nobody dares to call it a theory)	No?	Yes	Yes	Yes	No	No

We will focus our attention on the Special Theory of Relativity and Quantum Mechanics. Both sought to "reconcile" Newtonian Mechanics and

[7] General Relativity dismisses wave properties of matter as a necessary fundamental ontology. The only application of General Relativity that enlarged our action capabilities in the world was the contribution to the synchronization of the GPS system. However, the controversy on this subject is not finished. This is the reason why we use a question mark in the sixth column for General Relativity.

[8] The question marks in the last five lines of the first column will be understood in the following text. However, the question marks in the last two lines are mere consequences of three question marks on the fifth, sixth and seventh lines of the first column. The crisis in theoretical physics lies exactly here: The lack of a clear ontological postulate in the physical theories that emerged at the beginning of the 20th century prevented the emergence of a truly unified theoretical physics, culminating in an enormous mess.

Maxwell's Electromagnetism, the two major theories existing before the 20th century, under the new empirical evidence known then.

Newtonian Mechanics possesses a clear ontology. As we said before, there is a radical reductionism at the very bottom of this theory, a reductionism proposed by Galileo when he reduced the object of the new science to the mere description of the movement of corpuscles in space and time, forgetting the broad sense given to the concept of movement by the ancient Greeks. Newton clarified this reductionism when he proposed that the only significant property these corpuscles possessed was mass, i.e. the quantity of matter. They are localized, i.e. they possess a definite position in space for each instant in time. Space and time would constitute the stage where the corpuscles play their roles. These corpuscles suffer accelerations on their local movements under the action of forces according to the postulates of his theory. The ontology underlying Newtonian mechanics was the existence of a space considered as a stage independent of all the corpuscles that exist within it, and a time that flows independently of every change we can detect. The forces acting on these corpuscles can be divided into extrinsic and intrinsic. Extrinsic forces are the forces that act on the masses and are not produced by these masses. In that case, masses act as passive beings. They represent no more than the resistance that corpuscles offer to the action of this kind of forces. We call these masses inertial masses, and they are represented in the fundamental law of dynamics when the mass is constant, i.e.

$$\vec{F} = m\vec{a}$$

The mass, in this equation, is a mere actor, not an author.

On the other hand, intrinsic forces are the forces produced by the masses themselves. These are the forces represented in the law of universal gravitation, i.e.

$$\vec{F} = G\frac{m_1 m_2}{r^2}\frac{\vec{r}}{r}$$

These masses, which are called gravitational masses, are the authors of gravitational forces.

This is, in general, the underlying ontological framework of Newtonian mechanics.

Regarding electromagnetism, besides all the failed attempts to insert it into the ontological framework of Newtonian mechanics, trying to find the

properties of the mechanical ether[9] throughout the 19th century, the situation is radically different. In electromagnetism, the space was no longer a mere stage. It became an actor with physical properties. In electromagnetism, masses exist as the support of electric charges but are no longer the fundamental issue. But the existence of masses establishes the link to Newtonian mechanics because electric and magnetic forces act on charged masses as extrinsic forces. Charges are the sources of these kinds of actions, and this kind of action acts on masses that are charged as well. There is a permanent dialogue between them. The actions are supported by perturbations of the space between these charges. These perturbations of an underlying medium are often described using the wave concept. In electromagnetism, we deal essentially with waves that propagate in the framework of a space filled with the "subject of the verb to undulate".

Accordingly, there are two different and incompatible ontologies associated with both theories existing before the 20th century. What happened then? As stated above, both the Special Theory of Relativity and Quantum Mechanics wanted to "reconcile" these two theories, but followed different paths with different purposes.

We will briefly analyze the case of the Special Theory of Relativity first and the case of Quantum Mechanics later. We will give more attention to Quantum Mechanics because it gave rise to the wave-corpuscle dualism that is central to this text.

2. SPECIAL THEORY OF RELATIVITY

As we said before, when Einstein presented his Special Theory of Relativity, he was under the influence of Ernst Mach, which is why he followed a very formal path. He was not concerned with the unification of the two ontologies. Everybody knows that the equations used in Newtonian mechanics were an invariant for Galilean transformations, while, in electromagnetism, Maxwell's equations were an invariant for Lorentz transformations. Einstein's choice was to modify Newtonian mechanics in building the Special Theory of Relativity, whose equations were an invariant for Lorentz transformations. Following this exclusively formal path, he did not care about any kind of unification between the two differing and opposing underlying ontologies.

[9] We call it the mechanical ether because it would obey Newtonian Mechanics.

The two requests (postulates) of this theory are:

1. All physical laws are the same in all inertial frames of reference.
This means that the equations of mechanics and the equations of electromagnetism must retain their ability to describe the observable phenomena in any inertial frame of reference. They should maintain their form in every inertial frame of reference using Lorentz transformations.

2. The speed of light is the same in all inertial frames of reference.
One consequence of this postulate is that the speed of light is the maximum speed we can attain to transmit information.

Let us mention some anomalies. Franco Selleri wrote in 2003 that we can use transformations different from Lorentz that do not assert that the speed of light is the same in all inertial frames of reference. With these transformations, the famous equation $E=mc^2$ is preserved, alongside all the empirical agreement achieved by the Special Theory of Relativity without falling in the paradoxes such as the twin paradox. To show the unlikely conventional answer of the Special Theory of Relativity to this paradox, i.e. the twin that suffers the action of acceleration breaks the symmetry between the two twins, Selleri inserted a third twin that suffers exactly the same acceleration as the second, but runs a double path. In this case, he shows that it cannot be the acceleration that breaks the symmetry, because the third twin, according to the Special Theory of Relativity, will appear younger than the second twin and the only possible cause is the double path followed by him. Franco Selleri defends that we must admit the existence of a privileged referential in every situation. However, we think that Franco Selleri fails when he adopts so general a stance. We think[10] that there are cases where the situation mentioned by Selleri prevails. One of them happens with the Sagnac interferometer[11,12,13], mentioned by him, where we must adopt Galilean relativity to account for the phenomenon. The other can be found in

[10] J. R. Croca - Private communication.
[11] Franco Selleri, *"Lezioni di relativitá da Einstein all'etere di Lorentz,"* Progedit snc, Bari, 2003, pp. 174-177.
[12] Georges Sagnac, *"Sur la preuve de la réalité de l'éther lumineux par l'expérience de l'interférographe tournant," Comptes Rendus* 157 (1913), S. 1410-1413. Georges Sagnac, *"L'éther lumineux démontré par l'effet du vent relatif d'éther dans un interféromètre en rotation uniforme,"* Comptes Rendus 157 (1913), S. 708–710;
[13] Ruyong Wang, Yi Zheng, Aiping Yao, Dean Langley, *"Modified Sagnac experiment for measuring travel-time difference between counter-propagating light beams in a uniform moving fiber,"* Physics Letters A 312 (3003) 7-10.

a recent paper[14] by Günter Nimitz, a German physicist, who reports that he achieved the transmission of information with a speed 4.7 times greater than the speed of light, under a tunnelling effect. He was able to transmit Mozart's 40th Symphony under these conditions over a distance of 12 cm. Thus, there are, as far as we can see, two cases where Selleri appears to be right. In any other situation the second postulate works quite well.

Selleri's theoretical analysis, the peculiar case of a Sagnac interferometer, and the empirical result obtained by Nimitz, leads us to the conclusion that we must be very cautious when we analyze a theoretical project that does not convincingly meet all six criteria presented above. We are not saying that there are no important cognitive gains associated with the Special Theory of Relativity, but this crisis in contemporary physics compels us to consider all this from a new perspective. We are convinced that physicists need to abandon a widely spread instrumentalist stance to overcome the problems confronting physics.

The Special Relativity Theory helped us describe what happens in particle accelerators. We need increasing energy to accelerate a particle by the same amount of speed when it approaches the speed of light. The Special Relativity Theory helped us to explain, through the relativistic time dilatation, the exceeding measurement of the number of muons on the Earth's surface coming from space at high speed, because many more than expected were detected. Their short half-life of 1.56 microseconds would not allow it. These phenomena can be included in the fifth criterion that theories must observe. However, when we ask for an example of an instrument that we use nowadays and that could not exist without the Special Theory of Relativity, the common answer is the GPS system. This is because it is this theory, along with General Relativity, that enabled us to adjust the clocks in the satellites of this system. General Relativity foresees an increase in the beat of these clocks of 45,900 nanoseconds per day, while the Special Theory of Relativity foresees a decrease in the beat of these clocks of 7,200nanoseconds per day. Thus, it was necessary do delay them by 38,700nanoseconds per day in order to maintain the synchronicity between time measured by the clocks in the satellites and time measured by the clocks on Earth. Some may ask why not include nuclear reactions and nuclear plants as instruments that could not be built without the Special Theory of Relativity. The reason is that researchers at the U. S. Navy's Space and Naval Warfare Systems Center, or SPAWAR, in San Diego,

[14] Günter Nimitz, *Superluminal Signal Velocity and Causality,* Foundations of Physics, Vol. 34, No. 12, December 2004.

claim that cold fusion is now reproducible[15]. At a minimum, this shows that we still do not understand well enough what happens in this kind of reaction.

Croca[16] recently proposed a radical ontological unification within the framework of the scientific research program proposed by Louis de Broglie. This attempt will be comparable to the one that occurred in the 17th century. The ontological unification proposed by Copernicus, when he considered the Earth as a mere planet, i.e. that there were no longer two different worlds, the sub-lunar world and the supra-lunar world, led to an epistemological unification proposed by Galileo and, finally, led to Newton's theory. 17th century physics should follow the path to astronomy suggested by Plato. Plato suggested that the astronomical movements should be described using the concept of circular and uniform movement, because in the heavens we only observe one sort of change: the change of position. Moreover, if we assume that we observe, with the naked eye, about 1,000 astronomical objects, and 995 will suffer only a single, uniform circular motion within a period of 24 hours, it becomes easy to understand why Plato admitted that the supra-lunar world should be a part of the idea of circularity and uniformity. Thus was born Plato's scientific research program of astronomy to describe the only change that was observed in that world, i.e. the mere change of position. What Galileo proposed was that we confine ourselves to the changing of position as the object of physics here, on Earth. Not like the ancients, because they were not aware of Plato's radical simplification, but now wilfully, consciously. This radical reductionism leads us to the first modern physical theory: Newtonian mechanics and gravitation. The first clash to the success of this reductionism was Maxwell's electromagnetic theory as a development of Faraday concepts. As we said before, both theories rely on two antagonistic ontological postulates. The Special Theory of Relativity did not propose, as we said before, any ontological postulate. Besides, it maintained the use of inertial frames that makes sense in Newtonian Mechanics but is alien to electromagnetism.

Croca's proposal is inserted in the realistic and causal stance defended by Louis de Broglie, where matter possesses, actually and simultaneously, a corpuscular character and a wave character. Following this scientific research program, every material particle would be a complex structure interacting constantly with its surroundings, i.e. changing information and reacting concomitantly. This would enable us to reassess the second postulate of the Special Theory of Relativity. This postulate is generally

[15] Stanislaw Szpak, Pamela A. Mosier-Boss and Frank E. Gordon, "*Further evidence of nuclear reactions in the Pd/D lattice: emission of charged particles,"* Naturwissenschaften, February 2007, pp. 511-514.
[16] J. R. Croca - Private communication.

valid but there are some cases where it simply does not work, for instance, as we said before, in the tunnelling effect and in a Sagnac interferometer. We must reappraise the use of the concept of inertial frames of reference. We must substitute this concept, like Quantum Mechanics did, by the concept of measuring apparatus or by the concept of observer, not linked to any inertial frame of reference.

General Relativity integrated non-inertial frames of reference. However, it constrained itself to the same philosophical background, i.e. neo-positivism. Like the Special Theory of Relativity, it restricted itself to a mere mathematical framework without a solid ontological compromise. Both the Special Theory of Relativity and General Relativity keep themselves within the framework of physics confined to the mere description of the change of position. This means that they keep themselves, in large measure, within the radical reductionism proposed by Galileo.

From our point of view, this is the present situation. It is necessary to understand all this at a deeper level. But this will be impossible if we do not adopt, as we just said, a radically different ontological postulate. This will become clearer later. But before, we must talk about Quantum Mechanics.

3. QUANTUM MECHANICS

Quantum mechanics tried to "reconcile" Newtonian mechanics and electromagnetism as well. This necessity emerged from the study of the interaction of radiation (object of electromagnetism) with matter (until then the subject of Newtonian mechanics). The radiation spectra (emission and absorption) after 1885, the black body radiation (Planck's equation $E = \hbar\omega$, 1900), Einstein's introduction of the concept of photon[17] (1905) and Bohr's first model of the atom (1913) are products of that study. Louis de Broglie matter waves[18] ($\vec{p} = \hbar\vec{k}$, 1924) are an attempt that foresees a generalized and coherent conception of matter, that integrates both classical matter and

[17] Einstein adopted a different approach in the explanation of photoelectric phenomena. Here, he adopted an ontological compromise, introducing a corpuscle in the classical electromagnetic wave associated to the energy quantum. Einstein confessed later that a physicist should appear an incoherent person to philosophers. Einstein, in the Special Theory of Relativity, appears as a neopositivist. In the case just mentioned, he introduced an "atom" in radiation, a concept the neopositivists would reject as a metaphysical one.

[18] Louis de Broglie was a much more coherent physicist for philosophers. He tried always to adopt a non-naive realistic stance.

radiation. We must stress that the program he proposed appears as the most promising one, at least since the 1980s.

Bohr's atom model suffered great difficulties in the early 1920s. At that time, the need for a more satisfactory theory became evident. Two different paths were followed.

In Göttingen, Born, Heisenberg and Jordan, among others, tried, once again in a very formal way, to build a new mechanics for corpuscles, whose properties (position, velocity, energy, etc.) should be represented by matrices. This is why it is named matrix mechanics. The idea of using arrays came from empirical laws obtained for the spectra of radiation. These laws always contain two integers, one of the numbers to represent lines, and the other to represent columns. We must stress that in this exclusive formal approach the ontological compromise was absent as well. It can even be said that this approach adopted a hostile stance against any ontological compromise. However, it was in the framework of this heuristic process that Heisenberg reached the famous uncertainty relations. Immediately after having reached this result, Heisenberg went to Copenhagen to show it to Bohr (February, 1927). The reception was not a warm one: Bohr told Heisenberg that the result he obtained was mere mathematics. It was necessary to fill it with physics.

Bohr was aware that Schrödinger, all through 1926, had followed a totally different heuristic process. This new approach was almost exclusively formal as well. However, Schrödinger adopted an ontological postulate. He admitted that the wave concept was the primordial concept. According to Schrödinger, we should use only this concept in the construction of the new theory. This is why Schrodinger's approach is named "*wave mechanics*". We will briefly outline the heuristic process that he followed.

Hamilton, in the 19th century, had already told scientists that there was a great resemblance between two particular equations of classical physics. These equations were the Eikonal equation and the Hamilton-Jacobi equation. The Eikonal equation is:

$$\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 = n^2(x, y, z)$$

The solutions S(x,y,z,t) to this equation represent the wave fronts and, in the geometrical optics approximation, light rays perpendicular to them. The Eikonal equation is a simplification of the wave equation (the first equation in the next page) when the wavelength of light tends to zero, i.e. in order to deal with problems in which we can forget the wave character of

light. Thus, it can be said that the Eikonal equation may represent the movement of light corpuscles.

The Hamilton-Jacobi equation was used in classical mechanics to describe the movement of corpuscles under the action of conservative forces. It reads:

$$\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 + V(x,y,z) = -\frac{\partial S}{\partial t}$$

This equation describes the trajectory of corpuscles that obey Newtonian mechanics. These trajectories are perpendicular to the surfaces defined by the solutions S(x,y,z,t) of this equation, like in the Eikonal equation.

As stated above, the Eikonal equation is an approximation of the wave equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$$

Following this line of reasoning, Schrödinger tried to find a "wave equation" for material corpuscles that could be a generalization of the Hamilton-Jacobi equation for material corpuscles possessing wave characteristics, according to Louis de Broglie's original idea. The equation developed by Schrödinger was the famous equation that carries his name:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V(x,y,z)\psi = i\hbar\frac{\partial \psi}{\partial t}$$

However, Schrödinger was looking for a wave mechanics where corpuscles were absent. We must add that, in a series of papers published in 1926, Schrödinger had shown the formal equivalency of his approach to the approach followed in Göttingen.

Bohr was aware of all these facts, which is why he did not accept the meaning given to Heisenberg's "uncertainty" relations. He knew that the two fundamental equations of quantum physics, i.e.

$$E = \hbar\omega \quad \text{(Planck)}$$

$$\vec{p} = \hbar\vec{k} \quad \text{(Louis de Broglie)}$$

impose the unavoidable inseparability of quantities associated classically with corpuscles ($\boldsymbol{E}$ and $\vec{\boldsymbol{p}}$) and with waves ($\boldsymbol{\omega}$ and $\vec{\boldsymbol{k}}$). In the period from February 1927 to the fall of the same year, Bohr succeeded in finding an interpretation of quantum formalism using a principle that he named the complementary principle. This principle is the cause of what Popper called the schism in physics. This principle reduced an ontological problem into an epistemological one. The concepts of corpuscles and waves that are classical attributes of beings existing outside us became categories that belong exclusively to the cognoscente subject, i.e. the observer. This attitude is a transcendental escape of Bohr. Frequently, we note that physicists do not understand the deep meaning of the complementary principle. Often they don't even know of its existence. They reduce the complementarity principle to Heisenberg's "uncertainty" principle, related to the impossibility of measuring simultaneously and with unlimited precision the values of two conjugate variables, for instance, position $\boldsymbol{x}$ and momentum $\vec{\boldsymbol{p}}$. The debate on the consequences of the complementary principle led Einstein, with Podolsky and Rosen (EPR), to establish their famous paradox. It is helpful to look attentively to Bohr's answer to Einstein. Bohr said that the philosophical prejudices underlying the EPR paradox are incompatible with the philosophical foundations of Quantum Mechanics. Bohr never accepted the "battle" in the field proposed by Einstein. In order to understand this refusal, we must know Bohr's interpretation of quantum formalism, which calls for a deep understanding of the complementarity principle.

Bohr's philosophical convictions are a result of the profound influence exerted on him by his philosophy professor at the University of Copenhagen, a friend of his father, and his own friend when Bohr grew older: Harald Høffding. We cannot provide an accurate description of his philosophical stances in this text[19,20]. For him, the object of psychology was the study of the spontaneous behaviour of the human mind, and psychology was a phenomenology of the mind. Every law we would find in this behaviour must spread to every other manifestation of human thought, even to the most sophisticated ones, i.e. philosophy and science. The whole of human thought must be "*psychologically possible*". Høffding established a deep link between psychological functions, philosophical categories and scientific principles. He often mentioned this link in his works.

Høffding began his career as a philosopher in the fields of Religion, Morals, Ethics and Psychology. It was, however, in the area of psychology that he won international acclaim. So it is easy to recognize that psychology

[19] R. Moreira, *Contribuição para o estudo da génese do princípio de complementaridade*, PhD Thesis, UL, 1993.
[20] J. Faye, *Niels Bohr: His Heritage and Legacy*, Kluwer Academic Press, 1991.

plays a decisive role in Høffding's thought. He said that we could not engage ourselves in a detailed study of human thought without previously carefully considering the study of human psychology. Any study of human thought should begin with psychology. We should first try to know ourselves as an experiencing subject in order to know our own limitations. We can only understand them when we try to understand the purpose of the experiment, following the terminology adopted by Wundt. Let us hear Høffding himself:

> "The independence of psychology in the face of the theory of knowledge emerges from the fact that scientific knowledge can not free itself from the general laws of life of consciousness... Either the discovery, or the demonstration, are the result of psychic work. In terms of psychology, both should be possible... Thus, psychology is an introduction, a kind of Phenomenology of the Spirit."[21]

Psychological laws could not be violated in scientific activity. Høffding states:

> "In the Critique of Pure Reason, Kant proceeds synthetically. His 'metaphysical' deductions, in his 'Transcendental Aesthetic,' and his 'subjective' deduction, in his 'Transcendental Analytic', are actually psychological analysis through which the concept of form is separated from the concepts of experience and value. In my book *The Human Thought*, I proceed synthetically as well, attributing only a more decisive importance to the psychological and historical foundations. I wanted to show that the passage from intuition to judgment already contains the germ of scientific thought, and how this one, throughout the history of science, acquires a consciousness of himself that is increasingly clear. This is the reason for the division of my presentation into three main parts: functions, categories, problems." [Author's note: Psychological functions, philosophical categories, scientific principles][22]
> "Meyerson, from an analysis of the assumptions of modern science, returns to the opposition between identity and succession. Causality, in the strict sense of the term, implies, he said, a mutual equivalence of this type, the possibility of a replacement, and thus the identity. But according to him, this may be lawful without mutual equivalence; in which case substitution is not possible and this is why Meyerson did not use the term causality. Carnot's principle (or entropy)

[21] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, pp. 24-25.
[22] Ibid. pp. 25-26.

> expresses the importance of this difference. Meyerson finds in the history of science a continuing trend, an indomitable trend of human spirit in forming the concept of something that persists despite all the succession. But, as time cannot be eliminated, which proves Carnot's principle, setting a limit to the replacement of the states in nature confronts us with the opposition between identity and succession, related to the opposition between the quantity and quality, that transports us to the relationship between thought and sensation, which is the very object of Psychology. Here, once again, we go back of the principles to psychological functions through the categories."[23]

In examining in this light the thought of neo-Kantian Cassirer, he concludes that:

> "Cassirer even finishes his book[24] by saying that finally the opposition that exists between [Author's note: The theory of knowledge and psychology] disappears because 'the very psychology raises problems that should gradually look for their solutions in logic and its application to science."[25]

Thus, for Høffding, the way Cassirer addressed the problem would be similar to his own. But Meyerson and Cassirer are not the only philosophers used by Høffding to support this position. Poincaré is also referred to:

> "In the works of Henri Poincaré we also find the same triplicity [Author's note: Psychological functions, philosophical categories, scientific principles] we have just mentioned. His analysis of the theories of modern physics and mathematics led him to the opposition between continuity and discontinuity along the development of thought and science. The goal of psychology is, according to him, to consider the opposition that is constantly reborn between intuition and thought. This is the reason why Henri Poincaré[26] criticizes the possibility of a theory of knowledge totally independent of psychology: it will be as impossible as science without scientists."[27]

[23] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, p. 27.
[24] Cassirer, "*Substanz und Funktion,"* 1910.
[25] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, p. 27.
[26] Høffding refers to the "Dernières pensées,", pp. 139, 158.
[27] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, p. 27.

We must emphasize that Høffding highlights the fundamental opposition between the continuity and discontinuity categories assumed by Poincaré, and concluded decisively shortly thereafter:

> "If the existence of spontaneous thought did not prepare the existence of scientific thought, the science will miss the following cornerstone: nature. Science starts from the existence of spontaneous thought. Thus, there is a whole group of categories, the fundamental categories, with which the spontaneous thought works: synthesis and relation, continuity and discontinuity, similarity and difference, are forms without which any consciousness function cannot exist. They provide a definition of 'thought' in its broader sense, in the sense of Descartes when he said: 'I think, therefore I am'. The other groups are particular cases of the fundamental categories. In the classification and in the mathematical thought and logic emerge the formal categories, identity and rationality. Physical and natural sciences, psychology and sociology work with the support of real categories, causality, totality and development. Political economy, aesthetics, ethics and philosophy of religion have as their basis the concepts of value. A rational classification of sciences constitutes a unity with a systematic doctrine of categories."[28]

Contrary to Hegel, Høffding believed that scientific thought would not represent a complete cut with spontaneous thought and, therefore, with the contact with external reality: nature. The fundamental categories that are valid in spontaneous thought would also be valid for every type of knowledge, both specific and non-specific forms. The conclusions we would get from the study of spontaneous thought, namely through its psychological study, would spread inevitably to all types of knowledge, and, therefore, to scientific thought as well. This is a crucial issue. According to Høffding, every conscious activity must be "psychologically possible". Therefore, if a complementary principle was found to exist in psychology, this principle would spread to all activity of human thought, namely, to the domain of scientific knowledge.

This is the main point where this analysis differs from Faye's. Faye[29] admits that it is possible to distinguish in Høffding what he calls the "real me" and the "formal me", i.e. the "psychological me" and the "logical me", where the latter is a pragmatic version of Kant's "transcendental me". We would agree, but with strong limitations, defined exactly by the assumption

[28] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, pp. 27-28.
[29] J. Faye, *Niels Bohr: His Heritage and Legacy*, Kluwer Academic Press, 1991.

that any form of thought could overcome what is "psychologically possible". For Høffding, there is a limit from which the "formal me" would be completely subjugated by the "real me". The transposition of complementarity from psychology to physics happens through the adoption of a general epistemological framework where, beyond a certain limit, the "formal me" would be prevented from overcoming the limits imposed by the "real me". We will return to this issue later.

The complementarity principle reached by Høffding in psychology could be expressed by saying that it is impossible to see and to understand simultaneously. These are the two psychological functions. Consequently, it could be said that Høffding introduces a complementarity principle between the Kantian *a priori* forms of intuition and *a priori* forms of knowledge. At this point there is a very strong link between Høffding's thought and Kierkegaard's thought. As we know, when Kierkegaard proposed the concept of leap, he wanted to question Hegel's philosophical system. Hegel defended that we can reach the truth using our own rationality exclusively to solve the contradictions with which we are confronted. Although Hegel recognized the existence of a philosophy of nature by dividing it into three parts (mechanics, physics, and organic physics) he never dedicated much space to them in his works. Hegel's detachment from the material world is almost complete. For him, sensory data represented a devalued role. In his own words, he never became enthusiastic in viewing the Alps or a starry sky. Conversely, Kant always paid a renewed and increasing admiration to the starry sky and to the moral law living inside him.

> "If physics, says Hegel, should be supported exclusively by perceptions, and the perceptions should be no more than sensory data, physics processing would be no more than to see, to listen, to smell, etc., and then the animals could be physicists as well."[30]

Hegel's disdain for the accidental and finite as demonstrations of nature led him to find in reason, identified with reality and only in this one reality, the solution to all problems. Reason, as a manifestation of the infinite and universal, would necessarily reach the denial of the finite and the individual as manifestations of the contingent and uncertain nature. Hegel himself states:

> "The impotence of nature sets limits to philosophy, and what we can imagine as more inconvenient is that it should try to

[30] Enciclopedia delle scienze filosòfiche, trad. Croce, Bari, 1907, § 246, p.12.

> understand conceptually the mentioned accidentality and ... build it, imagine it ..."[31]

Of course, Hegel is not contradiction-free. He, too, in one way or another, refers to nature as something that is beyond the idea and, therefore, as something that could not be identified with the idea, i.e. with reason. But as we know, this is not the cornerstone of his thought, and he would gladly have bypassed this difficulty if it had been at all possible.

Høffding was aware of Hegel's stance and of the difficulties he experienced. Høffding criticizes him very clearly in a letter to Meyerson:

> "Hegel wanted to reject the irrational and, simultaneously, to keep the reality of the qualities. He did not see that this is impossible and he did not know the development of physics after Galileo."[32]

Contrary to Hegel, Høffding was intimately familiar with developments in physics and astronomy after the scientific revolution of the 17th century, and he was perfectly aware of the role Galileo played. Aristotle had considered two major kinds of changes, substantial changes, related to the generation or corruption of substances, and accidental changes. The latter category included three kinds of accidental changes: quantitative changes (change of size), qualitative changes (change in the characteristics), and local changes (change of position). Aristotle's physics intended to describe all these changes. Galileo reduced the object of physics dramatically. The only change Galileo was able to describe mathematically was the local change, i.e. the change of position. This stance was reinforced when Newton created the first modern theory. Almost a century later, Laplace believed that every change should be reduced to a simple local change. This is why he defended the vision of a deterministic world where free will would not exist.

In this attempt to reduce every change to local change, modern science rejected the existence of qualities within the Aristotelian meaning of the designation, i.e. non-quantifiable determinations of the substance. Classical science was looking for a simultaneously causal and spatiotemporal description of all phenomena. Thus, when Høffding stated that Hegel made a big mistake by saying that it would be possible to keep the reality of the qualities and reject irrationality simultaneously, he was clearly showing that,

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[31] Enciclopedia delle scienze filosòfiche, trad. Croce, Bari, 1907, § 250.

[32] *"Correspondence entre Harald Høffding et Émile Meyerson"*, ed. Frithiof Brandt, Hans Høffding et Jean Adigard des Gautries, Einar Munksgaard, Copenhagen, 1939, p. 8.

in his opinion, both things were incompatible. At this point, we must not forget Høffding's sentence:

> "Both [Høffding and Meyerson] detected an irrational residue that will remain forever, even when we made use of a strict rational scale, regardless of how large the progress of knowledge."[33]

That is an irrational residue that will always remain, despite all the progress that science may undergo. So, Høffding did not believe that reductionism, a characteristic of modern science, would always be possible. If we find that a small portion of reality could not be described using the category of causality (Newtonian causality), and within the framework of space and time simultaneously, we should state that qualities would exist, i.e. something would not be quantifiable. Surely, as we already said, all this has strong connections with Kierkegaard's thought. Kierkegaard's qualitative dialectics defended the same thought in the domain of rational activity. A manifestation of a volitional action represents, in Kierkegaard, a leap from a situation where there exists a world of possibilities to a state in which only one of these possibilities comes into existence. This leap would remain forever inaccessible to all rational activity. This represents Kierkegaard's major criticism of Hegel's attempt to reduce reality to thought, to the Idea. Something similar exists in Høffding. However, Høffding did not adopt Kierkegaard's stance without some criticism because this would represent the complete renunciation of the attempt to study the rational activity, thus removing, in this process, the scientific character of psychology. Høffding could not accept this. Høffding wanted to find, in a very precise way, where this irrational residue would exist by making use of his psychological studies. This is the reason why Høffding accused Kierkegaard of giving up on psychology too easily.

Høffding did not adopt Kant's classification of categories. For instance, he defended that we should reject the category of substance. This ontological scepticism is an important feature of Høffding's thought. Appealing to Leibniz's authority, he said that we never know from a substance more than the laws of its actions, and should therefore give up this category. He considered three pairs of fundamental categories in his classification of categories: synthesis and relation, continuity and discontinuity, similarity and difference. These categories would be used in every form of thought, including spontaneous thought, and this is why he

[33] *"Correspondence entre Harald Høffding et Émile Meyerson,"* ed. Frithiof Brandt, Hans Høffding et Jean Adigard des Gautries, Einar Munksgaard, Copenhagen, 1939, p. 153.

considered them fundamental. Høffding established a complementarity relationship between these three pairs of fundamental categories. The complementarity relationship stressed by Høffding as existent between the two psychological functions is, evidently, strongly connected to Bohr's complementarity principle in physics. But Høffding did it first.

We may be inclined to establish links between Høffding's complementarity concept and Kierkegaard's qualitative dialectics. This temptation led some authors to conclude that Kierkegaard influenced Bohr directly. However, great caution should be exercised because Kierkegaard looked to science with a kind of fear, as evidenced by his famous aporia "*Subjectivity is Truth*" in the Postilla,[34] i.e. the truth resides in the subject, not in the object, and represents a total incompatibility with any kind of activity in natural sciences. All through the 19th century, science pretended to be completely objective. Høffding, instead, knew very well the history of science and tried to weaken this aporia in order to make it compatible with natural sciences. When Høffding accused Kierkegaard of having given up on psychology too easily, he was saying that in his psychological studies he believed to have found support for the existence, not of a total incompatibility with natural sciences, but of an irrational residue that represents a limit to our ability to describe in a quantitative manner nature and our relationship with her, even when we make use of all our rational capabilities.

In his book "Relation as a Category", Høffding describes the relationship between the concept of reason and the fundamental categories. At a certain point, he says:

> "The relation between continuity and discontinuity is analogous to the one existent between synthesis and relation or constitutes a particular form of this one. The synthesis operates much more easily as there is less disruption... When the continuity reigns, intuition and reflection take the form of a river that runs regularly without eddies or cascades. No problem arises because no shutdown, no perplexity imposes it. In a better way, if we question ourselves, the question can be put as: why not? Why not continue the same path if there is sufficient energy to carry on. But in life and in science there are fault lines more or less clear and, therefore, opposition and resistance that force us to stop."[35]

[34] Johannes Climacus (Søren Kierkegaard's pseudonym), in *"Concluding Unscientific Postscript to Philosophical Fragments,"* 1846.
[35] Høffding, *"Philosophes contemporains"* Alcan, Paris, 1924, p. 196-197.

All this could have been written by Kierkegaard. After a few more sentences about the path taken so far by science, he adds, referring directly to the problem raised by the emergence of quantum energy:

> "In physics, the recent doctrine of Quanta, that the energy does not change in a continuous manner but by leaps, is based on mathematical laws according to which this exchange by leaps appears as necessary... they seek theoretical points of view to help them understand the discontinuity. Because such jumps can be ordered in series, the work to discover a new continuity has already started even if the continuity is more formal than real."[36]

And he continues:

> "These problems had already been posed by the most ancient Greek philosophy. The "uno" of Parmenides had simultaneously the meaning of continuity and identity; for him, there were no differences between them. On the contrary, Heraclitus and Democritus insisted on discontinuity. The struggle proceeded until modern times and it was it that piqued Platonism and its adversaries. Kant considered this opposition as a rational need. His doctrine of antinomies tries to prove that there exists an absolute limit to reason. The great founder of critic philosophy did not see that discontinuity cannot be the last word of thought other than under the form of a problem; so their "antithesis", which develops this point of view, is correct, contrary to their 'thesis' that supports the existence of absolute discontinuities."[37]

These words cannot be misunderstood: Høffding speaks within a strict epistemological framework. We must always look for continuities, however:

> "Continuity and discontinuity are correlatives that cater to each other. They designate different points of view and different operations; the history of science shows how much one or the other takes precedence, but in a way that the struggle between them is always reborn. Nobody has clarified this issue better than Henri Poincaré when he said: 'This fight will last as long as we do science, as long as mankind think, because it is due to two *irreconcilable needs* of human spirit, that this spirit cannot

[36] Ibid, pp. 197-198.
[37] Ibid, p. 198.

> abandon without ceasing to exist, *to understand*, and we cannot comprehend but the finite, and *to see*, and we can not see anything except the extension which is infinite..."[38]

According to this text, there exists a complementary (in Bohr's sense) relationship between the categories of continuity and discontinuity, and it is deeply linked to another complementary relationship between two irreconcilable needs of the human spirit: to see and to understand. These words from Poincaré, that Høffding adopts enthusiastically, are the touchstone to detect how Bohr reached the complementarity principle. It is presumed that Bohr read Høffding's book quoted here because he congratulated him on its title, something that would not be possible if Bohr had not read it. As stated above, according to Høffding, psychological functions are *to see* and *to understand*. And, still according to him, every law we detect in the spontaneous behaviour of human thought, the fundamental object of psychology, should overflow to every other level that human thought can reach. Høffding maintains that continuity and discontinuity are two *irreconcilable needs*. In the dawn of Quantum Mechanics, as stated before, there emerged a tension between the attempts of Max Born and Werner Heisenberg, on one side, and Erwin Schrödinger, on the other. However, if Born and Heisenberg adopted a hostile attitude against any ontological postulate, and their approach was a strict formal one, they had, as background, the classical mechanics associated with the concept of matter corpuscles linked to a discontinuous vision of nature. Instead, Schrödinger, even if his approach had also been almost formal, defended an ontological compromise in which the primordial concept was the concept of wave, associated with a continuous vision of the being. It is the conjunction of the irreconcilability of both concepts with necessity that characterizes Høffding's thought and Bohr's interpretation of the quantum formalism as well. Although Høffding presents this fight in a diachronic form, he never said that it would be impossible to reach a stage of the advancement of knowledge in which discontinuity would prove impossible to overcome. And if we recall the hypothesis that he raised when he discussed the issue of vitalism, saying that we should find "on the lower level of material existence anything analogous to the psychic, if we don't want to admit that the life of consciousness arises by a leap"[39] (and the influence of Spinoza prevented him from accepting this) we must consider, on the contrary, that he admits such a possibility, especially if combined with other statements he made,

[38] Høffding [1924], pp. 197-199. Poincaré's statement quoted by Høffding can be found in "*Les Conceptions Nouvelles de la matière*," in "*Le matérialisme actuel*," p. 67.

[39] Høffding, *La Relativité Philosophique*, Paris (Alcan), 1925, pp. 112-113.

stating that "physiology is therefore much more favourably inclined towards the principle of continuity than psychology can ever come to be...,"[40] which clearly shows that the behaviour of the human mind, which Høffding believed not to be perfectly continuous, could have a counterpart in the lower levels of material existence. The existence of the quantum of energy and the interpretation of quantum formalism reached by Bohr could never have disturbed Høffding's general epistemological stance. This interpretation has been defended by Faye, but we think that, this way, it is now much more evident.

As we said, Høffding considered these functions to be *irreconcilable needs* of human thought. They are *irreconcilable needs* that we cannot perform simultaneously. We can only understand what we saw, and never what we see. We can immediately conclude that, for Høffding, Kant's *a priori* forms of sensitivity (space and time) and *a priori* forms of understanding (categories, e.g. causality) are also two *irreconcilable needs*. This is the fundamental "law" he believed he had found in psychology and extended to philosophy. He criticized Kant and Kierkegaard, saying that neither paid sufficient attention to psychology. For Høffding, psychology was the key to understanding the difficulties we always experienced in our attempts to understand the world around us. He linked it in a manner similar to Kant's antinomic categories: continuity and discontinuity. This was how Høffding established a *complementarity principle*, without explicitly naming it, as an *irrational residue* in psychology, and extended it to philosophy. But he clearly established the unavoidable broadening of this principle to science. Science should be *psychologically possible*. He was not a scientist, and was unable to make this final leap. Bohr did it. When we read Bohr's communications in Como and those from the 5th Solvay Conference in 1927, we are immediately confronted with the statement that it is impossible to get a space-time description and a causal description of quantum phenomena simultaneously. That is, we cannot "*see*" and "*understand*" quantum phenomena simultaneously. And he added that we cannot attribute a wave or corpuscular character to a quantum particle simultaneously. The wave concept is associated with a continuous (non-localized) character of the being, and the corpuscle concept is associated with a discontinuous (localized) character of the being. The question that remains is: are these concepts properties of the being, a position that is defended by realists, or are they categories of human thought, i.e. we must use without being able to make them attributes of an external world?

Later, Bohr said that the difficulties we always felt in understanding what we observe, i.e. Høffding's *irrational residue*, are now expressed in a

[40] Høffding, *Os problemas da filosofia*, Lisboa (Lux), 1962, p. 36.

lucid mathematical form. This lucid mathematical form was the Fourier analysis. This analysis allowed Bohr to introduce the *complementarity principle* as the key to interpret quantum formalism. In a simplistic way, Bohr erected the Fourier analysis to an epistemological privileged analysis. He attributed to it an exclusive and fundamental epistemological status. We do not say ontological because the *complementarity principle* deals exclusively with our cognitive failures and not with any attributes of an outside world. This is the reason why Bohr, when he discussed the foundations of Quantum Mechanics, always referred to the operation of measure. For him, such an operation would reveal our cognitive failures. The possible properties of an external world would always be submitted to our cognitive disabilities. It is a common misunderstanding to consider the uncertainty relations as a consequence of the existence of an unavoidable perturbation due to the existence of a minimum value of the energy necessary to perform an operation of measure. This is not the point. To perturb something, it is absolutely necessary that something really exist before the operation of measure. But if we admit so, we are immediately admitting that Quantum Mechanics is an incomplete theory because Bohr's interpretation of quantum formalism would prove unable to describe the existence of a real state of the quantum object before the operation of measure. For Bohr, it has only a potential existence. This constitutes a common misunderstanding about the meaning of Bohr's interpretation of quantum formalism. We all know that Einstein never accepted Bohr's interpretation of this formalism. But we are unsure if he fully understood Bohr's interpretation. This is why those who mention Heisenberg "uncertainty" relation and link it with any measure apparatus forget the deep meaning of the *complementarity principle*. Only within the *complementarity principle* framework can we give to Heisenberg relations the meaning of indeterministic relations and not that of "uncertainty" relations. The problem with Bohr's Quantum Mechanics is exactly the lack of an ontological compromise. What Bohr did was a transcendental escape in a Kantian way, but within Høffding's framework. Wave and corpuscles are scientific concepts, linked to continuity and discontinuity as fundamental categories, and to the two psychological functions, i.e. to see and to understand. Between them there is a complementary relation. They are "irreconcilable needs" of human thought.

However, we must admit that Quantum Mechanics satisfies all the other five criteria proposed at the beginning of this text. Quantum Mechanics is a theory that fulfills the last criterion because it enables us to build new tools that would be inconceivable without it. Most developments in the study of condensed matter are due to Quantum Mechanics. Its application in theoretical chemistry has been very important. We owe to Quantum

Mechanics the existence of electronic microscopes and tunnelling microscopes, as well.

We can, and many do, ask the question: where is the problem? The problem is that both Special Relativity and Quantum Mechanics establish an epistemological division between the macro and the micro world. Nowadays, we think that we need to unite these two worlds, like it was necessary to unify the sub-lunar and the supra-lunar worlds in the 17th century scientific revolution. We must adopt an ontology common to both Special Relativity and Quantum Mechanics that would enable us to get a unifying theory, one that would unify corpuscles and waves in complex structures, which would also enable us to unify Newtonian Mechanics and Maxwell Electromagnetism under a single theory. The Dirac equation was the last formal successful pastiche done between the Special Theory of Relativity and Quantum Mechanics, but without trying to establish a unifying ontology, because neither Special Relativity nor Quantum Mechanics imposes it.

This is the main reason why 20th century epistemology introduced the concept of incommensurability (Kuhn), anarchic scepticism (Feyerabend) and ontological scepticism (semantic view). The currently dominant semantic view represents the current stage of a process that began in the 19th century with positivism, and successively spanned empirocriticism, neo-positivism and logical positivism. Both Special Relativity and Quantum Mechanics emerged within this epistemological framework. As we have just shown, Bohr's thought is rather more complex, but the greater number of Bohr's interpretation supporters accepted this framework, mainly because many of them did not fully understand Bohr's interpretation. Thus, we are now immersed in a kind of neo-scholastic or, similarly, immersed in a tautology between mainstream physics and mainstream epistemology. It is urgent to break it. It is urgent that we start a new physics revolution.

It is evident, today, that the fundamental problem of physics is the wave-corpuscle dualism. The "solution" to this problem proposed by Bohr in his interpretation of quantum formalism is now unsatisfactory, not only from a philosophical viewpoint, but from a strictly physical viewpoint as well. Presently, we know that the "barrier", i.e. the *irrational residue* expressed through the *complementarity principle* and its mathematical expression, the Fourier analysis, has been empirically surpassed. This was first shown by J. Croca in 1996[41], and constitutes a strong blow to Bohr's interpretation.

[41] J. Croca, *Experimental Violation of Heisenberg's Uncertainty Relations*, talk at the 5th UK Conference on the *Conceptual and Philosophical Problems in Physics*, at Oxford, Sept. 1996.

The Feynman Lectures on Physics state that:

> "The uncertainty principle 'protects' quantum mechanics. Heisenberg recognized that if it were possible to measure momentum and position simultaneously with a greater accuracy, the quantum mechanics would collapse. So he proposed that it must be impossible."[42]

We are not sure if Quantum Mechanics would collapse, but we are sure that the limits of its applicability would be finally established. However, the important issue in the above quotation is the implicit recognition that Bohr's interpretation of Quantum Mechanics would not survive, if it was possible to measure the momentum and position with greater accuracy than those imposed by Heisenberg's relations. Heisenberg[43] himself, showed, and after him, Karl Popper[44] and Andrade e Silva,[45] also showed, against Feynman's statement above, that in an actual operation of measure it was possible to achieve a greater accuracy than that established by these relations. The answer given by Heisenberg to this challenge, admitting this fact, was to show that we can never predict the result of a future measure with such precision. We think that this answer from Heisenberg is an auxiliary hypothesis that implicitly recognizes the failure of Bohr's interpretation of quantum formalism. The *complementarity principle* deals with our cognitive limitations that would always manifest themselves in an actual operation of measure and not only in the prediction of a future measure. Conclusion: nowadays, after the works of D. Bohm, K. Popper and J. Croca, Bohr's interpretation of Quantum Mechanics has been discredited. Unfortunately, many are not yet aware of this. Moreover, we now know that it is possible, not only to measure, but to predict the position and velocity of a quantum particle with a precision 350 times greater than that predicted by the Heisenberg-Bohr relations, using the so-called super-microscopes![46] This is an empirical fact.

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[42] R. Feynmam, *Lectures on Physics,* Vol. III, Cap. 1, p. 11.

[43] W. Heisenberg, *The Physical Principles of Quantum Theory*, Dover, New York, 1930.

[44] Carl Popper, *The Quantum Mechanics and the Schism in Physics,* Hutchinson, London, 1982.

[45] J. Andrade e Silva, Portugalie Physica, 4 (1966) 257.

[46] Ivan Amato, *Tool builders are pushing optical microscope vision to single-molecule sharpness,* Chemical & Engineering News, Volume 84, Number 36, pp. 49-52. Henri-Pierre Penel, *Le Microscope capable de voir les gènes*, Science et Vie, nº 942, Mars 1996, p 86.

David Bohm showed, in 1952[47], that a different interpretation of quantum formalism was possible by introducing a new one. But Bohm's interpretation of quantum formalism never achieved an experimental situation where the predictions of his theory were different from Bohr's. However, what Bohm achieved is an important historical event. For instance, it permitted the French Nobel Prize winner Louis de Broglie to go back to his initial attempt to congregate corpuscles and waves in a unifying ontology in physics, establishing a new scientific research program. It leads us to a new theory, with new equations.[48] Within this program it has been possible to envision experiments[49] where the predicted results are substantially different from those predicted by Bohr's interpretation of quantum formalism. These experiments will enable us, in principle, to establish the limits of applicability of Bohr's theory. This will provide arguments to pursue this scientific research program along with its underlying ontological compromise. Within this scientific research program, any quantum particle is a complex structure, constituted simultaneously by a θ wave and a ξ acron (corpuscle), similar to a pinnacle, guided by this wave. This implies the necessity to build a non-linear theory, as outlined by Louis de Broglie and more recently by J. Croca.[50] The main purpose of this scientific research program is to unify macrophysics and microphysics in an ontological manner, and surpassing the ontological and epistemological chaos in which physics has lived throughout the past decades. The commensurability between the descriptions of the micro-world and the macro-world becomes possible either ontologically or epistemologically. Louis de Broglie's scientific research program is not affected by Bell's theorem,[51] which discusses the attempts to recover a Newtonian description of nature. In this program, on the other hand, there are no material points with a precise

[47] D. Bohm, 1952, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables, I and II," *Physical Review* **85**: 166-193.
[48] J. R. Croca, *Towards a Nonlinear Quantum Physics*, World Scientific, 2003.
[49] J. and M. Andrade e Silva, C. R. Acad. Sci. (Paris) 290 (1980) 501 - F. Selleri, Ann.Found. L. de Broglie, 4 (1982) 45. - F. Selleri, Found. Phys. 12 (1983) 1087; 17 (1987) 739. - A. Garuccio, V. Rapizarda and J. P. Vigier, Phys. Lett. A 90 (1982) 17. - J. R. Croca, in *Microphysical Reality and Quantum Formalism*, eds. A. Van der Merwe, F. Selleri, G, Tarozzi, Kluwer, Dordrecht, 1988. - J. R. Croca, Ann. Found. L. de Broglie, 14 (1989) 323. - J. R. Croca, in *Waves and Particles in Light and Matter*, eds. Alwyn Van der Merwe and A. Garuccio, Plenum, 1994. - J. R. Croca, A. Garuccio, V. Lepori e R. N. Moreira, Found. of Phy. Lett. 3 (1990) 557. - J. R. Croca, R. Moreira and A. J. Rica da Silva, in *Causality and Locality in Modern Physics*, eds. G. Hunter et al., Kluwer Academic Publishers, 1998.
[50] J. R. Croca, *Towards a Nonlinear Quantum Physics*, World Scientific, 2003.
[51] J. S. Bell, Physics, 1, 195 (1965).

position and velocity. It deals with complex structures in permanent interaction with other complex structures, both having ill-defined frontiers. These new ontology and epistemology will enable us to glimpse the unification of Gravitation and Electromagnetism and, consequently, that of Quantum Mechanics and Special Relativity because both attempted to "reconcile" the two classical physical theories just mentioned without assuming a clear ontological compromise. The phenomena that General Relativity tries to describe will be a trivial consequence of this unification. We will see the world through different eyes. Recently, within the context of this scientific research program, a new principle[52] was proposed to unify the description of both classical world visions. We will talk of this principle in the next section. As stressed before, this is not a matter of mere philosophical taste. There are experiments to be done. We just need to look, once again, through the "monocle".

4. METAPHYSICAL CONSEQUENCES IN GIORDANO BRUNO'S WAY

The ontological compromise associated with the scientific research program initiated by Louis de Broglie, and developed by Croca and his group, enables us to foresee a possible great unification. When we say that a physical being, like a quantum particle, is a complex structure that is permanently interacting with its surroundings and reacting according to that interaction and to its own structure, we are entering into a completely new vision of the world. Theoretical biologists define life by three properties, namely, the ability to metabolize, to reproduce[53] and to evolve. A pre-biological physical being, as far as is known, cannot reproduce. But if we weaken the meaning of the verb to metabolize, perhaps we can say that pre-biological beings can, within this new broad sense, "metabolize". They are in permanent transformation, a transformation compatible with the level

[52] J. R. Croca, *"The Principle of Eurhythmy. A key to the Unity of the Physics"*, in S. Rahman, J. Symons, O. Pombo and M. Torres, *Unity of Science. Non traditional approaches*, Dordrecht:, Kluwer/Springer, 2009.

[53] Stuart Kauffman, in his book "*At Home in the Universe. The Search for Laws of Complexity," Oxford University Press, 1995,* defends that there are *autocatalytic systems* and that life represents a *catalytic closure*. These systems would represent a primordial form of reproductive systems that possesses proto properties of life systems. However, Stuart Kauffman confines his defense arguments to the field of biology, chemistry and physical chemistry. He is not aware of how the new physics we are proposing may contribute to his objective. But this would imply the adoption of a completely new world view.

of structure we are talking about. However, one thing we can be sure of: to evolve will be a true characteristic of pre-biological natural beings, i.e. of pre-biological natural complex structures. Otherwise, we would not understand the emergence of biological beings, i.e. of biological structures as a natural process. Croca's *The Principle of Eurhythmy*[54] enables us to foresee a more general principle that will remain in every level of complexity of natural structures. We are referring to a principle of persistence. Every emerging natural structure follows that principle: the emergence of a new structure, and its behaviour after this emergence, are expressions of an attempt to persist as a structure and of their ability to self-assemble. The goal of *The Principle of Eurhythmy* is to give a comprehensive view of the way a complex structure that corresponds to a quantum object, possessing simultaneously a θ wave and an acron (ξ), similar to a pinnacle, guided by this wave, should follow to persist. This is the ontological compromise that attempts to achieve a realistic and causal interpretation of quantum phenomena within Louis de Broglie's research program. Let us ear what Croca says:[55]

> "... Initially, the relation between the theta wave and the singularity was named by de Broglie as the guiding principle. Meaning that the singularity is guided through a nonlinear process, preferably to the regions where the theta wave has higher intensity.[56] Still this name does not give full account of the complex physical process involved. What really happens is that the singularity, or the corpuscle, necessarily immersed in the theta wave field when moving randomly from one point follows, on average, the best possible path. The best possible path is the one where the theta wave has higher intensity. So, in reality, it is not a simple action of guiding. The theta wave naturally guides the corpuscle, but this guiding action is of a very special kind. The theta wave, in reality, guides the singularity to the best possible path, the average path where the intensity of the theta wave is greater. The singularity avoids the

[54] *The Principle of Eurhythmy* is a reappraisal, in the framework of Louis de Broglie's scientific research program, of Albert Einstein's and Max Born's association of intensity of the wave *Ψ in one region of the space* with the probability of, in a future measure, detecting the particle in that region.

[55] J. R. Croca, "*The Principle of Eurhythmy: A key to the unity of physics"*, in the First Lisbon Colloquium for the Philosophy of Science - Unity of Science, Non-traditional Approaches, Lisbon, October, 25-28, 2006, pp. 9-10.

[56] Max Born has already said something apparently similar. However, he gave up achieving a realistic and causal description of quantum phenomena. Here, Croca, based on Louis de Broglie's original ideas, attempts to go further.

> regions where the intensity of the theta field is null because in these regions its very existence is in danger, in the sense that in such zones the corpuscle needs to regenerate the surrounding field at the expenses of its own energy. Therefore the motion of the corpuscle in the theta wave field follows always the best possible path, the most adequate path, that is, the average motion follows the principle of eurhythmy. It is assumed, of course, that any corpuscle, as long as it keeps existing, has an inner energy of its own and furthermore keeps always in motion with a natural velocity. The corpuscle moves incessantly, in the theta field, with an instantaneous huge velocity, called the natural velocity. Nevertheless, due to the chaotic nature[57] of the sub-quantum field the average velocity, which is the observed velocity, can go from zero to the natural velocity."

In this quotation lies the proposal for a new ontological unification, proposed initially by Louis de Broglie and developed by Croca in *The Principle of Eurhythmy*. At this point, it becomes imperative to clarify what is the so-called sub-quantum field. This concept was first introduced by Louis de Broglie. He maintained that any quantum particle is a real complex structure where we can apprehend a corpuscular characteristic and a wave characteristic, both parts of an inseparable monad.[58] Admitting this, he was faced with an unavoidable problem. If there is a real complex structure at the quantum level, then it must be an inferior level of reality from which that complex structure emerged. Evidently, since we are talking about one of the most important physicists in the 20th century, this step became necessary because of his realistic stance and also of some characteristics of the quantum formalism. We do not intend, in this text, to enter into the mathematical details of the problem, because these can be found in other texts.[59] However, it must be said that, in quantum formalism, namely in the Schrödinger equation, there exists the conjugation of two classical physics equations, the continuity equation and the Hamilton-Jacobi equation, with a supplementary term that was interpreted by Louis de Broglie and by David Bohm as representing the interaction of the quantum particle with this lower level medium, i.e. the sub-quantum medium. The supplementary term in the Hamilton-Jacobi equation was called the quantum potential.

[57] Croca uses *chaotic nature* in the sense that, at that level, we cannot foresee any correlations. This corresponds to an epistemological chaos and not to an ontological chaos.
[58] The term monad is of our own responsibility.
[59] J. R. Croca, *Towards a Nonlinear Quantum Physics*, World Scientific, 2003, pp. 65-80.

The sub-quantum medium can be regarded under two different metaphysical perspectives. They are metaphysical perspectives because they are behind all empirical information. We cannot report, for the time being, any interactions below the quantum level. From this point, we may look at the sub-quantum level as, using Anaximander's name, an undefined medium, i.e. in ancient Greek, the apeiron, a medium considered as an underlying chaotic level. This chaos can be considered as an epistemological chaos, or an ontological chaos, that is, it can be considered a chaos either because we cannot apprehend any interactions under the quantum level or because it is chaotic in itself. Evidently, nobody can answer this question seriously - it is a matter of philosophical opinion. We can always acknowledge that, in the future, we will be able to apprehend interactions at that level. Before the advent of the microscope we had no information about the existence of biological cells. Today, we are able to distinguish periods of time of 10^{-15} seconds, and two points in space separated by a distance of 10^{-10} meters. Fifty years ago nobody would have suspected this. Perhaps in the future we will be able to go even deeper. However, for the time being, we can regard the sub-quantum level as an Anaximander apeiron. This is the easiest metaphysical stance. Evidently, we may always push the sub-quantum medium beyond the limits of any empirical information that we may obtain in the future, far beyond the limits imposed by the current interpretation of Quantum Mechanics.

Going back to the quotation above, we must clarify a crucial point: it contains no deterministic stance. The new physics does not intend to recover the Newtonian description of the world. This would be insane. There are no *hidden (Newtonian) variables*. Complex structures do not have a precise position in space and their frontiers are always ill-defined. They are in permanent interaction with their surroundings and this interaction modifies their structure permanently. So, *Bell's Theorem*[60] does not apply to this new physics and to the underlying new vision of the world we are proposing. The position of the acron inside the structure of a quantum complex structure is always ill defined because the complex structure to which it belongs is in a permanent evolution. Matter is no more considered as consisting of mathematical points with properties like mass, charge, position and velocity. The behaviour of the complex system mentioned above is not completely determined by its theta wave. There are permanent interactions within any complex structure. As we said, the acron and the theta wave are only two perspectives of an inseparable monad. A quantum complex structure is not the linear sum of its parts. Its parts, the acron and the theta wave, are in permanent interaction. The theta wave is part of its structure and interacts

[60] J.S. Bell, Physics, 1, 195, 1965.

with other theta waves surrounding it. This interaction may lead to a decrease of the amplitude of the theta wave and, in this case, the acron will decrease its own amplitude in order to rebuild the conditions to persevere as a complex structure and, if this situation persists, it may lead to its own destruction, i.e. a situation where it would no longer be in a position to survive as a complex structure. In this description, there is an obvious enunciation of an incipient "free will", but this must not be confused with some sort of concession to a panpsychic stance. Perhaps some can associate what we are saying to the panexperientialism of Alfred North Whitehead.[61] In fact, we may agree with Whitehead when he credits all entities, not with cognition, but with phenomenal "consciousness". We used quotes because even this statement contains serious risks of misinterpretation. However, if this statement means the possibility of interacting with the surroundings and a certain propensity to survive, we may agree. Occasionally, this interaction and this propensity to survive may create new associations, i.e. new emergent complex structures; then, we may, once again, very carefully, agree. This tendency to survive is linked to the *principle of eurhythmy* that emerged from the empirical data that came to us from the deeper level where we can establish some sort of correlation, i.e. the quantum level. But we don't dare say that a quantum complex system possesses a consciousness - this is a dangerous term that can be misunderstood. However, the *principle of eurhythmy*, linked to the abandonment of the wave-corpuscle dualism, can be regarded as underlying all the non-deterministic evolutionary process. We will return to this point later.

Any ontology is always provisional. We can only accept it if the theory emerging from it obeys the six criteria established at the beginning of this work. As stated before, if we can draw a lesson from our cultural history, it would be: the world is, surely, much more complex than we think it is. A theory that meets these six criteria only allows us to say that there is some harmony between the world and what we think about it, i.e. what the theory says about it. The world is surely much more complex than the new vision of the world we are proposing; however, accepting this new ontological unification, we can reappraise, for instance, the famous controversy between Leibniz and Clark about the meaning of Newton's law of gravity. What can we say after the introduction of the *principle of eurhythmy*? We must say that the Sun is not confined to its appearance. The Sun must inevitably be here, i.e. the complex structure we call Sun is much more extensive than what we can directly observe. What we see is the light emitted by each one of the complex structures, of lower levels, composing

[61] A. N. Whitehead, *"Process and Reality" (corrected edition),* The Free Press, NY, 1978.

the "acronian structure" of the Sun. The Sun's "acronian structure" is the result of the ensemble of acrons of the quantum particles that constitute it. The Sun is, within this new vision, much more than that. The Sun's huge θ wave, composed of all the interacting θ waves of all the acrons that constitute it, exists inevitably in permanent interaction with the Earth's huge θ wave, and, at least, with all the huge θ waves of each planet belonging to the Solar system. The Earth "*feels*" the Sun, i.e. the Earth "knows" that the Sun exists, or in the usual words, the Earth receives the gravitational information that comes to it from the Sun, and follows its path according to the *principle of eurhythmy*. Evidently, we are talking about the present situation. The process followed by the Solar system, until the present situation emerged, was a step by step process, like any other evolutionary process. With "forward" steps and "backward" steps, but each following the *principle of eurhythmy*. We used quotes in the two adjectives qualifying the word *steps* to emphasize the non-deterministic process followed. Every individual step has a direct purpose. The purpose was not to create the Solar system as a whole, but rather to follow the propensity to survive of every individual quantum object that constitutes the Solar system. After the emergence of the present situation, the Earth has followed the "*better*" way, i.e. the way corresponding to the regions where the interference of its huge θ *wave* with the Sun's huge θ *wave* produces a maximum of intensity. The Earth does not have the "free will" to follow a path that differs dramatically from the current one. However, it does have the "freedom" to fluctuate inside the region where the interference of several θ waves gives place to a maximum of intensity. The Earth's translation, rotation, precession and internal movements possess fluctuations. Of course, the Earth is not free to "jump" from one orbit to another completely different one without the occurrence of a catastrophic event. Using a metaphor, even if we accept the existence of our free will, it does not enable us to fly. Our free will is limited by our internal structure and by our interaction with the exterior. If the wind were to blow strongly enough, it could allow us to "fly". However, in this case, we are talking about a catastrophic event for our scale.

Croca is able to show that the *principle of eurhythmy* encompasses the principle of minimum time and the principle of minimum action as particular cases in physics. The first is related to the wave character of matter, the second to the corpuscular character of matter. Within this new framework, this dualism must be abandoned. Croca's proposal shows that those principles are only different and partial perspectives of one general principle, i.e. the *principle of eurhythmy*. As we said, this constitutes the first step towards a new and bigger unification in physics, which can be extended, in principle, to every other branch of science, i.e. chemistry, biology, and even human and social sciences, because this principle gives us the hope that it

will be possible, at least in the simpler cases, to describe the change mathematically in the most elementary complex structures we can talk about, i.e. quantum beings. The extension of this principle to other scientific domains represents a reappraisal of our place in the world. We would be only an example of a complex structure, a natural structure that emerged because it found conditions that made the emergence of our species possible. However, every individual of our species is related to the history of the species he belongs to, i.e. that species and all the individuals that constitute it, carry with them all the history of the evolutionary process that led to its emergence and, unavoidably, after its emergence. Perhaps we can foresee a new version of Plato's reminiscence doctrine. The "Truth" that would exist in our soul, according to Plato, can be understood in the sense that "Truth" lies within us, because we are the culmination of a long and complex process that followed the non-deterministic way that processes should have followed in order to give birth to our, for the time being, viable species. Kierkegaard's famous aporia "*Subjectivity is Truth*" in his *Postscript*[62] is a different version of Plato's stance. Evidently, we must see Plato's "shadows" in the "cave" as the "detonator" of our "explosive" cognitive processes. We are always interacting unavoidably with our surroundings that, under this perspective, belong to our extended complex structure. We continue permanently to evolve through a similar process of evolution that led to our emergence. We are, unavoidably, in permanent interaction with our always ill-defined surroundings. So, subjectivity alone is not able to reach the "truth", unless we consider our subjectivity extended, precisely because it is in a permanent becoming, a becoming that needs the interaction with its exterior through an always ill-defined frontier. The frontier of natural complex structures is always ill-defined; this is one essential feature of the evolutionary process.

If the extension of this principle is effective, we can foresee the path every structure follows in order to survive. We can foresee the emergence of a new structure as the better way that a structure of a lower level finds in order to survive within new environmental conditions and according to its previous history, i.e. according to the information stored inside it or, similarly, according to its complex structure. In our case, we are not restricting ourselves to the information of which we are aware; we are extending this information to the deepest level of our beings. In every natural complex structure, if the interaction with its exterior, always ill-defined, modifies the form of the surrounding θ wave, it will lead that modified complex structure, i.e. the acron ξ and its θ wave, to pursue new links with

[62] Johannes Climacus (Søren Kierkegaard's pseudonym), in *"Concluding Unscientific Postscript to Philosophical Fragments,"* Ed. Robert L. Perkins, 1997, (1846).

other structures because they, in a first approach[63], would "seem" to turn its persistence easier. Of course, this new structure would not be something like a "life insurance". If the surrounding conditions, or the inevitable modifications within the new structure, no matter the natural level we are considering, would change, in a shorter or longer term, that persistence will no longer be possible. This will be the fate of every natural structure. In nature, there is nothing permanent; everything is in a permanent becoming with a permanent need to make "choices". Kierkegaard is probably the utmost representative of this situation at a human level with his works "*Either/Or*"[64] and "*The Concept of Anxiety*",[65] where he uses the term *dizziness of freedom*. Kierkegaard is a forerunner of a current of thought historically known as existentialism. If Kierkegaard, while analyzing human behaviour, implicitly attributed to our species a privileged status, the *principle of eurhythmy* and the underlying vision of the world removed all of its remaining privileged status. The human species is just an evolutionary accident, even if we, while members of it, can think about everything we have been discussing in this text. We are not above the world. We are just an occasional creation of it.

Our vision of the world, very close to Heraclitus', separates us from the scientific revolution of the 17th century. As stated before, this revolution carried inside it a radical reductionism. Galileo confined physics to the mere description of local movement. This reductionism emerged from the necessity to achieve a mathematical description of nature at that period. Galileo was not able to describe mathematically everything that Aristotelian physics wanted to describe. Aristotle intended to describe the substantial changes and the accidental changes, with the latter divided into qualitative, quantitative and local changes. Galileo reduced the object of physics to the very last one: the local change, i.e. the change of position or the movement, as we call it now. Descartes integrated Galileo's epistemology in a more consistent philosophical system. Like Galileo, Descartes considered mathematics a source of more reliable knowledge. This conviction, in addition to the results achieved by Galileo, led to a widespread vision of the world as a huge mechanism.[66] It should be a mechanism because, according

[63] Ahead we will introduce the designation of "weak teleology."

[64] Søren Kierkegaard, "*Either/Or: A Fragment of Life",* translated by Alastair Hannay, Abridged Version, Penguin, 1992.

[65] Søren Kierkegaard, *The Concept of Anxiety*, Princeton University Press, 1981.

[66] A mechanism which, in Descartes' time, would necessarily have been simple. Today we build much more complex instruments. For example, a computer behaves in a much more complex way. This complexity led to the emergence of new areas of research, e.g. artificial intelligence, cognitive sciences and the sciences of complexity.

to Descartes' method, we must divide the problem into as many parts as necessary until we reach the ability to solve it. Of course this means that, at this point, we can apply a mathematical description, but a mathematical description that should be linear because the possibility to solve it imposes the possibility to divide it, and this would only be possible if the whole would be equal to the sum of its parts. This is the deep meaning of a linear description of a system or of a process. This is the deep meaning of Descartes' method. However, Descartes, as a philosopher, was aware of the differences between an organism and a simple mechanism. This is the origin of the thought that the pineal gland is the link through which spirit and matter, or mind (soul) and body interact, or the thinking substance and the extended substance.[67] The simplistic mistake of considering the world as a mechanism is deeply connected to the mathematical techniques used. The result attained in the 17th and 18th centuries, with Newton's physics, was the vision of the world as a huge mechanism ruled by an empiricist view of Newton's laws, i.e. Newton's laws interpreted through John Locke's empiricism. The Laplacian determinism[68] was its unavoidable consequence. There would be no place for free will even at the human and social level. Until the beginning of the 20th century, physics followed this path. However, physics through the 19th and 20th centuries began to show the naivety of this belief. Physics became far more complex than the Newtonian paradigm had imagined. The electromagnetic theory that emerged in 1873, when Maxwell published his *Treatise of Electricity and Magnetism*, constituted the first blow against this vision of the world. Relativity and quantum mechanics constituted the final blow against it. Quantum mechanics, however, raised the puzzle of a deeper dualism: the wave-corpuscle dualism. This was a signal that has never been satisfactorily understood. Bohr's interpretation of quantum formalism led to the schism in physics, as Popper called it. Bohr's interpretation of quantum formalism constituted a transcendental escape. Waves and corpuscles are only categories that our mind must use to describe the phenomena; they do not really exist. This attitude contains a risk: if we do not believe in the existence of something, we give up searching for it. The new proposal we are talking about, within Louis de Broglie's scientific research program, developed in Lisbon by Croca's group, demonstrates a capacity to foresee a deeper understanding of nature. With the new unifying theory proposed by this scientific research program, we can describe quantitative phenomena that are far more complex than the mere change of

[67] In fact, Descartes introduced three substances: The infinite substance (God), the thinking substance and the extended substance. In this text, we refer the last two.
[68] P. S. de Laplace, "*Théorie analytique des probabilités,*" 1813.

position in the restricted field of physics. This is the strongest argument on its behalf.

The emerging physics we are proposing is a process that had been announced, as we said before, by Whitehead[69] when he left the concept of mechanism and embraced the concept of organism. This concept, or natural complex structures, naturally emerged from the characteristics of the problems in the field of biology, even before it was considered as a separate branch of science. For us, an organism, whatever the level we are talking about, from quantum to biological or sociological, is a complex structure that emerged either from the sub-quantum level or from the quantum level on. Any natural complex structure is composed of several natural complex structures of a lower level, interlinked in a way that "seemed" to them the better way to persist as structures, in a specific environment. No natural complex structure will ever be equal to the sum of its parts. Not only its structure but its interaction with its ill-defined exterior, from which it can never be separated because there is an intimate interdependency between both, they are quantitatively and qualitatively different from what exists in natural complex structures belonging to lower levels. This new emerging structure will never be static. It will always be in a constant process of adaptation to every change in the environment. Its behaviour will always depend on its own structure and on its complex interaction with the environment, which are always inextricably linked. These complex interactions presuppose, precisely because the complex structure survived, that the new emerging natural complex structure has the capacity to get information and to process it. All structures follow this principle but, because it results from a non-linear interaction between those complex structures with their ill-defined surroundings, some would survive for a longer period of time than others. This interaction will stimulate modifications in the structures that survive a longer period of time, i.e. that structures will be "learning" throughout that longer period of time, in a very wide sense. That structure persists precisely because it is able to "learn", and to "learn" always implies an adaptation. However, if the environment suffers a huge change, the persistence of that complex structure may become impossible. In this case, it will be destroyed.

A Neo-Darwinist may disagree with this statement. This stance should not be confused with a simple Lamarkism. We think that Neo-Lamarkists are

[69] A. N. Whitehead, *"Process and Reality" (corrected edition)*, The Free Press, NY, 1978. Whitehead was a British mathematician and philosopher who was associated to a school of British Emergentists like J. Stuart Mill, C. Dunbar Broad and Samuel Alexander. George Henry Lewes gave it a philosophical sense in his work *Problems of Life and Mind (1875)*. The most popular nowadays is Samuel Alexander.

correct, just at the Neo-Darwinists are too. Both concepts should be applied to all evolutionary processes. The adaptation we are talking about shall exist as suggested by Neo-Lamarkists, even if it should be researched not only at a biological level. Perhaps we must understand what we usually call life only if we understand emergence, i.e. the interactions at a deeper level that led to the emergence of life. However, there is no teleology-based evolution. We are touching a crucial point here. There is only a "weak teleology" based evolution, which is the way to reconcile Neo-Darwinism, Neo-Lamarkism and the vision of the world underlying this new emerging physics. Neo-Darwinists interpret mutations as completely random events.[70] We agree that mutations do not follow a teleologically predetermined way. But we think that they may follow a "weak teleological" predetermined way. What we think of today as random steps can be, in the future, considered epistemological random steps and not ontological random steps.[71] Evolution began before the emergence of life. If the new ideas in physics are more correct than the ideas connected to the old physics, biologists should listen carefully to what the new emergent physics is saying. They should not exclude the possibility that mutations have been caused by interactions on a deeper level. Evidently, the interactions that we are considering are not deterministically correlated to a strict biological "logic"[72]. This way, the resultant mutation may be contrary to the probability of the persistence of the mutant. The underlying "logic" is not concerned with it. This is the reason why biologists say that there is no project in the evolutionary process. We agree completely with them. The *principle of eurhythmy* can be the key to solving this riddle, admitting that there is a sort of reaction behind mutations that may not be an exclusively biological reaction. That is,

[70] Stuart Kauffman, in his books: "*Origins of Order: Self-Organization and Selection in Evolution,*" Oxford University Press, 1993 and "*At Home in the Universe. The Search for Laws of Complexity*," Oxford University Press, 1995, defends that a principle of self-organization should mingle with chaos and necessity. His view of the world is therefore similar to our own. However, he confines his analysis to the field of biology and chemistry, with some incursions into other scientific fields like economics, but he is not aware of the new emerging ideas in physics. He only mentions the mainstream ideas of the physics that is in a deep crisis.

[71] Completely random events can only be considered if we admit the existence of an ontologically chaotic sub-quantum level that can always exert, at least potentially, its influence, whatever the level we are considering.

[72] We know that biologists know that, for instance, radiation can produce mutations. However, radiation is only a sort of interaction. For instance, mutations may occur, as is already known in biology, by molecular decay, but we must not exclude that even deeper interactions may occur.

perhaps it is not the living organism as a whole that responds to external stimulus, but some complex structures belonging to the living organism, which is a complex structure of a higher level, and that reacts (in a "weak teleological" form) to the external stimulus without being "concerned" with the persistence of the living organism as a whole. As we can see, there are no conflicts between the new physics and the process of natural selection. However, we defend that there is always a "weak teleological" adaptation.

The *principle of eurhythmy* gives a basic meaning to the term "learning", which is why we can foresee the possibility of achieving a quantitative description of it. This is, another feature of this new proposal. We can, in principle, give a quantitative account of these phenomena that can be no more reduced to the mere change of position. The change of position would be only one feature of a much more complex process. The change of position would be no more than a particular change linked, like any other change, to the search for a better way that a natural complex structure at quantum level follows according to its propensity to persist. Every change, including structural changes, is always linked to this propensity. Perhaps the most fundamental natural law in the world is the propensity to survive. But the ending of every natural complex structure would never be a "happy ending". No complex structure will persist forever.

This new vision of the world, as we said before, adopts a new and more radical ontological unification. That is the reason why we chose Giordano Bruno's name for the title of this section. We know that, after Copernicus proposed his heliocentric system, Giordano Bruno, taking this proposal seriously, and with no empirical evidence,[73] claimed that the stars we see in the sky would be Suns like our Sun, which must have planets around them and, perhaps, some of these planets would support life as it exists on Earth. That is, Giordano Bruno, as a philosopher, carried a radical ontological unification to its limit. It was a deeper unification than Galileo was prepared to adopt. Galileo accepted Copernicus' system because it blew the ontological distinction between the sub-lunar and supra-lunar worlds. He needed this unification to support the new epistemology, i.e. the mathematical description of the movement of bodies at the surface of the Earth. He wanted to follow the method the ancients had adopted to describe the movements of the celestial bodies. Galileo was mainly a physicist. His goal was to build a new physics coherent with the new vision of the world. Bruno, on the other hand, was a philosopher. So he searched for a deeper ontological unification. He went even further, reaching, as we know, a

[73] This is not our case. In physics, as we said, there are experiments proposed. They should be done.

pantheistic stance.[74] For him, matter and spirit were only two different perspectives of an inseparable monad. Accordingly, he fought the dualism between matter and spirit. After Bruno, Spinoza identified the *res extensa* and the *res cogitans* as well. Leibniz[75] recognized the difference between a mechanism and an organism, putting the individual, as an indivisible and invariable monad, at the centre of his system. He presented his philosophical system in opposition to Descartes' mechanistic system. However, as we said, Descartes maintained that dualism and, with some improvements, it lasted for several centuries as the most widely adopted stance. Even the materialistic philosophers of the 19th century retained a distinction between matter and ideas. For them, ideas are a superstructure built by matter and, as such, ideas would be a product of matter. They do not possess the same level of complexity. Ideas for Marxism are much more of an ideology, i.e. commonly accepted ideas to justify certain relationships of production according to a certain development of the productive forces. We would agree if we used ideologies exclusively and not ideas. Ideologies are an ensemble of ideas accepted by a group of persons and used to justify a given social organization. For us, like for Bruno and Spinoza,[76] ideas and matter belong to complex structures we call living creatures. Individual human complex structures possess ideas as part of their own structure. Ideas belong to their complex structure and suffer a permanent evolution, because the whole complex structure suffers a permanent evolution. Ideas are a fundamental characteristic of each individual human creature. There are innate "ideas" and acquired ideas. From our point of view, innate "ideas" are not linked, like Chomsky defended, to an innate knowledge of certain grammatical structures. He is talking about the exclusive framework of complex structured human beings. We are using the concept of innate "ideas" in a much broader sense. This is the reason why we used quotes. We are hereby considering the genetic process that led to what we now call ideas without quotes.

[74] We can be considered pantheistic, if being pantheistic means to say that the world "acts by itself," in Spinoza's sense. The pre-established harmony of Leibniz is another possible sense to which we can assign the "acting by it." However the metaphysical question remains: Why does the world exist? What is the primordial cause? This is probably the greatest question we may have. But this is a metaphysical question to which nobody can honestly answer.

[75] Leibniz was the first modern philosopher to use the concept of organism.

[76] William James defended the existence of a single substance. William James introduced the concept of emergence in the 19th century. He refutes also the existence of two different substances: the *res extensa* and the *res cogitans*.

We should not identify the concept of "ideas" with Richard Dawkins'[77] memes. These "ideas" represent a change to the *inner structure* and, eventually, to the *extended structure*[78] of the complex structure. The brain belongs to the *inner structure* and a change in the *inner structure* may cause a change in the *extended structure*, i.e. books, computers (our *extended* "*θ*" *waves*[79]) and, eventually, society, a higher-level complex structure that belongs to our *extended structure* as well. Introducing the concept of Richard Dawkins' memes creates a new dualism between genes and memes. This will be some sort of hardware (genes) and software (memes). In an organism, we cannot separate two distinct parts; they are interlaced in an inseparable monad. The *principle of eurhythmy* implies rejecting any sort of dualism, even if it seems to make sense in biology.

For us these are innate "ideas" because no complex structure can emerge without them. The concept of innate "ideas" is linked to the possibility of the very existence of any complex structure, whatever the level we may consider. Any complex structure exists because it is able to interact with the exterior, to treat the information they get from that interaction, and to act accordingly. Evidently, the interaction that we are talking about is not as complex as the ones we, human complex structures, can have with our surroundings. However, it is enough to admit that a complex structure can interact with its exterior, whatever the level of complexity we are talking about, to conclude that this complex structure is in a permanent becoming, because this interaction will always be transforming that complex structure. This is the process of "learning" associated with the *principle of eurhythmy*. This transforming interaction is deeply related to the concept of acquired "ideas". When a complex higher-level structure emerges, it carries within it, in its complex structure, innate "ideas" and the acquired "ideas" of the complex lower-level structure. For it, the ensemble of innate "ideas" and acquired "ideas" of the complex structures from which they emerged constitutes their innate "ideas". It is able to persist exactly because it possesses "ideas" that are now innate to it. Innate "ideas", in the sense considered here, exist from the very beginning of the pre-biological evolutionary process. In the same broad sense, as stated, acquired "ideas" are a product of the interaction with the external world after the emergence of any complex structure. "Ideas", in this very broad sense, is equivalent to a change in the characteristics of a complex structure.

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[77] Richard Dawkins, "*The Blind Watchmaker,*" New York: W. W. Norton & Company, Inc, 1986.

[78] See footnote 52.

[79] We use quotes because we refer to a much higher level of *θ waves* (achieved through the evolution) of interaction with the outside world that a human being as a complex structure is capable of doing.

Matter and ideas are, as stated, two different perspectives of Bruno's inseparable monad**.** This inseparability characterizes this kind of complex structures. However, as far as is known, only complex structures with a certain degree of complexity are able to produce what we may call ideas, (without quotes, because we are using here the concept with the usual meaning) and it was this feature that led us to commit the mistake of considering ourselves as living beings in which some part exists outside matter, or in permanent contact with the transcendent. This led those who are convinced that we are the only beings that produce ideas to consider that we are above matter, i.e. to consider ourselves as creatures created in the image of the God that created us. Our recent cultural history has led us very far from this simple vision of our species. We are just a product of a very long process of evolution, where the fight to persist and, inevitably, biological chance, have played the major role. This evolution gave rise to creatures like us that are able to produce ideas, which can be expressed through techniques, common language, literature, poems, sculpture, painting, theatre, music, photography, film, philosophical systems, theoretical biology, physical theories and science in general. We have wittingly mentioned in first place one way to express the capacity to produce ideas that is not exclusive to our species, but is shared with other species closer to ours, even in a broader sense. For instance, crows in New Caledonia are able to build hooks from twigs to capture worms hidden in holes in trunks. Certain monkeys are able to coordinate the use of three different tools to perform one action, like cracking the shell of certain fruits. So, the ideas we use are a product of a very complex evolutionary process. They are means that natural complex structures, similar to us, use to increase their ability to survive. They are also a product of a non-deterministic continuous fight for survival. This being said, we are not dismissing the important role played by chance. Chance is a consequence of the non-existence of teleology in this evolutionary process. There is only, if we are allowed to speak so, a "weak teleology" that is compatible with the ability to "foresee" the near future and, consequently, the best way for a complex structure "foresees" its own survival. This capacity to "foresee" the near-future must be compatible with the level that a certain complex structure belongs to. Only an omniscient complex structure would be able to avoid the role played by chance. But this kind of being simply does not exist. Ideas are no more than an inner tool that complex structures like us use. They are a tool that emerged from a long non-deterministic evolution guided by adaptation attempts and natural selection. Ideas are a more sophisticated version of "ideas" through a long non-deterministic evolution guided by adaptation attempts and natural selection. They may increase or not our ability to survive. However, ideas

are not a shield that prevents us from extinction. We cannot be sure that our species is a viable one. Our species is just an evolutionary accident.

If we maintain a clear ontological distinction between matter and spirit or ideas[80], we are opening the door to every sort of irrationalism. To defend that ideas are transcendent and not inherent to the world is to half-open the door to the permanent action of God in the world. There are only complex structures, possessing "ideas" or ideas as part of their complex structure. These complex structures as a whole are unavoidably connected to the way they react to the interaction with their exterior. How many deaths have been caused by some ideas? We must remember the consequences of the Nazi ideology that was particularly dangerous from the beginning. However, any ideology, even if it intends to adopt an altruistic standpoint, is potentially dangerous if it is considered in a dogmatic form. They become myths, and myths or religions are responsible for an astounding number of atrocities. In the past and even today, we are watching the result that such radical ideas produce. Ideas can produce, in fact, huge consequences. An idea is, as we just said, a kind of inner tool that complex structures such as ourselves possess. And, as any other tool, it can be used successfully or unsuccessfully. Successfully if, *a posteriori*, they increase the ability of the complex structures to persist jointly with these ideas, because they belong to that complex structure. Unsuccessfully if, *a posteriori*, they decrease the capacity of the structures to persist jointly with these ideas, because they belong to that complex structure. We say *a posteriori* because we must remain consistent with what was said previously about the "weak teleology". This "weak teleology" is an attempt to clarify what we call knowledge or, at our level of complexity, philosophical and scientific knowledge. We can never be sure that our ideas and our actions will be successful. Only an omniscient complex system would be sure *a priori* of the far future. However, such a complex system would not be natural. It does not exist.

Science represents our greatest attempt to increase our "weak teleology". We are natural complex systems and all these systems always have a partial "knowledge"[81] for one main reason: their interaction with the exterior is always incomplete. It depends on their capabilities to interact and these are always constrained by their own complex extended structure. A natural complex structure is a consequence of the particular evolutionary path that led to it. It is very unlikely that any future evolutionary path will lead to complex structures having the ability to get a perception of the world as a whole. This would only be possible if these complex structures were to

[80] Whenever we use this word without quotes, we are attributing to it its current meaning.

[81] We use quotes here because we are using this term in its broader sense.

encompass the entire universe in order to apprehend it in a complete way. However, this would only be a necessary condition and never a sufficient condition to achieve it. It is not enough to apprehend, we need to understand.

We are natural complex structures that are able to produce ideas. Ideas belong to our complex structures and constitute one feature that characterizes them. Complex structures, at this level, produce ideas as part of their complex structure. In their interaction, ideas constitute the primordial information they can share. Ideas, we repeat, belong to their structure. This is a consequence of the abandonment of the deepest dualism, i.e. the wave-corpuscle dualism, arising from the scientific research program proposed by Louis de Broglie and from Croca's *principle of eurhythmy*.

We must note that: 1) if, at the deepest level we are able to speak of, i.e. the quantum level, we admit that natural complex structures are monads that interact among themselves through their θ waves that belong to their complex structure; 2) if we accept the existence of biological evolution as a fact; 3) if we do not admit that life could have emerged as a transcendental action, then we must conclude that θ waves are the inner tool that complex structures at the quantum level use to "communicate" with their ill-defined surroundings, which, associated to the tendency to persist described in the *principle of eurhythmy*, led them to create new emergent complex structures of a higher level that are able to use more complex means to communicate with their ill-defined surroundings, and so on. Once again, we must be very careful. Evidently, we are not defending a pre-determined evolutionary process. The "weak teleology" is always present, preventing us from foreseeing a distant future. The renunciation of the wave-corpuscle dualism leads us to the renunciation of the dualism between body and mind and to the renunciation of the dualism between matter and spirit. There are just evolving complex structures guided by a "weak teleology". For us, like Bruno said, matter and spirit are no more than two perspectives of an inseparable monad.

Myths, art, philosophy and science are the major demonstrations of the education skills that a complex system can achieve. According to the *principle of eurhythmy*, every complex system can "learn", and this "learning" process is strongly connected to the possibility of an evolutionary process. This "learning" process is mainly connected to the propensity to persist that every complex structure follows, whatever level it belongs to. To "learn" is to be able to "choose" a path in a permanent evolutionary process. A "choice" is erroneously linked to the concept of "*free will*". The concept of free will emerged from the behaviour of human complex structures. However, whatever the level under consideration, different complex structures of the same level may "choose" different "paths". They "choose" different "paths" because they are really structurally different. Similar

interactions with the surroundings may produce different "choices", because two complex structures, even when belonging to the same species, are always structurally different. They are different either in an innate way, or in an acquired way. They are different because any learning process always has two components. This can be the result of a process developed throughout evolution before the emergence of a complex structure (innate culture) or it can be the result of the interaction of the complex structure after its emergence (acquired culture). The probability that two complex structures of the same level are completely equal is zero, i.e. it is an impossible event. Even two mono-zygotic twins cannot be completely equal at the moment of conception.[82] They become even more different during the gestational period, and inevitably even more so throughout their lives. They may choose different behaviour even when subjected to the same stimulus.

We must stress that this "choice" exists from the lowest level of existence we can conceive for the time being. To "learn" is related to a change in the structure of a complex structure. This is linked to a "learning" process that can extend the structure of the natural complex structure to its exterior.[83] The recently introduced concept of *extended mind*[84] represents the awareness of this feature at the human level. We consider that our structure possesses "inner" tools and "outer" tools, and they all belong to our *extended structure*[85]. The outer tools must include the society to which we belong, which is a tool as well, along with every tool that society produces. Every emergent complex structure must be considered a tool for the complex structures from which it emerged. Evidently, we are using the word tool in a very broad sense. For instance, some human beings, we call them scientists, belonging to the structure of a human society, constitute a *machine-tool*, metaphorically speaking, because they are able to produce other tools like, for example, scientific theories. A scientific theory that obeys the six criteria we described at the beginning of this text is a *machine-tool* as well, because it enables us to build other tools, and this is the ultimate criterion we can conceive of to validate this theory. This is the reason why we stressed the importance of the last two criteria, in particular the last one. A scientific

[82] A biologist may be adverse to this statement, but a physicist such as me must adopt this stance. We must look for the differences, because they inevitably exist.

[83] The concept of exterior must be carefully faced. It is difficult to define, within this new vision of the world, a sharp border to any complex structure. For instance, as mentioned earlier, the complex structure of the Sun will be much wider than what our senses enable us to apprehend, and even what current theories admit.

[84] Clark, A. & Chalmers, D. J., *"The Extended Mind*," Analysis 58 (1), 1998, pp. 7-19.

[85] This is a concept we need to introduce because any complex structure always has an ill-defined border.

theory increases our action in the world and it can increase our ability to survive as well, constrained always by our "weak teleology". This is the reason why we can only evaluate the success or the failure of a scientific theory *a posteriori*, i.e. after satisfying the three empirical criteria that we mentioned. As we just said, this stance is deeply correlated with the two last empirical criteria to validate a scientific theory as we defended in the beginning of this work. Evidently, our unavoidable "weak teleology" cannot assure that the use of these theories, and the use of the new technologies, only conceivable after the emergence of these new theories, implies an increase in our probability of survival. The purpose in the short-term is to do it, but the long-term result may be the opposite. Not because of the theories, but because of their misuse. We must evaluate permanently the results of their use. If we would persist, our evolutionary process should always follow this path. We are not sure if our species is able to do it, at least in an efficient and quick way. We can never be sure *a priori* that any emergent complex structure will be successful. The emergence of a new tool represents a structural change in the complex structure where the new tool emerged. This way, the emergence of a tool represents the emergence of a different complex structure that includes the new tool.

At this point, we must consider two groups of human-scale tools. Let us consider some examples of tools that belong to these two different groups: a) hammers, nails and screws, and b) GPS, radio telescopes and tunnelling effect microscopes. The items in the first series don't have an underlying scientific theory. They are but a sophisticated version of twig hooks made by crows of New Caledonia, or the three tools used simultaneously by certain monkeys. However, they are a demonstration of our superior technical skill. Evidently, the monkeys do not possess metallurgical techniques. The items in the last series only became conceivable after the emergence of scientific theories like Newtonian mechanics, electromagnetic theory and quantum mechanics. This constitutes the major characteristic of our species - scientific theories, along with art, philosophy and even myths, are exclusive characteristics of our species. This is the main reason why we became the dominant species on Earth. And this is the main reason why we, erroneously, thought that we were creatures nearest to the "Gods" that supposedly created us. Our species, along with its myths, art, philosophy and science, are exclusive products of an evolutionary process where complex structures fought permanently for persistence. So, scientific theories are a fight for survival, a fight against our "*weak teleology*". They are an attempt to improve our, always unavoidable, "weak teleology", and they emerged because the evolutionary process led to the emergence of brains like ours that enabled us to consider a world far more complex than our senses may apprehend. They led us to search for the

causes behind the facts. Facts became phenomena.[86] The first attempts to search for causes led to the emergence of myths. The concept of the existence of gods is a development that came from our permanent fight to survive. Everything that seemed to increase our probability of surviving became associated with beneficent gods. Everything that seemed to decrease our probability of surviving became associated with malevolent gods. When we hear the statement: "it looked like the work of the devil", we must translate it into "something happened that decreased something's probability of surviving". For instance, the speaker persistence regarded in its broadest sense, i.e. its own survival as a complex structure, the survival of another similar complex structure deeply linked to the speaker, belonging to human society (another complex structure) to which both belong; the survival of the complex structure (human society) to which he belongs; the survival of a particular characteristic of the complex structure (human society) to which he belongs, like, for instance, a myth or, in its broadest sense, its culture and, consequently, the survival of the complex structure (human society) to which he belongs, and, consequently, his own survival.

Philosophy emerged to question these naive beliefs. However, much of the philosophical thought remained largely influenced by these beliefs. For instance, even in the consolidation of the scientific revolution of the 17th century, Descartes introduced, in his *First Meditation*,[87] the evil demon hypothesis to signify his ignorance of his own origin and, in particular, the uncertainty associated with his knowledge, i.e. his ignorance. Descartes is useful in helping us see the deep correlation between the "beneficent God" with knowledge and the "malevolent God" with ignorance, that is, with everything that does not improve our "weak teleology", everything that wouldn't make it stronger or, in other words, less "weak". Descartes helped us to correlate the "beneficent God" with our increasing likelihood to persist, and the "malevolent God" with the failure to increase our probability of surviving.

The emergence of art is deeply linked with the emergence of myths. In the Renaissance, Masacchio was criticised by his friend Brunelleschi for painting a peasant on the cross. In fact, he painted a real peasant, but he needed the cross to do so. That is, even when art tried to represent a human being, it had to pay a "price" in order to be acceptable within the complex structure (human society) in which it began to emerge.

A society, such as a human society, is a complex structure of higher level and, simultaneously, constitutes our *extended structure* and,

[86] In the sense of Kant.

[87] R. Descartes, "*Meditations on First Philosophy,"* translated by John Cottingham, Cambridge: Cambridge University Press, 1996.

consequently, a new tool. Every human being, every human society, every living being, every molecule, every element, every proton, every neutron, every electron, every muon, every photon, every quark (if they are more than just mathematical concepts), the Sun, the Earth, the Solar system, they all are natural complex structures. Every complex structure, i.e. every natural structure, will only survive if in "harmony" with the lower-level structures that are interlinked within it, and with structures of lower, equal or higher level that exist outside it, to which it is inevitably interlinked as well. This "harmony" will never be completely achievable. We put the word harmony in quotes because every natural complex structure, whatever the level it belongs to, acts according to a "weak teleology". This kind of "harmony" can only be verifiable *a posteriori* due precisely to its "weak teleology". The natural selection exists from the deepest level we can conceive. It is important to stress once more, at this point, that the objective is not to defend any kind of panpsychism. The "weak teleology" of the Solar system is several orders of magnitude weaker than our own. Evidently, we are constricted, for the time being, to our Solar system and, concomitantly, we depend on the behaviour of the Solar system according to its "weak teleology". The Earth experiences the action of the Sun and acts according to the *principle of eurhythmy*, following the way where the interference of the θ waves of the Sun with its own θ waves creates a maximum intensity. Evidently, the θ waves belonging to the Moon will interfere as well, and we cannot neglect their action, such as the phenomenon of tides. The study of the interaction of the Sun, the Earth and the Moon is the classical problem of the three bodies in Newtonian mechanics. The θ waves belonging to each of the other planets of the Solar system, along with their moons, will disrupt this main interaction, but these disturbances will all be very weak. However, we must not forget that every one of these interactions is non-linear and, it is, therefore, impossible to foresee the long-term future of the Solar system. In the long-term, every natural complex system will cease to exist. This applies to complex structures that survive only for a few femtoseconds, as well as to complex structures that may survive, always in a permanent becoming, for billions of years. Our "weak teleology", however, enables us to foresee things such as those mentioned above. Our "weak teleology" is less weak than that of the Solar system. But we are constricted to the Solar system's "weak teleology". The Solar system, like any other complex structure, behaves according to the *principle of eurhythmy*. Its behaviour is not concerned with our need for survival. We are aware that an unknown comet, belonging to the Solar system, can, in the future, destroy every life form on Earth. It almost has happened in the past, and may happen in the future. However, our "weak teleology" leads us now to look for this type of comet in order to destroy it before it hits the Earth. This is only possible

because we have created physical theories that enable us to foresee possible future events like this, and to possess the means that may make possible, at least theoretically, such a protection. This does not mean that we can rule nature. Nature is far more complex than what we may think about it, i.e. our theories or, in other words, our "weak teleology". The belief that we can rule nature is a modern myth. This myth led to our present social organization. We are now feeling the nefarious consequences of this myth, as well as its ecological damage and social imbalances. Our "weak teleology" did not allow us to foresee all the consequences of our past choices. Once again, this stance moves us away from any sort of panpsychism. We do not believe in a holistic vision of the Universe, where the Universe possesses a "soul". Evidently, this being said, we do not even believe in the existence of our own soul. The tendency to survive is something in common with every complex structure. We cannot communicate with the Earth in the way panpsychism admits. Evidently, we interact with the Earth in a very complex way. We can even destroy the natural resources that exist on the Earth, like we have been doing. Otherwise, the Earth may produce a worldwide extinction through a huge volcanic event like it has happened in the past. For the Earth we are meaningless.

We must learn from our history. Our history shows us which of our past actions have led to disastrous results. We should use our distant and recent historical memory as a primordial source of learning. In this case, the word learning is used without quotes because it represents the traditional concept we are used to. Learning is a crucial process. The role of the school is to get the youth acquainted with the best achievements by the best individuals of our species who lived before us. Every human society that disregards this necessity runs a greater risk of failing. Evidently, even being, as members of a human society, careful learners, we would not be safe from any unforeseen event. This is an unavoidable consequence of our "weak teleology". Every one of us is a nonlinear natural complex structure, interacting nonlinearly with other nonlinear natural complex structures with which we are interconnected. Our unavoidable "weak teleology" is a consequence of such a complex process.

The concept of human society is used here in its most general sense. A human society can be a simple family, a village, a town, a country, a nation, a "union" of nations like the European Union or the United Nations, a civilization, a political party, a religion, a football team, a company, the Mafia, and so on. For instance, the European Union arose from a lesson taken from our recent past: World War II. The creators of the European Union intended to expand the traditional economic, cultural and political tensions between European powers in order to avoid a possible future war. They also intended to fight one consequence of World War II: the decline of

Europe's influence in the world. Of course, that influence can be exerted no more under colonial or neo-colonial domination. Our influence must be exerted between pairs and not from a "pulpit". Even so, we are right now feeling the difficulties of such a process. We are being confronted with the possibility of the collapse of the EU. Nobody knows what will be the result of the present crisis. There are tensions within EU. There are tensions with nations that do not belong to the EU. There are economic tensions between the EU and other surrounding natural social complex structures like the USA, and emergent world powers like China and India among others. This is the kind of problems that political science is always confronted with. The novelty comes from physics, which is starting to be confronted with problems of similar complexity. The *principle of eurhythmy* looks to natural physical objects that emerged through an evolutionary process, whatever the level it belongs to, where adaptation and natural selection play a dominant role, as natural complex structures interacting permanently in a nonlinear way with their ill-defined surroundings, i.e. physics is starting to deal with problems that, until now, we thought were confined to the study of complex natural structures of a higher level.

As we said, the Mafia is another kind of (human) social natural complex structure. It emerged because its members thought that it was the structure they ought to create in order to increase their chance of survival, i.e. the survival of the reduced group of persons who control power within such an organization. Every new emergent social natural complex structure depends on the inner structure of the members of this new organism: the ensemble of their ideas, the information they got throughout their lives, or, in one word, their culture or the lack of it. A gangster organization acts essentially as any other natural complex structure. Its actions target its own survival. However, in order to increase its "weak teleology", at this level of complexity, they usually try to infiltrate the police institutions, political institutions and legal economy, in this case to "launder" money. A generalised *principle of eurhythmy* carries them towards this kind of action. But they are constrained by their "weak teleology", deeply connected to the complex structure they created. The ideas of their members, i.e. their culture, constitute a primordial aspect of that complex structure. They must fight against other similar social complex structures, and against different social complex structures. Let us use a metaphor: its "acron", i.e. its core, must spend a lot of effort to maintain its "theta wave", that is, its protective belt. Their ill-defined surroundings (the infiltration of other institutions turns it diffuse) are mostly hostile. As we said, every complex structure will always have an ill-defined frontier. This is a characteristic of every complex structure. However, this particular complex structure needs to develop an excessive effort to survive. This is a crucial point.

We do not know if our species is viable. There is always a tension in natural complex systems. There is no deterministic process towards a harmonious future. There is no tendency to a path toward "perfection", i.e. toward God. This was, for example, the naive stance of the Jesuit priest Teilhard de Chardin[88] within the Christian framework. Our species, even if able to create things like myths, art, philosophy and science, is a mere consequence of a long natural evolutionary process that can be stopped, either by an external action like a catastrophic geological disaster or a catastrophic astronomical disaster, or by our own unsuccessful actions. As we just said, a gangster structure will be confronted with other similar complex structures and with society as a whole. Everybody knows that there are human societies, even States, dominated by organizations similar to the Mafia. These are known as failed States. They are undermined by internal conflicts and by an overall distrust.

The present financial and economic crisis leads us to suspect that we have been ruled by a global powerful financial Mafia. If it is not an organized Mafia, at least the economic paradigm underlying the actions of the most important financial institutions leads us to suspect as much. Neo-liberalism, defending the decreased role of the States, as institutions to which we give our power for a certain period of time, as it happens in a representative democracy, decreases objectively the quality of democracies. This has led to the dysfunction, as we are just observing, of the education system and of the justice system. The education system only functions well for those who have sufficient money to choose good private schools. A human society that dismisses the essential role of schools dismisses the huge social costs of ignorance. In the case of the justice system, we must consider the axiology behind the laws that it must apply. This axiology is built according to a neo-liberal view of the world. Since the neo-liberal vision of the world is dangerously incorrect, its axiology only reproduces, like a sounding board, this dangerously incorrect ideology. Everything, and everyone, that supports this vision of human society, with its "unavoidable" behaviour, is promoted, and everything, and everyone, that opposes this vision of the human society, is depreciated. These two systems, education and justice, are the most important systems supporting democracies. The education system, because it puts us in contact with the results obtained by our most outstanding ancestors. A good school bequeaths to us our most important legacy. A good education system should put us at the forefront of every branch of knowledge. A good education system is understood to be an accessible efficient education system for anyone who wants to learn. We mention the justice system because it must provide security to all of society,

[88] Teilhard de Chardin, *Le Phénomène Humain*, Éditions du Seuil, 1955.

punishing those who do not obey, not to biased laws, but to balanced and transparent laws. We consider balanced and transparent laws those that integrate the concept of social and environmental crimes. It must be a crime to artificially[89] and dramatically increase inequalities in human societies. Any environmental damage that decreases the Earth's resources and biodiversity must be considered a crime. In particular, the justice system must punish everybody, even the most powerful members of a society, whenever they commit crimes. It must be said, at this point, that we defend the freedom of initiative. Consistent learning will improve it. We do not believe in a centralized economy. A centralized economy will, inevitably, lead to the creation of a "legalized" Mafia, and lead human societies to stagnation. However, defending the freedom of individual initiative does not mean that this freedom should not be framed by accepted and transparent rules. Rules should be imposed by our democratic representatives, i.e. by ourselves, and not only by an abstract substantive: the "market" or, in other words, by multinationals, mostly financial, that try to achieve an absolute control of the world economy. The market may be a useful tool in economics. For instance, it translates value into price. We agree with the effectiveness of the empirical laws of the market. The economy should use market laws, but only up to a certain limit. This is even supported by the current chairman of the Federal Reserve, Ben Bernanke, when he said in an interview to the CBS program *60 minutes*, at the peak of the present economic and financial crisis, that we must prevent banks and other financial institutions from becoming too big. The amazed interviewer stammered: but this is exactly the opposite of what we said till now! Ben Bernanke shrugged resigned. Besides, even some neo-classical economists have defended that if an oversized bank cannot obey market laws, i.e. cannot enter into bankruptcy, because this will lead to a financial and consequent economic meltdown,[90] then it must be nationalized. However, when a financial institution is a multinational, what does this mean? There is not an international democratic institution with enough power to assure that such a financial institution would be effectively controlled by it. Meanwhile, people in all regions of the world where these multinationals are active will be affected by their uncontrolled actions, aiming solely to maximize profits. This is the underlying economic logic of the current world economic system. They do not want to internalize, at least according to our unavoidable current "weak teleology", all the inherent costs, such as social and environmental costs. Multinationals, i.e. their boards and their shareholders are not

[89] As we said before, there will always be natural inequalities between two complex structures of the same level; there will never be two equal human beings.
[90] As was the case of Lehman Brothers Holdings Inc.

interested in that. It is a dramatic mistake to transform our democracies into uncontrolled "marketcracies". The defenders of neo-liberalism, defending this vision of human society, say that the world will always be like this: imagine a region populated by foxes and rabbits. If the number of rabbits increases, the number of foxes will increase because they find more food. After a certain point, the number of foxes increases so much that the number of rabbits decreases. The unavoidable consequence will be the decreasing number of foxes. And the process restarts. This is acceptable if we are talking of foxes and rabbits; this process is a consequence of the "weak teleology" of foxes. We are not foxes. They despise an evolutionary process that keeps repeating. We are able to foresee further. If we want to increase our chance of survival, we must use our scientific knowledge, our philosophical knowledge, in a word, our culture. Foxes possess a culture as well. They are natural complex structures that exist because they got enough knowledge to make their existence possible. But they are constrained to their "weak teleology", weaker than our own. The "market", free to act, possesses a potential nefarious characteristic. Those who defend the submission of human societies to the uncontrolled empirical laws of the market want to deal with complex structures, like us, who created art, philosophy and science, as if we were foxes... This is unacceptable. Midas, the Greek mythical character, had the power to turn everything he touched into gold. The "market", free to act, is an abstract personage that has the power to turn everything it "touches" into filth. The Earth included.

There are no truly effective economic theories yet. Economics possesses only a few empirical quantitative laws. There are even no reliable economic models. No one could dare to call them theories. The only possible honest stance in economy is to confess ignorance, and they should not be ashamed of doing so. The systems they study are complex, and they still have not created, in the economic area, concepts and quantitative techniques to deal with these objects in an effective way. Nowadays, there are research fields that try to study complex systems. The so-called theories of complexity are attempts at studying complex systems like living beings and economic systems, but they are not yet in a condition to achieve an adequate theory. The novelty is that physics is now able to handle this type of systems, i.e. quantum complex structures. However, this means looking at the world in a completely different way. This means a scientific revolution.

If we do not use our culture effectively we will hike to an announced disaster. This will lead our increasingly global civilization towards a very likely global conflict. Our recent experience showed us a lesson about the uncontrolled behaviour of huge financial institutions. The ensuing social and economic disorder left the overwhelming majority of the people with a bitter feeling of insecurity. These people are apprehensive about the current

unstable economic, social and political environment because they cannot be sure that they will be able to avoid a global conflict. They know that, if that comes to pass, casualties will not be in the tens of millions, like in World War II, but in the billions. All this will happen because some insane people, with a world outlook similar to that of a Mafia, think that the world really is as simple as they think it is. The market is only a concept, associated with an empirical law: the law of supply and demand. Human society is not a mechanism; it is an organism, i.e. a complex structure. Like any other organism, its functioning is non-linear and is hugely complex. We must use many more parameters, and even if these will always be insufficient,[91] it would be better than the current situation. At least, we know that we must impose ethical constraints in every human activity. These ethical constraints had been imposed probably since some complex biological societies emerged. The members of these complex biological societies do not act exclusively in an innate manner, i.e. in an unconscious manner. Their actions are the fruit of a conscious process of learning. Evidently, there is an innate manner of acting that guaranteed their survival immediately after its emergence. This happens in any other complex structure, even when it belongs to a lower level of complexity. We may consider it as an evolutionary, unconscious process of "learning". But after a certain threshold of complexity, there is an acquired behaviour reached through a conscious process of learning. This became possible after the emergence of central nervous systems powerful enough to allow it.[92] It is well known that, in complex societies of chimpanzees, there are political events. They solve these political events, always in order to maintain the minimum harmonious functioning of those societies. Sometimes, the solution means the elimination of a leader, if that leader shows itself too violent. In human societies, which are much more complex that these chimpanzee societies, ethical constraints were first introduced through myths, like creation myths, mythical beings as ethical paradigms and, finally, religions. All this linked to making power sacred through the emergence of theocratic societies. Even

[91] We must not forget the unavoidable "weak teleology." The scientific revolution we have been talking about just intends to improve it a little.

[92] However, even a unicellular living being like a paramecium demonstrates an elementary ability to "learn," probably much closer to the ability to "learn" we may find in complex structures of lower level. An ability whose existence the *principle of eurhythmy* already allows us to glimpse. In fact, some experiments have shown that a paramecium changes its behaviour when subjected to a radical change in the environment. When this change in the environment is removed, they do not return to the previous behaviour. Harvard L. Armus, Amber R. Montgomery and Jenny L. Jellison, "*Discrimination Learning in Paramecia (P. caudatum)*," The Psychological Record, 2006, 56, 489-498.

today there are human societies where religious traditions, or the cult of personality, are used to make sacred a diffuse power of a class of individuals in the first case, or the power of an individual who represents a class of individuals in the second case. Currently, the most advanced human societies are secular and do not accept the cult of personality. The most advanced human societies do not accept the sanctity of any power. In a democracy the power lies with the people, who accept to transfer it temporarily to some representatives. These representatives must give a scrupulous account of their activities throughout that period of time.

Human societies were able to produce art, philosophy, science and, through science, sophisticated techniques. All these are powerful tools that societies must make accessible to the majority of its members, through an efficient education system. This will create the minimum conditions to make possible the emergence of a new axiology, translated into a system of laws that would provide a less dysfunctional behaviour by our societies. We must use all these victories to regulate our societies. It is of the utmost importance that we change the current functioning of economic relations. As we said, human societies should create a new axiology and, consequently, new systems of laws. We all agree that we must be very rigorous in punishing a murderer. We all agree that human life must be strictly protected. However, huge social harm, caused by those who have chosen their actions egoistically, acting like a Mafia, should be punished at least as rigorously as a murderer, because they are already responsible for the loss of a greater number of lives, and in order to prevent the continuation of a behaviour that will lead to a huge number of deaths in a more or less near future. They are socially dangerous.

The "invisible hand" of John Smith, without a strong democratic control under this new axiology, will lead us, as we just said, to an announced disaster. Belief that a simple empirical law of economics is enough to control the behaviour of a whole human society is also a consequence of the reduction of a complex organism, not to a sophisticated, but to a simple mechanism controllable by a single parameter. Democratic States are the only institutions that, at least theoretically, can assure, within our "weak teleology", its harmonious behaviour. We need the "visible hand" of the States. Reducing the role of democratic States to a residual form is insane. This way, we repeat, multinationals will have the control of human society. Their executive boards are not democratically elected by all the people affected by their actions, but exclusively by their shareholders. This constitutes a loophole in any democracy worthy of the name. Multinationals should be under the control of a truly democratic institution of global dimension. This kind of institution does not exist yet. The United Nations should be such an institution, but it must be completely restructured. It must

become a truly democratic institution, where the people of the world feel represented. Moreover, it should have an effective power to enforce transparent rules; rules in which all people feel represented. This may be a naive desire, but this is the only effective solution. Meanwhile, we must wrestle an effective power from the already existent democratic states. They are the only institutions that can control, at least locally, the functioning of the multinational companies including the main ones, i.e. global financial institutions. Otherwise, the world will remain on the way to an announced disaster. "Foxes" must decrease their number the hard way... It would be better to avoid this foretold disaster, but we are not sure that this will be possible. It is terrifying to admit that we cannot avoid the suffering of a massive extinction in a near future.

As we said before, if we want to specify the main characteristic of our species, we should mention that it acknowledges that the world is much more complex than our senses apprehend. It was this characteristic that enabled us to create myths, art, philosophy and science. Let us be worthy of this heritage. Evidently, philosophy and science try to overcome myths; but even myths show a great capacity to persist assuming new and more sophisticated forms. The *principle of eurhythmy* imposes a new worldview and, consequently a new form of acting. Perhaps the ideas we present in this text will help us to create the base for the emergence of a more effective scientific knowledge. In science, we must adopt the most complete freedom to conduct research. In science, we must also adopt a new axiology that should be a consequence of the six criteria enunciated at the beginning of this text. In this manner, we may avoid the disproportionate allocation of resources to infertile research lines because they do not fulfill the mentioned six criteria. Perhaps it would be reasonable to look for other lines of research that do not require substantial allocations of financial resources, but which may prove, in the short term, far more effective. Maybe in this way we will be able to reach, in a shorter time, a more effective scientific knowledge. Scientific theories and their technical consequences, jointly with philosophy and art, are the most sophisticated features of a future unified stronger culture. We use the term unified culture as opposed to the conception of "two cultures" mentioned by Snow.[93] These "two cultures" must be permanently interconnected. This will increase the probability of the emergence of new and more effective complex social structures. New complex social structures that, we hope, would enable us to face the future with less apprehension.

[93] C. P. Snow, "*The Two Cultures*," Cambridge University Press, 1960.

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ON EURHYTHMY AS A PRINCIPLE FOR GROWING ORDER AND COMPLEXITY IN THE NATURAL WORLD

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Summary: Eurhythmy is a general concept behind the attempt to provide quantum physics with a causal basis, following the guidelines on this subject first enunciated by Louis de Broglie in the 1920's. The following paper does not deal with the formal treatment of eurhythmy, dwelling instead on philosophical considerations that concur to link fields so distinct as physics, mathematics, biology, and economy. Arguments dealing with a natural and progressive formation of order are presented as a starting point for investigating the roots of causality in the world at large, and in the quantum realm in particular.

Keywords: Eurhythmy, akron, order, entropy, complexity, progress.

1. INTRODUCTION

The indeterminism and non-causality contained in the current (orthodox) quantum physics formalism have been shown to be a consequence of the linear assumptions attributed by this theory to the wave character of any quantum being. This being is represented within this formalism by a linear sum of harmonic waves, infinitely extending over space and time. This linearity in turn permeates the natural world as a whole, since the quantum being is everywhere all the time, with the consequent non-separability of quantum beings.

To cut this Gordian knot, a new non-linear causal alternative, here designated as *hyperphysis*, assumes that waves have a finite spatial and temporal content, and to every quantum entity or *akron* (which concentrates the entity's matter and energy), and in fact, to any object is associated a finite wave, which "carries" the ondulatory properties of the akron, and explains the usually assumed "dual" character of quantum beings (wave or

"particle") , not mysteriously, but physically, because waves have the capacity to divide and reunite before or after obstacles are met.[1]

The new analysis of quantum phenomena relying on the existence of an objective reality has been successful, using the more appropriate mathematical description of wavelets, instead of the usual Fourier series expansion of the orthodox theory. This alternative approach to physics has posed an intriguing demand, the need to admit a principle, under the name "eurhythmy", basically implicating that akrons do manifest an average "preference" to choose the best pathway, one where they can maximize the action resulting from their energy content, including their wave portion.

An objection to hyperphysis could naturally arise, bringing in the accusation that eurhythmy is a purely metaphysical delusion. This statement is however based on a false appreciation of how physical theories are formed, ignoring what has already been pointed out elsewhere, that most of what passes now to be the state of art in physical sciences, including orthodox quantum physics, is hidden behind quite metaphysical postulates, and does not rest upon some truly "objective" and exact criteria.[2]

However, we prefer to waver on this discussion of eurhythmy in the direction of the appropriateness or not of appealing to metaphysics as a normal route to establish a body of knowledge, a theory that does gradually become more and more objective in science. We will instead endeavor to discuss this matter by showing that, contrary to the *ad hoc* resources implicit in the non-causal orthodox quantum physics, such as the collapse of the wave function, the eurhythmy principle is on line with major trends in the philosophical thought that at crucial times were used by scientists to advance new knowledge. Even though at this stage and in this paper we can only present a summary introduction to the subject, where we will not be able to discuss the issues in detail, we hope to provide arguments to heuristically justify the eurhythmy principle, as being compatible with an objectively real and causal natural world, and therefore consistent with hyperphysis.

For this purpose, we begin with by remembering some of the visions already offered by myth-makers a long time ago, of course taking into account that this type of thought often made use of supernatural elements, but on the other hand pointing out that it also had a deep embedding in the

[1] Many articles have now been published, widening up the applications of the new causal nonlinear physics; here we refer the reader to two founding works which set up its methodological content: José R. Croca, *Towards a nonlinear quantum physics.* Singapore: World Scientific, 2003; José R. Croca and Rui N. Moreira, *Diálogos sobre física quântica - dos paradoxos à não-linearidade.* Lisboa: Esfera dos Caos, 2007.

[2] Michael Redhead, *From physics to metaphysics.* Cambridge: Cambridge University Press, 1996.

intuitive knowledge of reality, which allowed for some truthful descriptions, valid even when translated to modern times.

2. MYTH AND CAUSALITY

Before ancient philosophers could start examining in more depth the problem of understanding the origin and composition of the natural world, myths tried to provide a rational answer, based on causal reasoning, even though operating in a fantastic form, which is perhaps the stronger reason for the continuing interest men have shown in them.

According to the "Theogony" of the ancient Greek Hesiod (around 700 b. C.), Chaos was the beginning of all universe, and this Chaos was also the space, which was not a void. Out of this space and the elements it contained there appeared Gaia, the Earth, and the Night, both limiting the action of Chaos. Gaia created Uranus, the starry Sky, and Night created Ether, the light of the heavens, as well as the Day, which alternates its appearance with their mother, the Night. In Chaos there came into being Eros, the universal love. Gaia, Night and Day were spontaneously born, whereas Eros and the other divinities were born through fecundation.

In Chaos all the elements of the natural world were latent, although unorganized; only after their organization the cosmos we know was able to come about. For this order to manifest itself it was necessary to attribute to Chaos the pairs of oppositions and contrasts which are the cause of everything: heavens and earth, day and night, cold and heat, dry and humid, soul and matter, and finally love, which also operates in pairs – males for the generation. History started to exist and unfold beginning with the birth of the son of Heaven and Earth, Chronos, the time, as a process of opposition between order and disorder.

This opposition process propitiated a conflict that unraveled in the ensuing battles between the Titans, representing the older order of gods and the new generation of Olympic gods, and which was next followed by the war between these gods and humankind, a race that was the final result of the process. The new creature, man, was a rather refined species, as compared to other living beings, capable of creating by itself a new order, after first learning how to use the fire and the techniques, and thus discovering science, philosophy and the arts. So, as also told by other major myths beyond the Greek world, ever after each new era commenced, another order prevailed. Human life represents the constant victory of order amid natural disasters which abound in the natural world; of course, we know that still later on there were combats among the humans themselves, a process that corresponds perhaps to the gestation of a future new order.

With the mythical description of creation of successive order levels, we are introduced to what became a major concern: why is order preferred to disorder? Do these successive historical levels announce a constant trend to evolve in a certain direction, or on the contrary is such evolution mere illusion? Is there no teleological content to be observed as well in the unfolding of the history of science, or knowledge in general?

From this vantage point, and before tackling up these questions more closely we note down the recurrent demand in our point of view of an "arrow of order" that closely follows the arrow of time.

3. STRUCTURE FORMATION

We would like for a while to branch out of our foregoing discussion and focus on our present *status quo* of a few points of physical theory, where we meet with a seemingly simple question. Even contemporary physics, despite its arsenal of formal mathematics, when referring to the "particle" picture of an electron, usually depicts it as an electric charge uniformly distributed on a surface – like the surface of, say, a sphere. Otherwise its self-energy goes to infinity as the sphere radius approaches zero – and indeed the classical electron has a quite small radius, of the order of 10^{-13} cm. As a charge of equal electrical sign repels other charges of the same signal, the obvious unanswered question is: why does an electron simply not blow up?

We have applied to the electron itself the known fact that the repulsive force between two particles of equal charge q, apart at a distance r, was demonstrated by Coulomb (1785) to follow what is called Coulomb's law $F= q^2/r^2$. The same problem is met when considering not just the charge of a single being, but also between two distinct entities. If the distance r between them diminishes tending to zero, the force will become infinite, as in the case of protons held together in the atomic nucleus. As there is no real explanation that accounts for the nucleus not exploding away, a *deus ex machina* assurance for the stability of the nucleus was introduced so that at such small distances a "strong force" was postulated to exist binding the particles, so as to supplant their repulsion. A creative hypothesis in this sense, though distantly related, was already advanced in 1763 by physicist Father Roger Boscovitch. His idea was that binding forces of matter can be attractive in the immediate short range and subsequently repulsive.[3]

[3] Roger Boscovitch, *A theory of Natural Philosophy.* Chicago & London: Open Court, 1922; pp. 39 -43.

What is the difficulty with those assumptions regarding the nucleus or the electron? Fundamentally there is nothing in the natural world as a point-like, Newtonian particle. To avoid these infinities one must acknowledge that a "particle" needs an internal structure, and is therefore not a "particle".[4] It is useless to assume that some of these "particles" are in fact made of further smaller building blocks, such as quarks, for these also face the same problem, it only takes the explosive content one scale down.

The question gets in fact more complicated and deeper than that, as any collection, even of "particles" in the old sense of ball-like beings, assumes a collective character, thus creating some sort of new structure, which is not the mere sum of its constituent parts. Reality is, in other words, non-linear in many of its aspects, even though we tend to judge it to be linear, thereby simplifying it for our immediate purposes – only because of this, we may think of a new structural level as a replica of the older level in another scale. Surely at a point of our attempts to describe the reality of the natural world, it pays off to consider a new order simply as the result of the building up of repeated properties of the older order, only magnified by a scalar factor. However, if we do not take care with the approximation introduced, we will be obfuscated and forget that this procedure was only a crouch to let us walk in difficult circumstances. We will come back to the question of inner structure in a while.

4. What Happens if the Change to a Higher Order is not Deductible from the Previous One

The pre-Socratic philosopher Zeno appears to have been one of the first to note that the line is more than an infinite collection of points, so that the summation of particulars is never a complete description of the whole.[5] The consequences were explored by him in the famous homonymous movement paradoxes. This was a valuable insight into why the rules (or "laws", as we are used to call them) that are employed in a domain are prone to introduce fallacies in a set of a superior order. Laws are therefore not eternal dictates, and they are useful only inasmuch as they provide us with a workable approximation - usually a linear one, since for simplicity sake we

[4] Few physicists have attempted to ascribe to quantum beings an internal structure. One exception was Winston Bostick, in "The morphology of the electron", *International Journal of Fusion Energy,* vol. 3, nº 1, January 1985, pp. 9-52.

[5] Geoffrey S. Kirk & John E. Raven, *Os filósofos pré-socráticos*. Lisboa: Fundação Calouste Gulbenkian, 1979, pp. 297-298.

tend to consider linearization as a natural extrapolation in our daily experience.

This same difficulty of the collective versus the individual reappeared in the 19th Century, in the wake of a movement that swept the Western world as it unleashed great passions in several areas, Romanticism. Science witnessed this stormy clash particularly in what was labeled as *Naturphilosophie*. This is not the place to develop the theme, which is very complex and stretches far out of the usual (bad) reputation of "romantic" given in the arts and science.[6] The Romantic conception had a lot to do with major issues that erupted in the physical and biological sciences. Let it suffice for our purposes here to recall one of the consequences of *Naturphilosophie*, the bitter opposition in physics between "atomists" and "dynamists".[7]

The former physicists were associated with Newton and his followers, who viewed reality as made of point-like atoms, whose effects could be linearly summed up, and of course this sum could make use of refined techniques as the integral calculus. The latter ones were in favor of a holistic approach, and insisted that reality is a unified whole, so that the process must be a dynamical one, not reducible to its parts.

Lagrangian mechanics, which appeared during this time, for example, is a dynamist approach, and different from the Newtonian treatment of mechanics as a collection of point particles, even if the atomists could arrive at the same numerical results. In the collective, the particle is determined by the whole, not the other way round. Similarly, an electric current is different from a simple collection of electrons running parallel to each other. In a different level, and also showing up as a distinct collective, stable plasma structures are generated out of "singularities", the plasma vortex filaments (diamagnetic plasmoids), a phenomenon which is observed in galaxies, and also in our Sun's corona in the form of convection rolls in the penumbra of sunspots, as well as in high-energy physics laboratories.[8]

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[6] The subject is dealt with by, among others, Andrew Cunningham & Nicholas Jardine (eds.), *Romanticism and the sciences*. Cambridge: Cambridge University Press, 1990; and Robert J. Richards, *The Romantic conception of life – Science and philosophy in the age of Goethe*. Chicago and London: The University of Chicago Press, 2002.

[7] The details are in Armin Herman, "Unity and metamorphosis of forces (1800-1850): Schelling, Oersted and Faraday", *Symmetries in physics (1600-1980): Proceedings of the 1st International Meeting on the History of Scientific Ideas.* Bellaterra: Universitat Autònoma de Barcelona, 1987.

[8] See W. Bostick, *op. cit*. (4), p.40, and also Winston Bostick, "The pinch effect revisited", *International Journal of Fusion Energy,* vol. 1, nº 1, March 1977, pp. 1-54.

Another way to look at the passage from a collection of separate individuals to a higher-order collective was given by a keen thinker also associated with German Romanticism, Johan Gottlieb Fichte, around 1800: for him even highly-ordered systems of thought are derivable from some first principles which cannot be demonstrable from within the systems themselves. In other words, the description of a system transcends the inside constituents of the system. This reminds us immediately of Gödels theorem about the incompleteness of axiomatic logical systems, a point to which we will return in the next section.

The "first principles" inside a system are called postulates. They are not demonstrable, but they must be acceptable. It was in this sense that hyperphysis first postulated that our Universe follows the eurhythmy principle. As indicated above, to have a better view of this principle, and its motivation, one may profit to rise above its original introduction in hyperphysis quantum physics, and seek how it can transcend this application as part of a more general formulation.

5. HOW ORDER ENSUES FROM NOT-ORDER

In modern usage, chaos was a synonym for disorder, until the more recent mathematical theory of chaos showed that what in the natural world looks like apparently lawless behavior is in fact entirely governed by causal rules, although our knowledge of these rules is generally statistical.[9] It was demonstrated that for natural phenomena, even under regimes considered to be chaotic, regularities develop over time and they end showing up. Evidently here and there phenomena exist which deviate from order, but in general they tend to cancel one another for larger ensembles, so that with the passage of time even the randomness of "chaos" dissipates, and displays an underlying fine and ordered structure.

Mathematically, "chaos theory" enabled the contact among fields that seemed quite apart, such as fractals and the topology of "strange attractors". We would like here to add to the achieved interdisciplinarity some foundational problems such as Cantor's transfinite numbers, with their non-countable and non-recursive properties, which are akin to the emergence of new laws out of an "old" repertoire.[10] The transfinite sets are exactly

[9] Ian Stewart, *Does God play dice? The new mathematics of chaos.* London: Penguin, 1990.

[10] Roger Penrose. *The emperor's new mind – concerning computers, minds, and the laws of physics.* London: Vintage, 1990.

homologous to the statement of Zeno: they are a collective which is more than the mere juxtaposition of their individual elements.

We think that the formation of such transfinite numbers (Cantor's "alephs") has to do with the difference between a machine and the human mind, as innovations are a prerequisite for any system with fixed rules to continue developing itself. Machines do not develop new rules, as men do – and to grasp what a transfinite set consists of would be undefinable to a sequential Turing machine like a computer. The essence of this impossibility was what Gödel showed (1931) in his famous theorems on the necessary incompleteness of any logical, formal or axiomatic process.[11] No consistent system of axioms whose theorems can be listed by an "effective procedure" (an arithmetization process) is capable of proving all facts about that system. There will always be statements about the system that are true, but not provable within the system, thus leading to a standstill. On the other hand, if such a system is capable of proving certain basic facts, it cannot prove the consistency of the system itself. At a certain point it will always be necessary to introduce some rule which was not included in the theorems or assumptions before. Machines cannot effectively introduce new procedures, but man is able to do so. This is the basis of creative processes observed in some animals, especially mammals, and to the highest degree in men.

This revelation has a profound implication in the well-known discussion of the dilemma between freedom and necessity. As many philosophers proposed in the past, freedom does not mean the absence of rules, but that given certain rules the best behavior for an individual is the one that maximizes the welfare of a community of individuals – and sometimes that decision must take into account future consequences as well. Spinoza's ethical treatment of true ideas above human passions arrived at a similar conclusion, and in this perspective, free human behavior is determined by what has been called "natural law", and not by a set of fixed codes such as Roman law.

Necessity on the other hand is manifested when just following an order or impulse, as it would be the case with Kant's notion of a categorical imperative. Freedom arises when the will determines by itself that it wants to obey a law, or to disobey it. Even if the result may be the same the process is different. That is also how Marx interpreted the issue when he envisaged that socialist man would rationally regulate interchange with the natural world, with the least expenditure of energy, and one day would pass from the realm

[11] See Ernest Nagel & James Newman, *Gödel's Proof*. New York: New York University Press, 1958. Gödel arrived at these results by examining natural numbers, but the concept of logical completeness of formal, axiomatic systems is quite general.

of necessity into the realm of freedom, where the development of human energy became an end in itself.[12]

The issue of freedom also intervenes in another feature that characterized our universe. It is known that symmetry plays a crucial role in several physical, chemical and biological problems. Symmetry is essential for maintaining a given result, yet asymmetry ensures that processes go on without eternal cyclical repetition - otherwise there would not exist the condition called "progress", which is essentially what is implied in observed changes that reshape reality. The asymmetric intervention unbalances the expected outcome, so that it functions as a tool for the introduction of innovations.

Given this capacity for continual renewal, it is no wonder that we find these features of change in the masterworks of art, like in painting, or in the great classic musical compositions. This also implies that our own minds function in this form – we create habits, but all of a sudden we "discover" something new, and we are again confronted with the fact that the human intellect is capable of ever more discovering how the world functions - merely because the human brain is likewise made of the same building blocks as the universe.

In the animal world, maximum freedom corresponds to maximum capacity for change, which are both associated to something we call complexity. The peak of this capacity was attained by the human being, a species capable of developing activities such as science and art, and which became conscious of its own history. Man achieves through these abilities an intensified unification of opposites, freedom with necessity, consciousness and unconsciousness, the self with nature, individual with community. We should then neither fear nor be ashamed of a great doses of anthropocentrism; doing otherwise would negate history.

A series of thinkers devoted their attention to better understanding why in the biological realm it became possible to have this conscious expression of knowledge. Kant, for example, despite his strong opposition to the teleological reasoning applied to the natural world, arrived at one of the main results we are touching upon, namely he realized, as other thinkers before him, that a living being can be understood as a result of a plan, whereas a machine displays design, and yet none of its parts is the cause of the others.[13] For example, the kidney is not the cause of the heart, nor is the heart the cause of the kidney, but they both function together, so they are in fact cause and effect of each other. This is quite puzzling if we do not

[12] Karl Marx, *Capital*. London: Lawrence & Wishart, 1974, vol. 3, p.820.

[13] Immanuel Kant, *Introdução à crítica do juízo. Os Pensadores,* vol. XXV. São Paulo: Abril Cultural, 1974.

assume some purposefulness in the universe, which in turn brings against us the old charge of teleological reasoning. Teleology after the 19th Century has become anathema in the living sciences, nowadays under the accusation of "creationism", but all other options imply diving away from the problem. By ignoring issues of purpose, which needs not to be a direct result of a divinity - since purpose can arise as a property of the configuration of our universe's elements, as we will shortly return to - opponents of teleology embrace themselves a kind of religious position, equivalent to the admission that this problem is out of reach for the human mind, therefore it is not a real scientific problem.

Another relevant contribution of the *Naturphilosophie* movement lies in the work of its chief proponent, Friedrich Wilhelm von Schelling, who around 1800 dared to state a bolder idea: that everything, including the so-called "inorganic" is alive. He did not have the benefit of the scientific advancements we do now, but he understood that a purely atomist, mechanistic approach would fail to capture nature's operations even in the non-living realm.

We therefore shall dedicate a part of our efforts to evaluate how the inorganic world is also pregnant with order. The periodic table of chemical elements is one of the best instances where we can capture some of this order, and we should be aware that such order did not appear all of a sudden. It must have had a history, an evolution of its own, and the cosmological and astrophysical evidences we now possess do point to a later formation of the heavier elements in the universe, a process that went on until the radioactive elements - and beyond - were created. This has been compared as parallel to the formation of our solar system, with its ordered sequence discovered by Kepler, who compared it to a musical scale (the universal harmony).[14] Such a musical content is not only metaphoric, but real, as the acoustic effects of the oscillations we perceive in our brain as harmonic scales and combinations of tones replicate different layers of order that go from the chemical elements to astronomical structures. The idea was later recaptured in the 18th Century by Johann Gottfried von Herder, who understood our solar planets as resulting from nebular chaos, but according to laws of order.

The emergence of order is an invariant characteristic of our universe – a perhaps better name than a "law"-, be it organic or inorganic. As a whole and given sufficient time, the natural world is anti-entropic (or "negentropic"), a condition already observed in conjunction with the

[14] Jonathan Tennenbaum, *Energia nuclear – uma tecnologia feminina.* Rio de Janeiro: MSIa, 2000, pp. 35-48.

phenomenon of life by Schrödinger.[15] A resultant characteristic is that such systems are never at equilibrium, except at local regions and for a short while. Death is a relative equilibrium for a living being, but this the trivial case. Therefore, only during a more limited time span one will find "equilibrium" (or better saying, dynamic quasi-equilibrium) conditions, where entropy is locally conserved or even increased.

Given these preliminary remarks, we call the attention to the necessary precautions to be taken around what until these days has been considered an immutable and irrevocable statement, namely the second law of thermodynamics, whereby entropy constantly grows. It is in this context that we will now introduce the discussion of something that already appeared above, the concept of progress. Ideas like "progress" and "complexity" were once much used by science, but lately they have faced discredit both in the life sciences and in the social sciences, due to their supposedly "evil" anthropocentric accent and obvious teleological implications.

There was a recent proposal made by Jorge Wagensberg for mathematically defining progress in terms of life sciences.[16] In his description that makes use of communication theory, a living system exchanges matter, energy and information striving to keep itself alive, according to a continuity equation. If H(X) is the Shannon entropy (or average information) of a source X, H (X | Y) is the entropy of source X conditioned by source Y, and H (X,Y) is the joint entropy of sources X , Y, then

$$H(X,Y) = H(X) – H(X|Y) = H(Y) - H(Y|X) = H(Y,X)$$

This identity, when applied to a living system, can be interpreted as "The complexity of a living individual, less its capacity for anticipation in relation to its environment, equals the uncertainty of the environment, less the sensitivity of the environment in relation to that particular living being".

Using this equation, a mathematical and quantitatively measurable definition of progress is possible. There is progress whenever a living being improves its perception and knowledge of the environment, thus modifying its capacity to act correspondingly. Living beings capable of more progress in a given timeframe may also be called more "complex".

What happens if, instead of living beings as Wagensberg did, one now considers akrons, such as an electron? According to hyperphysis, an akron

[15] Erwin Schrödinger, *What is life? - The physical aspect of the living cell.* Cambridge: Cambridge University Press, 1969 (1944), pp. 72-91.

[16] Jorge Wagensberg, "El progreso: un concepto acabado o emergente?", in José Tono Martinez (comp.), *Observatorio Siglo XXI – reflexiones sobre arte, cultura y tecnologia.* Buenos Aires: Paidós, 2002, pp. 197-230.

concentrates the energy of a quantum being, which moves in the chaotic sub-quantum environment, and has associated with it a wave ("theta wave", or "empty wave", in the proposal first made by De Broglie). Well, an electron alone means nothing, because it always appears in a context of other entities, as for example an atom, ions, a pair of electrons, etc. In this case, the interactions among such entities are what matters, for the akrons and their interactions constitute a process. It is in principle also possible to speak of the entropy of an electron in relation to its environment (thus comprising whatever may exert action upon it – other akrons, radiation, etc.), as well as the multiple choices this environment poses to the electron, and how the environment will interact with the electron. We could then foresee a similar continuity equation governing the electron behavior as if it were a living being, which could be interpreted as:

"The complexity of an akron, less its capacity to anticipate the environment action, equals the uncertainty of the environment, less the sensitivity of the environment in relation to the akron".

What we are implying here is that the akron, by following a determined behavior, and an optimizing eurhythmic one, has more chance of success in its action than with a different behavior. Now, this formulation vindicates Schelling's idea that the inorganic world is also "alive", in the sense that it is not static – real equilibrium is nowhere to be found, even a stone lying on the ground has plenty of transitions occurring inside it, and in the long range the stone will "move", either by internal processes or by external forces. In this sense, and given the proper meaning to words, lest they make no sense at all, we feel free to refer to the "life" of the inorganic world, which is a metaphor for the akron – we can thus say that an akron persists in its existence by following eurhythmy, and this translates what happens as an average behavior, as if the akron in fact contributed to a causal behavior, and did not behave randomly as in the orthodox view of duality and wave collapse.

Let us return to the akron behavior and consider, for example, a photon. When passing from an environment to another with a different refractive index, it has more chance to proceed well on its "life" (persistent existence) as a photon if it follows what is known as the refraction law (probably the real process is much more complicated than that, but let this suffice for now). Knowing as we do that such a path is the one associated with less travel time (a consequence called "Fermat's principle"), we envisage that akrons that exhibit such optimized behavior will "live" longer, a result that brings as consequence that they can also succeed better in the tendency to form larger (more complex) ensembles, like a ray of light, where interactions will be more numerous and more complex than with a single

photon. That is also fundamentally why we declared before that an electric current is more than a mere collection of electrons traveling together.

The trend to follow most of the time the best (optimum) paths is thus related to our concept of eurhythmy. Of course we introduce anthropormophization in our description, and care should be taken with words like "best" or "optimum", but then this description is more readily assimilated. This property of the natural world had already been perceived by Leibniz, when he stated that ours is the best possible world, where there reigns a pre-established harmony.[17] These principles allow us to find the maximum area under a curve that has the smallest length among others. The application found by Leibniz and others was straightforward: as we know from the history of science and technology, the modern world was radically changed by the application of maximum and minimum techniques, and the infinitesimal calculus was a major contribution to our present existence – a demonstration that progress exists and we are indeed complex beings.

All of the processes we have been hinting at are real, not ideal, so that both in the natural world as in human existence, there will always be unpredicted events, as it is impossible to know beforehand all variables and initial conditions of such complex processes. Simple ignorance is an explanation for the resulting chance-like events; however there is in chance a more intimate reflection of the evolution of the physical universe shown by chaotic processes, because even ordered chaos is subject to fluctuations that can be examined only by assuming stochastic variations, as is done by hyperphysis.[18]

It is therefore quite possible that some entity or set of entities deviates from the eurhythmical behavior, so that they end up evolving towards more disorder (greater local entropy). However, we insist that the general trend is to follow the eurhythmical pathways, and the result of this along the universe's history has been a world of order formation amidst chaos, and the consequent reduction of global entropy.

As noted before in relation to what is called physical "laws" of the natural world, the assumption of growing entropy for parts of the universe that one supposes to be isolated is a simplification, not an iron fixed commandment. This simplification has been useful as an approximation to understanding and manipulating the physical processes, but it fails when one takes into account the integration of the parts, their continued existence, and their implicit greater complexity. For this purpose, one should emphasize our

[17] Gottfried Wilhem Leibniz. "Zur prästabilierten Harmonie", in *Hauptschriften zur Grundlegung der Philosophie.* Hamburg: Felix Meiner, 1966 (1902), Band II, pp. 272-275.
[18] Stewart, *op. cit*. (9), pp. 261-301.

conclusion, that eurhythmy leads in the long range towards negentropy and progress, in the broader definition we gave before to "progress".

Eurhythmy is such a powerful ingredient of the natural world that we conjecture it must also be responsible for the very formation of structures internal to akrons. As mentioned before, electrons and other quantum entities may well have internal parts, where one could also observe the same eurhythmic processes going on.

With these presuppositions we would like to rephrase the problem of freedom versus necessity, examined before. We had already observed that this problem becomes more acute when a collective is formed out of individual entities – how is it possible to maintain individuality in a multitude? Communist physicists in the early times of the Soviet Union, like Igor Tamm, Yakov Frenkel, and Lev Landau, worried about this dilemma, as well as David Bohm in the USA, and they all proposed that particles in a collective are different from the aggregation of isolated particles. [19] The question of a particle's freedom was reminiscent of the then recent dispute in Russia between Bolsheviks and Mensheviks.

Even akrons, such as electrons, may have a kind of "free will", when confronted with the eurhythmic path. An akron "chooses" the path with the least energy expenditure and the maximum energy throughput, in what we anthropomorphically call a "decision process", because if it does not do so, its energy will be spent away with no support of the "collective" will. The resultant average behavior is what we call the application of the eurhythmy principle. The conflict between the individual and the collective is also one of choosing how to act with the greater thrift of overall entropy, and in essence it is not different from the options for men to decide on freedom and necessity.

6. UNITY AS A PRINCIPLE

Another ideal encompassing the Romantic conception of life was the search for finding a deep unity passing through the multiply variegated aspects of the world, the old problem of "the one and the many".[20] According to Schelling, diverse natural phenomena might be regarded as transformations of polar forces, like attraction and repulsion. This belief was

[19] See the following articles by Alexei Kojevnikov: "Freedom, collectivism, and quasiparticles: Social metaphors in quantum physics", *Historical Studies in the Physical Sciences*, vol. 29, nº 2, 1999; "David Bohm and collective movement", *Historical Studies in the Physical Sciences*, vol. 33, nº1, 2002; "Lev Landau: physicist and revolutionary", *Physics World*, June 2002.

[20] Richards, *op. cit.* (6), p. 112.

behind the impulse to discover what was common between electricity and magnetism, something that turned out to be the first breakthrough which united those two branches of physics, in the famous compass needle experiment by Oersted (1820). This success prompted the adepts of Schelling's philosophical system to find other kinds of unity.

That was also the assumption behind the much earlier demonstration of the brachistochrone construction by Jean Bernoulli (1697).[21] The original problem was finding a path for a particle, which under the effect of gravity must pass from point A to point B, not on the same vertical plane, in the briefest time. Bernoulli very boldly equated this problem with another one, of a light ray that must traverse in the shortest time through a medium whose refractive index is proportional to the continuously changing velocity of the light ray.

Apart from the efforts of *Naturphilosophie,* the quest for unification is a quite natural one, and the goal of understanding apparently disparate phenomena is built upon a basic common question: are there invariants in the universe? As recalled before, one candidate for such invariance is the process of creation of successive orders, every order becoming more perfect than the previous ones, a process that has otherwise been known as progress. Nature follows a trend, which is one of growing complexity as time passes, from the inorganic (physical, chemical processes) to the organic. To accomplish this there is an indefinite succession of stages, and every step forward is in itself beautiful. We have already stated that fundamentally the universe is not cyclic due to asymmetry, even though repetition concurs for its maintenance, so that the universe does not "wear out" and die – on the contrary, its overall entropy diminishes. These characteristics are part of the invariant associated with the formation of order.

We recall that for Kant the unity of consciousness implies that the world and the "I" are viewed by the same lens, which is responsible for attaching a unified appearance to the world. Natural laws have to be logically isomorphic with the ways in which we mentally reconstruct the natural world, and that is the reason why our minds have cognitive powers that are suitable to understanding nature. The only observation we add to this reasoning is that there is no necessity for supposing that our minds function in a linear way.

Going back once again to the problem that Cantor confronted, we can say: infinity is more than an infinite collection of numbers, it is a different entity, a new "set", which is to be apprehended not as a static collection, but as a process. That is the reason why Cantor preferred to call the result of this

[21] David E. Smith. *A source book in mathematics.* New York: Dover, 1959 (1929), pp. 644-655.

process not as the traditional infinity, but as something he designated with a different word, the "transfinites" (or his "alephs"). The infinity of the real numbers is greater than the infinity of countable numbers, their quality is different because they are formed according to different processes.[22] Transfinites are ideas that can be quite real, just as all mathematics always has roots in the physical, natural world. The possibility of our mind being itself an open set of the same order of cardinality as all the possible transfinites is very great, since our thought cannot be of a level below what it is capable of thinking. This implies that the order of the human cognition is at least of the same order of the process of formation of Cantor's alephs.

The unity of mind and the natural world should be expected, inasmuch as the brain is itself part of the natural world. This unity of mind and reality has persuaded men, even though not in an explicit way, to attempt looking for other traits of unity, and such a belief in the existence of this additional invariant, namely unity, is peculiar to a universe which was capable of originating a species that has historically demonstrated its trend towards increasing knowledge.

7. KNOWLEDGE

How can we make contact with the infinity of transfinites, the infinity of Cantor's alephs? This possibility is realized through knowledge – or as Spinoza put it in careful words, through the "intellectual love of God". New knowledge occurs as an intuition into the core of reality, where man perceives himself to be part of this infinite reality. This is the reason why man was once characterized as *imago Dei*, a concept that first appeared in the Hebrew Bible and came down to us through the apostles and patristic fathers of the Church, as Augustine. New knowledge is similar to self-creation, and this lies at the origin of creativity – man as creator establishes a new order, not derivable directly from the previous collection of information.

Also for Schelling truth is the adequacy of the images of our understanding to archetypes of reality – which function much like the Platonic ideas or essences; and beauty is the manifestation of unity between universal and particular, species and individuals. In terms of the objects of cognition, science and art are ultimately the same. The greater scientists and artists have been able to capture how ideas (archetypes) are related to each other, a variant for the unity quest discussed before.

[22] Penrose*, op. cit*. (10), pp. 108-114.

8. WHAT WORLD IS COMPATIBLE WITH THE EURHYTHMY PRINCIPLE?

At this point, we can repeat an ages-old question, to which we expect to have shed some new light: why does something exist, instead of nothing? This problem is also related to the eurhythmy postulate of hyperphysis we begged for at the beginning. Stated in a different manner, the question is: would it be possible for the world to follow a principle other than the eurhythmic one, and would the world just go on?

First of all, we established that some principle must exist, otherwise anomy and anarchy would be permanent, and beings, including akrons would have only an ephemeral duration. We observe quite the contrary in the natural world: relatively long-term stability, causal relationships, the existence of physical "constants" like G or h, – even though these may change over time and with changes of scale – relative constancy of life on the Earth, as evolutionary changes generally occur in what appears to be a long duration for our human scale. Given this long duration, it is with difficulty that we meet the consciousness of not-being (as in death) – definitely we perceive the world as made by beings, and this is a world of things that are, instead of things that are not.

We may thus risk an answer as to why the eurhythmy principle can be the most adequate to account for our natural world. Using as an image the descriptive mathematical resource known as phase space, we state that the eurhythmic path, when represented in such space, is that one which minimizes the energy consumption, and simultaneously concurs to lower the overall entropy of the system described in the phase space. That is the deeper reason to justify the existence of physical processes such as that involved in light reflection (shortest distance) or light refraction (shortest time), and so many other physical "laws".

Moreover, the eurhythmic path is not only the best solution, it may be unique in its capacity of meeting extreme criteria. Any other concurrent transition in the phase space would be defeated by the eurhythmic one, because after a certain time alternative solutions die out and end by excluding themselves. Otherwise, for a given set of initial conditions there would be at least two possibilities for realizing a certain process, and as a result causality would be destroyed (under given circumstances the same causes must produce the same results), indeterminism would surface, and one could not be certain as to which one of those two possibilities would result. In other words, if the solution were not unique, the universe would be unstable, and chaos would be completely random, lawless – all that is contrary to what is observed. The best and the unique solution in the long range is the eurhythmic one.

What we have here discussed does not imply determinism. Anyway, determinism would rank as a poor description, not adequate to the phenomena of the natural world, since determinism is just an approximation for reducible systems, where it is possible to subdivide a whole in separate parts, and thereof extract a linear solution as merely a sum of those parts. In the more general cases, non-linearity demands that eurhythmic processes be determinant. Eurhythmy is a postulate that we are beginning to understand what it is good for. As hyperphysis develops, we are confident that we will meet more complex descriptions of the natural world, whose existence we are now barely beginning to suspect of.

* * *

ELEMENTARY POEM
By Gildo Magalhães – to his good friend José Croca

Of all things the sea is deepest,
Oscillating, penetrating, mingling with air
To return as pure drink.
The living multiplies in water,
A magic mirror of itself.

Yet though the water carries away the earth,
The grains land elsewhere, bounding the water
To give us the solid ground –
I wish to be in its fine salts
And erect grand works to be long admired.

New vibrations harmonize in the air,
So withering away both earth and water,
An epic labyrinth soaring high
As source of invention, and transparency
Bows and breathes respect with air.

Combustion however leads to the cosmos,
Never-ending energy, transfiguration
That kills air, water and earth –
It is a matter of wonder how fire
Lights up and strongly burns.

Alas, we always come back on our steps:
How can so many elements
Just become a playful one,
Enticing games of hide and seek,
Ever changing, comforting, mystery?

BETWEEN TWO WORLDS. NONLINEARITY AND A NEW MECHANISTIC APPROACH

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Summary: What is the philosophical meaning of adopting a fundamentally non-linear perspective about reality? Either from an epistemological or from an ontological point of view, the meaning and the scope of such an approach involves numerous challenges. Here we will focus particularly on the tension between two fundamentally different ways of conceiving the world, and on the major consequences of this tension for the revaluation of the classical mechanistic philosophy.

Keywords: mechanism, atomistic-eleatic ontology, dynamical systems theory, nonlinearity, organization, emergence.

1. TENSION BETWEEN TWO WORLDS

Mechanism and its atomistic materialism – as defended by Galileo, Descartes and Boyle – was undoubtedly a paradigmatic ontological and epistemological model in the history of modern science and philosophy. According to this model, all phenomena must be explicable under the light of their different efficient causes, which are ultimately reducible to the dynamics of local (mechanical) movements of material bodies. Accordingly, the explanation of the nature and behavior of a composite system should be obtained (taken as a machine) through its decomposition into parts (as its pieces) in order to locate and determine their individual properties, as well as their relations and respective local dynamics.

Any phenomenon should thus be explained as the effect of the linear composition of its individual parts or causes and their operations, since it is assumed that the existence and the nature of such parts or causes can always be taken as prior to, or independent of, their different relational contexts. As an essentially analytical doctrine, Atomism regards any composed

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phenomenon as an additive or linear aggregate of fixed elements with their fixed properties and arrangements.

With the development of contemporary science, this model has known, however, serious problems, some obstacles and limitations. Promoted in part by the emergence of Cybernetics during the 30s and 40s of the twentieth century, sciences such as Physics, Biology, neurosciences and even social sciences gradually began to develop new kinds of approaches able to account for different types of processes and systems, that were since then called 'complex', given the sheer quantity and variety of elements and relationships involved, the quantity of exchanges of information, the different kinds of causal links involved, and their non-linear structures and dynamics. Finally, with the advent of Cybernetics, the notion itself of 'machine' was revised and amended.

The spread of this new types of approaches, however, not only was not always welcomed (and often not even well understood), nor their practical and theoretical management was easy. After all, compared to the centuries that the history of modern science already has, these new approaches – although with visible results in many areas – are still at an early stage of maturation. Hence, its impact continues to be felt today in the controversy generated around the methods and epistemological models of each individual discipline and its various models of explanation and reduction. Ultimately, the problem is to assess the meaning and scope of these new kinds of approaches, and to determine their consequences for a renewed Philosophy of Science and for human knowledge in general[1].

It goes without saying that we are talking about the *ontological* justification of the linear or atomistic kind of approach. Obviously, no one denies the incontestable practical successes that sciences always provide when they can overlook or neglect, in specific cases, the causal influences of interactions and the different degrees of dependency on specific modes of organization, so as to obtain linear solutions to local problems. The question is rather to project such local successes to a global conception of reality, stubbornly insisting on wanting to solve any kind of problem under the light of that atomistic and linear model of intelligibility, assuming its universal scope.

We will mainly focus our analysis on this metaphysical tension. But first let us try to reveal what is really involved in the atomistic ontology on which the paradigmatic linear worldview of classical mechanistic doctrine is founded.

[1] Cf. E. Morin e M. Piatelli-Palmieri (dir.), *L'Unité de l´Homme*, Seuil, Paris, 1974; and P. Dumouchel & J.-P. Dupuy (dir.), *L'Auto-organisation: de la physique au politique* (Colloque de Cerisy), Paris, Seuil, 1983.

2. THE LINEAR WORLDVIEW: THE ATOMISTIC-ELEATIC ONTOLOGY

Although often forgotten, the materialistic and atomistic metaphysics of Democritus and Leucippus that modern science has inherited, not only expresses the well-known mechanistic ontology of the local movements of material bodies, but also some of the founding principles of the *Eleatic Metaphysics* and of the *Parmenides's doctrine of being*.

Ironically enough, the most radical version of materialism is heir and hostage of one of the most idealistic doctrines in the history of Western Philosophy.

Ancient Greek atomism, as it is well known, is part of a movement that tries to depart from the Eleatic monism and from its new conception of Being, seeking to rehabilitate the rational study of physical reality. Indeed, while for all the Ionian physicists, 'being' designates the plurality of singular things (*ta onta*) that the human experience captures, in Parmenides', and for the first time in the history of philosophy, the notion of Being will be expressed as singular (*to eon*), pointing to Being in general, total and unique - the being that the philosophical *logos* must be able to grasp: the pure unity in the form of an abstract self-identity, postulated as a logical principle of reason.

As Jean-Pierre Vernant has noted, the doctrine of Parmenides marks the moment when is proclaimed the *contradiction* between, on the one hand, the change and the becoming of the sensible world (the Ionian world of *physis* and *genesis*), and on the other hand, the demands of logical abstract thought. The principle of non-contradiction requires that being *is*, and not-being *is not*. Thus, the study of natural reality, in its dynamism, plurality and mutability, is void of any rational sense: how could anything change, if that means that it will *stop to be* something in order *to be*come something else? The reason can have no other object than the ideal, immutable and self-identical Being. Becoming, transformation and diversity are seen as irrational and illusory.

The Being does not change, does not move. Being is conceived as a Whole, as an Absolute Unity, homogeneous, indivisible, timeless, motionless and unchanging, about which no speech in the form of particular predicative propositions is possible, since those very propositions would destroy the presumed Unity and Oneness of Being. As from unity plurality cannot come, also from being non-being cannot come, and vice versa. How

is it possible to explain, in these circumstances, the change, the becoming and the plurality of our natural world? [2]

The Greek Atomism, by resuming the Ionian pluralism, and thereby seeking to make possible a new rational study of natural reality, is part of a movement of reaction to the Eleatic ontology. However, as it also happened with Empedocles and Anaxagoras, this reaction was led simultaneously with a positive reception of some fundamental principles of the Eleatic doctrine. Specifically, the atomistic doctrine will preserve:

(1) The rejection of the testimony of sense experience;

(2) The denial of objective reality for the qualities above ultimate (atomic) reality;

And, although claiming a pluralist ontology and acknowledging the reality of emptiness, Atomism will be in line with the Parmenidian doctrine of Being in respect to:

(3) The intrinsically immutable and unalterable nature of the true being – now represented by the atoms (indeed, each atom is conceived as a Parmenidian unit: immutable, indivisible, uniform, and full – there is no emptiness inside each atom);

(4) The impossibility of the production of units from certain kinds of composition (no new unity can come from a given atomistic plurality);

(5) The impossibility of new emergent qualitative phenomena in the sphere of being (the new, as being, would then come from the non-being); and

(6) The rejection of the objective reality of genesis and corruption (since being is immutable and eternal).

In a strategy of *compromise* between the requirement to account for physical reality, on the one hand, and the requirements of the Eleatic Metaphysics on other hand, Greek atomists will seek to *conciliate* the Parmenidian doctrine of Being with the recognition of a multifarious, diverse and changeable world. How was it possible? Simply by adopting a linear model of composition, distinguishing two levels of reality:

[2] Cf. Jean-Pierre Vernant, "Du mythe à la raison: la formation de la pensée positive dans la Grèce archaïque" [1957], *in* J.-P. Vernant, *Myhte et Pensée chez les Grecs*, Paris, François Maspero, 1985, pp. 373-402.

– That of the true Being, at the level of atoms and void; and

– That of the Epiphenomenal and superficial reality, at the level of the mixture of atoms.

Thus, all entities, properties, behaviors and processes that arise from the atomic level must be fully explained in a causal-mechanical way, according to the quantitative differences of the first layer of reality – that is, in the light of the intrinsic properties of atoms (geometric shapes and volumes), their local movements (in chains of contacts, collisions and reactions, movements of approach and departure), their relative positions, order, and combinations. Any compound is thus solved as a certain aggregative association (*synkrisis*), mixture (*mixis*), or combination (*krasis*) of certain primitive elements (the *stoicheia*), and each compound will be distinguished from others solely by the linear kind of ordering or arrangement (*taxis*) of their various parts.

By virtue of their respective geometric shapes, some atoms are interwoven. Some are angular, others curved, some concave, others convex. For example, atom-shaped hooks can connect to one another to form the shape with which hook fits, then forming a complex or a cluster of two atoms. However, these processes of entanglement never match the formation of real new units (that is, new kinds of entities). Just as a unity cannot come from a plurality, a plurality also cannot give rise to a unity. This Eleatic principle is upheld by the Atomism in the outright denial of *coalescence* processes – atoms only come into contact, thus retaining their shape and individual identity intact[3].

For this reason, the objective reality of the processes of change, becoming, genesis and corruption of the compounds is never recognized. Firstly, the compounds never become entities in their own right, with their own identity. Therefore they cannot become subjects with their own biographies. In the terminology of Aristotle (*Met.*, VII, 1041a, and VIII, 1045), a compound can never transcend the mere nature of a sum (*pan*) of its parts, to become a whole (*holon*) with a unity, cohesion and identity on its own. Secondly, as we have seen, the true Being is in itself unchangeable, eternal and incorruptible. In order to accommodate this Eleatic principle, the Atomists will say that what seems to be the genesis and the corruption of

[3] Cf. W. K. C. Guthrie, *A History of Greek Philosophy*, Cambridge Univ. Press, vol. II, 1978 (1965); and G. S. Kirk & J. E. Raven, *The Presocratic Philosophers. A Critical History with a Selection of Texts*, Cambridge, Cambridge University Press, 1957, pp. 319 and 400-426.

certain composed entities (of living organisms, for example) is *nothing but* the illusory effect of the local motion of their constituent atoms in terms of their associations (*synkrisis*) and separations (*apokrisis*) or dissociations (*diakrisis*).

It is relatively easy to discern the survival of the basic atomistic opposition between the true and ultimate reality of the atoms and the epiphenomenal reality that supervenes on their combinations, in different clothes in the history of philosophy: on the one hand, the level of the primary qualities, the physical qualities, or the first order qualities; on the other hand, the epiphenomenal world of secondary qualities, second-order properties, or supervenient properties.

The lesson that all microphysicalist kinds of reductionism always tried to draw is that all supra-atomic (or supra-quantum) reality is formed from different modes of linear or atomistic kinds of compositions – modes of composition that, so understood, do not change the intrinsic and absolute nature of the basic physical constituents. It is this thesis that *justifies*, metaphysically, the attempt to universally apply the model of linearity, its principle of superposition, and its micro-reductionism. And it is also this thesis that allows one to say that engendering compound phenomena can never cause the emergence of anything new or qualitatively different from the set of the ultimate micro-physical properties. Atoms (or quantum entities nowadays) should therefore tell the whole story of our reality – they are the only protagonists, or star performers, in the history of natural reality, and everything can be fully reduced to them, or to their laws.

It is this '*nothing-but-ism*' kind of reductionism (both ontological and epistemological) that the non-linear theory of emergence refuses. That is the reason why Paul Humphreys is right when he notes that "[e]mergence has considerable philosophical importance because the existence of certain kinds of ontologically emergent entities would provide direct evidence against the universal applicability of the generative atomism that has dominated Anglo-American philosophy in the last century". And he adds: "By generative atomism is meant the view that there exist atomic entities, be they physical, linguistic, logical, or some other kind, and all else is composed of those atoms according to rules of combination and relations of determination". If Emergentism thus proves to be a correct account of reality we can safely establish "the failure of various reductionist programs, especially that of physicalism", and we may realize that "our universe is more than an ontologically modest combinatorial device"[4].

[4] P. Humphreys, "Emergence", *in* Donald Borchert (ed.), *The Encyclopedia of Philosophy* (2th Edition), New York, MacMillan, 2006, vol. 3, pp. 190-194.

3. A NON-LINEAR WORLDVIEW

Will someone uphold this Atomistic-Eleatic ontology today? It seems unlikely. And yet many still defend theories and types of approach that only this ontology can support.

Indeed, the linear worldview, paradigmatic in all history of modern science, is ontologically committed to the atomistic-Eleatic Metaphysics. Only assuming this metaphysics can one defend and justify the *irrelevance* of the causal processes of organizational interaction in the real world, and thereby assume the independence of natural phenomena in relation to their specific modes of organization. The thesis here at issue is the pre-determined and intrinsically immutable nature of the ultimate level of physical reality – either populated by the classic atoms or by the new quantum entities. In this sense Atomists may consider that any supra-atomistic phenomenon is nothing but the additive and epiphenomenal effect of the combination of its different parties or of the joint action of their different causes. Nothing qualitatively new can therefore emerge in the linear Atomistic-Eleatic world.

An organizational and non-linear approach to reality stands against this way of conceiving the world.

In a positive characterization, non-linearity means dependence on certain modes of organizational interaction. Thus, a non-linear phenomenon is a phenomenon, which existence and identity depend on the existence of a certain kind of organizational interaction between its component parts or individual causes. Likewise, linearity means independence of specific modes of organization. To explain the existence of a linear phenomenon suffice to determine, in an atomistic way, the individual nature or behavior of its parts or causes, and then (following the principle of superposition) sum the individual results, in order to obtain the nature of that phenomenon – interactions play no role here, or at least they can be easily neglected with no serious problems[5].

So, will someone hold up this linear Atomistic-Eleatic ontology nowadays? Maybe the question is ill-posed. Maybe we should rather ask how can one justify a treatment and an analysis of most natural phenomena *as if* reality *were* the natural world described by the Atomistic-Eleatic Metaphysics.

[5] Cf. G. Nicolis & I. Prigogine, *Exploring Complexity*, New York, W. H. Freeman and Co., 1989, p. 59; and D. G. Luchinsky & A. Stefanovska, "Nonlinearity, Definition of", *in* A. Scott (ed.), *Encyclopedia of Nonlinear Science*, New York-London, Routledge, 2005, pp. 647-649. See as well the 'Introduction' written by A. Scott to this Encyclopedia, pp. vii-xi.

In this connection, we recall a passage from Susan Oyama's book, *The Ontogeny of Information*. In the context of a critical evaluation of the different ways in which DNA is given a special role in explaining the ontogenesis, Oyama says:

> "'But wait', the exasperated reader cries, 'everyone nowadays knows that development is a matter of interaction. You're beating a dead horse'. I reply, 'I would like nothing better than to stop beating him, but every time I think I am free of him he kicks me and does rude things to the intellectual and political environment. He seems to be a phantom horse with a thousand incarnations, and he gets more and more subtle each time around'"[6].

This phantom horse is but the tacit linear worldview. And this 'everyone nowadays knows' is just one of its contemporary reincarnations. Of course, everyone nowadays knows that genes are not sufficient to explain the development of organisms, but the issue here at stake is the widespread attribution of a special directive, formative and privileged power to genes. Indeed, this genetic reductionism that Biology is facing today is just one of the many effects of the linear atomistic thought[7]. This does not mean that the demand for reductionist explanations in Biology, as in any other sciences, should be abandoned. As I. Cohen, H. Atlan and S. Efroni wrote, "scientific reduction has been the key to the identification and characterization of the elements – the cells and molecules – that constitute living organisms. The power of modern biology must be credited to reductive analysis. Our point is that reduction to component parts is only the beginning of wisdom. The essence of biology, like that of other complex systems, is the emergence of high-level complexity created by the interactions of component parts"[8].

The same applies to the expectation, so often delusional, about the Human Genome Project. Most scientists who welcome this project are well aware that an absolute genetic determinism is obviously unsustainable. But as Lewontin stated, if we seriously assume the idea that internal and external

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[6] S. Oyama, *The Ontogeny of Information. Developmental Systems and Evolution*, Cambridge, Cambridge University Press, 1985, pp. 26-27.

[7] As Oyama says, "[j]ust as traditional thought placed biological forms in the mind of God, so modern thought finds many ways of endowing the genes with ultimate formative power, a power bestowed by Nature over countless millennia." (*Idem*).

[8] I. Cohen, H. Atlan, S. Efroni, "Genetics as Explanation: Limits to the Human Genome Project", *Encyclopedia of Life Sciences*, Chichester, John Wiley & Sons, 2009 (URL: http://www.weizmann.ac.il/immunology/ iruncohen/reprints/2009/ 502.pdf).

both co-determine the organism, then we cannot seriously take the belief, often repeated, that the human genome sequence is the 'code of codes' or that it constitutes, as Walter Gilbert defended, 'a vision of the *Holy Grail*', which would show us what is to be a human being[9]. And once again, this is not to deny the achievements of such a reductionist research project, but simply recognize that "the limitations of the project are only the limitations of the genome itself"[10].

Likewise, everyone nowadays knows that just as there are no organisms without environments, there are no environments without organisms. However, the *adaptive* conception of evolution is still widespread as if evolution could be reduced to a simple process of adaptation of the organisms to their environments, that is, as if they were independent of each other, and as if the nature of the environment was already given at the outset, determined in its smallest details and unchangeable. But it is evident that "the environment is not a structure imposed on living beings from the outside but it is in fact a creation of those beings". As Lewontin says, "the environment is not an autonomous process, but reflection of the biology of species"; thus, "the metaphor of adaptation must therefore be replaced by one of *construction*"[11]. Living beings and their environments co-exist in a relationship of *mutual specification* and *co-determination.*

We must therefore move from the trivial admission that there are relational processes between parts and causes in the real world, to welcome and genuinely explore an organizational perspective of the whole natural reality. This is precisely what has been defended by the dialectic (R. Lewontin), interactionist and constructivist (S. Oyama), or enactivist (F. Varela) approaches, not only against mysterious holisms and linear atomisms, but also against the standard interactionism that, although recognizing the importance of interactions, it has always the tendency of conceiving the *relata* as separate and independent things with a prior identity to (and above) their relational contexts[12].

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[9] R. Lewontin, "The Dream of the Human Genome", *in* R. Lewontin & R. Levins, *Biology Under the Influence. Dialectical Essays on Ecology, Agriculture, and Health*, New York (NY), Monthly Review Press, 2007, p. 243.

[10] I. Cohen, H. Atlan, S. Efroni, *Idem*.

[11] R. Lewontin, "The organism as the subject and object of evolution", *Scientia* 118 (1983), pp. 75 e 78.

[12] *Vide* Lewontin, S. Rose e L. Kamin, *Not in Our Genes: Biology, Ideology and Human Nature,* London, Penguin Books, 1984; R. Lewontin, *The Triple Helix: Gene, Organism and Environment*, Cambridge (MA), Harvard University Press, 2000; S. Oyama, *The Ontogeny of Information*, ed. cit.; and F. Varela, E. Thompson and E. Rosch, *The Embodied Mind: Cognitive Science and Human Experience*, Cambridge (MA), MIT Press, 1991, cap. 9.

3.1. TOWARDS A NEW MECHANISTIC MODEL

Through the development of different studies of an increasingly wide range of different types of dynamics and systems with their processes of self-organization, feedback and non-linear causal interactions, the impact of the so-called paradigm of complexity became so deep and intense that it forced a rethinking of the traditional epistemological and ontological approaches of science. One good example is, nowadays, the ongoing project of reevaluating the classic mechanistic model and its particular way of analyzing and explaining natural phenomena. Given the limits of the old deductive-nomological model of scientific explanation (largely promoted by logical positivism), and given the collapse of the old atomistic scheme of analysis and reduction, mechanism is now forced to recreate itself in order to survive as a model of explanation.

Non-linear interactions, pattern formation, structural and functional organizations have now come to occupy the center of attention. This fact challenges the classic atomistic and mechanistic models of analysis, according to which all we need to understand a given system is to decompose it in its different parts, determine their individual properties and behaviors, and then combine them in a linear way, in order to reconstitute the whole system and explain its properties. As David Bohm wrote, "the most essential and characteristic feature of mechanism" was "to reduce everything in the whole universe completely and perfectly to purely quantitative changes in a few basic kinds of entities (...), which themselves never change qualitatively"[13].

The question is whether the creation of a new kind of mechanism is possible, and if so, if it will be enough to cope with the complexity of reality. The challenge is to devise a *non-linear organizational* and *dynamic* kind of mechanism free from the Atomistic-Eleatic Metaphysics. It is precisely in this sense that S. Glennan defines the concept of mechanism under the light of the complex systems theory, and W. Bechtel seeks to build a dynamic mechanistic model of analysis and explanation. Consider, thus, two of the new definitions of mechanism currently proposed:

> "A mechanism for a behavior is a complex system that produces that behavior by the interaction of a number of parts, where the interactions between parts can be characterized by direct, invariant, change-relating generalizations"[14].

[13] D. Bohm, *Chance and Causality in Modern Physics*, London, Routledge, 1999 (1957), p. 47.

[14] S. Glennan, "Rethinking Mechanistic Explanation", *Philosophy of Science*, 69 (2002, Supplement), S344. For yet another definiton, see P. Machamer, L. Darden

> "A mechanism is a structure performing a function in virtue of its component parts, component operations, and their organization. The orchestrated functioning of the mechanism is responsible for one or more phenomena"[15].

A mechanism can therefore be analyzed in four different aspects or dimensions:

> *i*) a phenomenal dimension: they are no mechanisms *simpliciter*, only mechanisms for certain phenomena, and mechanisms are the agents that *do* things;
>
> *ii)* a componential dimension: as a composed system, each mechanism has component parts and component operations;
>
> *iii)* a causal dimension: a mechanism is a dynamic entity, because its component parts act and interact generating a system of causal chains;
>
> *iv)* an organizational dimension: the component parts of a mechanism, and its different kinds of operations, are interrelated and organized spatially, temporally, structurally and functionally[16].

Calling a certain kind of explanation mechanistic is obviously underling the fact that we treat natural systems in a similar way to how we treat the machines produced by our own technology. The goal is always the same: to explain the behavior of a given system as a function of the causal activity of its component parts. But a mechanism, as defined in the light of contemporary complex dynamic systems theories, has very little to do with the atomistic mechanism of classic materialism. What we want now is a mechanist model that can take into account the dynamic and non-linear aspects of the different structural and functional kinds of organizations – therefore not restricted to the classic efficient and material causes, and to a static, linear and deterministic perspective.

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and C. Craver, "Thinking about Mechanisms", *Philosophy of Science*, 67 (2000), p. 3.

[15] W. Bechtel and A. Abrahamsen, "Explanation: A Mechanist Explanation", *Studies in the History and Philosophy of Biology and Biomedical Sciences*, 36 (2005), p. 423.

[16] Cf. C. Craver and W. Bechtel, "Mechanism", *in* Sarkar & Pfeifer (ed.), *Philosophy of Science: An Encyclopedia*, New York, Routledge, 2006, pp. 469-471.

They are as many different kinds of mechanist models of analysis and explanation, as many different kinds of machines we can devise and build.

The Cartesian model of a machine, for example, thought for the animals, has two aspects: an articulated structure of moving parts such as levers, wheels, pulleys, pipes and valves, and a principle of movement generation, as a source of heat in a steam engine, thus resorting to a kind of principle to generate heat to power the animal-machine. In this respect, Descartes anticipated the theory of metabolism in considering the heat as the force of motion in the animal-machine – the heat generated by combustion of food. Thus the theory of metabolic combustion complements here the mechanistic theory of the anatomical structure[17].

In Spinoza, on the other side, we have not a mechanism in the traditional sense of a theory of the local motion of bodies – as we find in Descartes, Pascal, or Huygens – but a theory of chemical or biophysical composite bodies as open and dynamic complex systems, disposing of what we call nowadays non-linear structures. However, a dynamic system, as we conceive it today, cannot be confused with a hydraulic machine model, where a separate individual power source, analogous to an inner fire, is necessary to activate its parts[18]. Therefore, these different mechanistic models should not be confused.

Henri Atlan, for instance, asked: what kind of system is capable of generating its own program (goals, meaning, task, or meaningful data)? In other words, what is a "natural machine"? Atlan rejects the notion of a classical computer as a natural system, thus rejecting the use of classical computer as a metaphor for living organisms. However, Atlan suggests that under the influence of recent work in automata network theory, the cellular mechanism may be designed as a 'multi-layered parallel computer network' embedded in the global geometrical and biochemical structure of the cell, the elements of which would be biochemical reactions and transports[19].

Whether or not this analogy turns out to be valid, at least this is enough to show that the newer machines can work as metaphors with which one can talk about genes in ways that are compatible with the findings of the new molecular biology, and in particular, in a way that it enables one (as Atlan) to argue against genetic reductionism and genetic determinism[20].

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[17] Cf. F. Duchesneau, "Modèle cartésien et modèle spinoziste de l'être vivant", *Cahiers Spinoza* 2 (1978), p. 272.

[18] Cf. H. Atlan, *Les Étincelles de Hasard – vol. II*, Paris, Seuil, 2003, pp. 206-207.

[19] Cf. H. Atlan & M. Kopel, "The cellular computer DNA: Program or Data?", *Bulletin of Mathematical Biology* 52 (1990), pp. 335-348.

[20] Cf. E. Fox Keller, "Is There an Organism in This Text?", *in* P. R. Sloam (ed.), *Controlling Our Destinies. Historical, Philosophical, Ethical, and Theological*

Anyway, for a mechanistic explanation to be implemented it is necessary to fulfill at least three kinds of operations as different phases of one process. Firstly, it is necessary to *decompose* a given system into different parts (structural decomposition) or into separate operations (functional decomposition), since the scientist must know what the relevant parts of a system do. Secondly, it is necessary to carry out a *localization* process in order to determine the link between the different parts and the different types of operations and functions. Finally, one needs a kind of synthetic *recomposition* or reconstitution of the organization of each whole, since the ultimate objective is to determine the specific mode of organization among the parts and their operations that generate the system as such, its properties and its behaviors[21].

It is precisely here that the essential characteristic of this new kind of mechanism lies: between the parts of a system and the system itself as a whole there is the *organizational and intermediate level* of the different forms of causal interaction and mutual dependence that explain the very existence and persistence of the system.

3.2. A MECHANISTIC MODEL FOR EMERGENT PHENOMENA

In a generic way, a mechanism's model of a given system represents its component parts, their relevant operations, and the systemic organization of these parts and their operations, that is, the ways in which parts and their operations are orchestrated to produce the phenomenon (property, behavior) that we want to explain. Therefore, we must be able to identify and locate different parts, their respective actions, and the overall organization engendered by the network of local interactions among those parts and their operations. In this sense, it seems clear that the inability to carry out the operations of localization and decomposition of a system necessarily drags with it the impossibility of subjecting this system to any kind of mechanistic explanation.

In the simplest cases, one can identify clear separated components and attribute to them independent operations; the organization is linear, and components preserve their own identity and integrity through their relations. In other cases (the most common, by the way) the organization plays a more

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Perspectives on the Human Genome Project, Notre Dame (Indiana), University of Notre Dame Press, 2000, pp. 288-289.

[21] Cf. W. Bechtel & R. Richardson, *Discovering Complexity: Decomposition and Localization as Strategies in Scientific Research*, Princeton (NJ), Princeton University Press, 1993; and C. Craver & W. Bechtel, "Mechanism", *ed. cit.*, pp. 469-478.

important and decisive role. This is the paradigmatic case of the so-called complex systems, that is, open systems with non-linear structures and dynamics, which can exhibit causal feedback loops and self-organizing processes, generally ignored by the classical mechanistic science.

Here come the fundamental problems of mechanism. First, "[t]he more intrinsically integrated and the more nonlinear a self-organizing system is, the more localizability and decomposability will fail"[22]. Bechtel and Richardson have recognized this problem: may there be types of systems in which a mechanistic analysis, even if properly renovated and updated, might fail? Perhaps in systems where components are not sufficiently distinct, and where the behavior to be explained is generated above all by the structural and functional organization of the component's interactions, in terms of an "*emergent mechanism*"[23].

For example, how can a highly interactive non-linear complex system as the brain be subject to such decomposition and localization processes? How can one identify the specific contributions of each individual brain area when we know the inherent plasticity of such a system?

This is one the main objections to the mechanistic project, raised by the supporters of the so-called dynamic approach, inspired by the modern dynamical systems theory, and therefore using its mathematical methods, tools and geometric concepts: differential equations, phase (or state) spaces, trajectories, attractors, basins of attractors, landscapes, bifurcations, etc. A dynamic systems model takes the form of a set of evolution equations that describe how the state of the system changes over time. Therefore, its explanatory process will focus on the internal and external forces that shape the different phase trajectories[24].

The question is whether the organizational complexity and the highly interactive non-linear dynamics that characterize a system as the brain constitute an insurmountable barrier to any decomposition and localization heuristics, or if it only confronts us with the need to further develop a new mechanism.

According to Stuart Glennan, the kind of problems mentioned above do not give reason to give up of a mechanistic approach, but instead they are a good reason to acknowledge the failure of some traditional explanatory

[22] A. Chemero & M. Silberstein, "After Philosophy of Mind: Replacing Scholasticism with Science", *Philosophy of Science* 75 (2008), p. 18.
[23] W. Bechtel & R. Richardson, *Discovering Complexity*, ed. cit., pp. 199-244.
[24] Cf. E. Thompson, *Mind in Life. Biology, Phenomenology, and the Sciences of Mind*, Cambridge (Mass.), Harvard University Press, 2007, p. 11. See also R. F. Port, "Dynamic Systems Hypothesis in Cognitive Science", *in* L. Nadel (ed.), *Encyclopedia of Cognitive Science*, vol. I, London, Nature Pub. Group, 2003, pp. 1027-1032.

strategies and to develop a new type of analysis able to take into account "emergent mechanisms"[25]. We should seek alternative ways to develop a mechanistic program, rather than to just give up from any kind of *causal* explanation.

According to W. Bechtel, there are some reasons for optimism. On the one side, the new mechanism is consistent with the discovery of a large number of complex interactions that give rise to non-linear dynamics. On the other side, it should be stressed that it is precisely when we recognize the parts of a given system and their interactions that we are in a position to apply the tools of dynamic analysis. Likewise, it is when researchers begin to formulate hypotheses about the different contributions of particular areas of the brain that they are able to discover its plasticity and to incorporate it into their theories. So, it seems there is more complementarity than contradiction between mechanistic and dynamic approaches[26].

Due to the pressure felt by the clash of old models of analysis of non-linear phenomena, some researchers began to try to explain the complexity of biological and neural systems with a set of more sophisticated tools. The component processes in complex systems often behave in a non-linear way, and the non-linear interactions that they involve often generate emergent and unpredictable phenomena. In this sense, the identification of a mechanism will become a condition of possibility for the very practice of a dynamic analysis, and both can complement rather than oppose each other. Bechtel, for example, notes that both perspectives can support each other to the extent that the information about the mechanisms and their components can help us to better formulate the dynamics and, conversely, dynamic analysis can help us to discover the most relevant mechanisms[27]. Moreover, even advocates of dynamic approach, such as Kelso and Engstrom, point out that dynamic *patterns* require pattern *generators* – i.e., causal mechanisms – and vice versa[28].

The challenge always seems to be this one: the causal explanation of how the system behaves lies mainly in *how* its component parts and operations are *non-linearly organized*.

The natural attitude of the researcher seems to be always starting by assuming that the internal relations of a system, and its state changes, can be addressed in an additive or linear manner. First of all, this approach is

[25] Cf. Glennan, "Mechanisms", in S. Psillos and M. Curd (ed.), *The Routledge Companion to Philosophy of Science*, London-New York, 2008, pp. 382-383.
[26] Cf. W. Bechtel, "Decomposing the Mind-Brain: A Long-Term Pursuit", *Brain and Mind* 3 (2002), pp. 239-241.
[27] Cf. *Idem*, p. 240.
[28] Cf. Kelso & Engstrom, *The Complementary Nature*, Cambridge (MA), MIT Press, 2006, p. 95.

obviously the easier one. So we prefer to study systems where its various component parts and operations can be treated as if they followed a linear or sequential order of steps, so that the contributions of each component could be analyzed separately. According to Craver and Bechtel, in part this may be common because human conscious cognitive activities are typically serial – "humans proceed from thinking of one thing to thinking of another"[29].

But this is not always true. Natural systems are not always organized in this way. Operations, actions, interactions, relations and functions are often dependent on each other, making it impossible to understand the behavior of many phenomena by imposing a linear order. Relations between parts are not always mere interactions, but causal interpenetrations that transform them - their activities and behaviors being dependent on each other. In such cases, the behavior of a system depends on the non-linear interactions of its component parts and operations at different hierarchical levels of organization[30].

In any case, it seems that the distinction between the different component parts, areas or regions of a system, and the identification of its various operations and functions, is absolutely necessary in order to be able to causally explain the genesis, the evolution and the behavior of any composed system. Otherwise, we risk falling into a simple *phenomenological description* at the macroscopic level of the system taken as a whole, being able to say *what* happens, but unable to explain *why* it happens so.

One thing at least seems obvious. Whatever the success of such a renovation project of mechanism to reach the explanation of the various complex systems, the explanatory model cannot clearly be thought in terms of the old "Lego-philosophy of atomism or mechanism"[31].

We need to assume right from the very beginning a non-linear way of approaching reality – in other words, we need to assume from the very start a fundamentally organizational and interactive way of thinking about our world.

[29] Cf. C. Craver & W. Bechtel, "Mechanism", *ed. cit.*, p. 474.
[30] Cf. W. Bechtel & A. Abrahamsen, "Explanation: A Mechanist Explanation", *ed. cit.*, p. 435; C. Craver & W. Bechtel, "Mechanism", *ed. cit.*, p. 472; and Glennan, "Mechanisms", *ed. cit.*, pp. 378-379.
[31] A. Chemero & M. Silberstein, "After Philosophy of Mind: Replacing Scholasticism with Science", *ed. cit.*, p. 19.

4. MECHANISM AND QUANTUM PHYSICS

Nevertheless, the tension between the linear (atomistic) and the nonlinear (organizational) worldviews does not arise only when we talk about life and mind. The problems generated by contemporary Physics to the classic mechanist doctrine and, particularly, to its ontological assumptions are not new. For far too long it was observed that even during "the period of the greatest triumphs of mechanism, Physics began to grow in new directions, tending to lead away from the general conceptual framework that had been associated with the original form of the mechanistic philosophy"[32]. D. Bohm mentions in this context Maxwell's theory of electromagnetism, the kinetic theory of gases, and the initiation of the use of statistical explanations for the laws of thermodynamics and other macroscopic properties of matter. Above all, the introduction of the Faraday-Maxwell concept of the electromagnetic field was a decisive turning point to a new physical conception of material reality that – according to Ernst Cassirer – clearly defies "the assumption that every whole must admit of being understood as the 'sum of its parts'":

> "(…) the field itself no longer admits of being understood as a merely additive whole, an aggregate of parts. The field is not a thing-concept but a concept of relation; it is not composed of pieces but is a system, a totality of lines of force"[33].

At this point Cassirer refers to Hermann Weyl's interpretation:

> "For field theory, a material particle, such as the electron, is merely a small area of the electrical field in which the strength of the field assumes enormously high values and where there is, consequently, an intense concentration of field force. This scheme of the world reduces to a complete continuum. Even atoms and electrons are not ultimate unchanging elements shoved willy-nilly by bombarding forces of nature, but are themselves subject to continuous, extended, and delicately flowing changes"[34].

Thus, the presumable *stoicheia* of physical reality are no longer conceivable as Parmenidean entities and as atomistic parts immune to their interactive contexts. The Atomistic-Eleatic mechanistic doctrine of classic

[32] D. Bohm, *Chance and Causality in Modern Physics*, ed. cit., pp. 39 ff..
[33] E. Cassirer, *The Logic of Humanities* [1942], New Haven and London, Yale Univ. Press, 1966, p. 166.
[34] H. Weyl, *Was ist Materie?*, Berlin, J. Springer, 1924, p. 35.

materialism collapsed at the very level of quantum physical reality. The new ontology is, in its essence, organizational. The world is a universal system of interrelationships, and what we conceive as individual entities are organized systems of local relationships that behave as *relata* in relation to other organized systems of local relations.

This, however, does not mean that the essentially linear approaches have been overcome in Physics. According to D. Bohm, the problems that Physics faced during the nineteenth century were resolved by means of a series of accommodations and modifications, which retained, however, the essential characteristics of the classical philosophy of mechanism[35].

We will not seek to analyze here the history of the theoretical developments of Quantum Physics and its relation with the history of the mechanistic theory. Let's rather see how the new philosophy of mechanism relates to the new *causal, local and non-linear Quantum Physics* (henceforth NQP), as developed by J.R. Croca[36], in the spirit of Broglie's work.

4.1. THE COMPLEX AND NONLINEAR NATURE OF QUANTUM ENTITIES

How does NQP relate to the (new) mechanistic model? On the outset, it seems that this question can get two different and contradictory answers. On the one hand, the rehabilitation of *locality*, *individuality* and *causality* (against the standard Copenhagen view) obviously meets the demands of one of the most basic requirements of classic mechanism. However, the decision to take a radical *non-linear* approach will find the same kind of problems that we already mentioned, regarding the confrontation between the study of complex systems and the mechanistic model of analysis and explanation.

Orthodox Quantum Physics is based on 'Fourier ontology', according to which the basic building blocks of the universe are *harmonic plane waves*, *infinite in time and space*. Therefore, orthodox Quantum Physics is essentially based on a linear, non-local and non-causal approach to physical reality. Rejecting Fourier's Ontology, the NFQ affirms the possibility of conceiving quantum particles as local systems in space and time, described by a new mathematical tool called *wavelet local analysis*. The quantum particle is mathematically described by a wavelet solution to a non-linear master equation that describes both extended and localized properties.

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[35] Cf. D. Bohm, *Chance and Causality in Modern Physics*, ed. cit., pp. 65-67.

[36] Cf. J. R. Croca, *Towards a Nonlinear Quantum Physics*, London, World Scientific, 2003; and J. R. Croca e R. N. Moreira, *Diálogos Sobre Física Quântica. Dos Paradoxos à Não-Linearidade*, Lisboa, Esfera do Caos, 2007; and J. R. Croca, "Local Analysis By Wavelets Versus Nonlocal Fourier Analysis", *International Journal of Quantum Information*, Vol. 5, N^{os} 1 & 2 (2007), pp. 88-95.

This new conception of quantum particles as local entities in time and space fits well in the classic mechanistic model, since it establishes the discrete nature of the so-called building blocks of the universe. However, these particles are now conceived as very complex, dynamic and non-linear structural organizations. That is, the supposed elementary micro-entities of quantum physical reality are *not* elementary, but have instead a very complex internal structure.

What was once called a quantum particle, sometimes observed as a corpuscular entity, sometimes observed as an extended entity (a wave), is now defined as a complex system actually *composed* of an extended, yet finite, part – the *theta wave* – and a very localized (ideally punctual) structure – the *acron* –, both immersed in a *subquantum medium* called 'chaotic' because we do not know (so far, at least) its component parts, and therefore their dynamics and their causal relationships.

In this model, the acron represents the area of greater energy density of the quantum particle system. However, through a non-linear causal process, the acrons are guided by their theta waves to the areas where the theta waves have higher energy intensities. Quantum particles are thus conceived as complex, dynamic and non-linear systems. To take into account their nature and behavior we need to explain:

– not only the structural interactions among acrons and theta waves, through which they influence and modify reciprocally;

– but also the interactions between each quantum particle (as a system composed of an individual acron and its theta wave) and its external environment, that is, its relation with other quantum particles, and with the subquantum medium.

Nevertheless, although complex and non-linear, these systems lend themselves to a conceptual decomposition and localization – operations required, as we saw, by any mechanist analysis. Otherwise, how could we *identify* and *distinguish* between the different natures and behaviors of the two component parts of each quantum particle (the acron and the theta wave), and how could we *locate* their different causal contributions to the global behavior of each particle as a complex system?

If there is something in this approach that goes directly against the traditional mechanist model is rather the fact that *organization* plays the crucial role on the behavior of physical entities, and the fact that structural organizations formation always involves the occurrence of *non-linear causal processes*.

This means that Physics, and the new mechanism, must acknowledge the existence of other kinds of causality, beyond the traditional efficient and material causes. We refer here to the well-known *formal* (= structural or organizational) causality that contemporary dynamic approaches also

accommodate in an explicit way. Besides the causal power exercised by the material component parts, we must also take into account the causal power exercised by the *structural organization* of those component parts on the overall behavior of the system, and on their own individual behavior as well.

Robert Bishop, for example, recently addressed the well-known problem concerning what is customary called 'downward causation' (better named *structural causation*), under the light of an analysis of the Rayleigh-Bénard convection phenomenon, concluding that the constraints exercised by Bénard cells on the movements of individual fluid elements – reducing the number of degrees of freedom of their dynamics – act "as formal or structural causes"[37]. Although the fluid elements and their individual dynamic are necessary for the structural organization's formation and for the Bénard cells dynamics, they are *not* sufficient. Hence the need to appeal to the notion of formal causality.

That is, one feels the need to take into account not only the causal contribution of the elements, as component parts of a given system, for the existence of that very system, but also the causal power exercised by the system's structural organization where ultimately lies the explanation of the very existence, behavior and evolution of the system as such.

But the acknowledgement of such kind of formal causality is hardly surprising, given the planned new mechanism. As Chemero and Silberstein noted, unlike the old mechanistic doctrine, "modern mechanistic explanation places much more emphasis on the dynamic organization, contextual features and interrelations of a given mechanism"[38].

The formal cause, nevertheless, is not the only kind of Aristotelian cause that contemporary science, to a certain extant, rehabilitates – providing, evidently, the revision and the liberation from some of the metaphysical assumptions attached to their original definitions. We mean, of course, the final causality. This topic will lead us to the end of this brief essay on the challenges currently facing the mechanistic philosophy of science within a nonlinear and dynamical conceptual approach to reality.

[37] R. Bishop, "Downward causation in Fluid Convection", *Synthese* 160 (2008), p. 242.

[38] A. Chemero & M. Silberstein, "After Philosophy of Mind: Replacing Scholasticism with Science", *ed. cit*., p. 18.

4.2. THE PRINCIPLE OF EURHYTHMY: FINALITY AND SELF-ORGANIZATION

Within the framework of its NQP, J.R. Croca has more recently proposed a general unification of Physics, rooted on a general principle called eurhythmy[39].

This principle is defined as an "organizing fundamental principle" of the behavior of physical entities, according to which all quantum systems spontaneously tend to their self-preservation through their orientation to regions where the energy density is higher. This is a physical principle of economy and optimization regarding the individual existence of physical entities, and it is directly linked to the model of quantum particle proposed by the NFQ.

As we saw, according to this model, a quantum particle is a complex system consisting of an extended part (the theta wave) and a very localized part (the acron). The energy of the particle is almost entirely concentrated in the area of the acron. According to Louis de Broglie, the motion of a corpuscle is determined by the wave propagation (*onde pilote*, *pilot wave*) to which it is associated, according to the so-called guiding formula (*formule du guidage*). Accordingly, through a non-linear causal process, the corpuscle is guided by its wave to those regions where the energy intensity of the wave is higher. However, according to J. Croca this physical process is quite complex and therefore it must be deeper analyzed. When moving randomly in his theta wave field the acron follows, *in average*, the best possible path, the most adequate path – that is, the one that leads to where the theta wave has higher energy intensity. In this sense, one can say that the acron usually avoids the regions where the intensity is null, because in these regions its very existence is in danger, in the sense that in such zones the acron needs to regenerate the surrounding field at the expense of its own energy. In short, among all possible trajectories of their phase spaces, acrons tend preferentially to move to areas where there is a greater intensity of energy, where their persistence will find the best conditions to be ensured.

This principle of eurhythmy is neither an *a priori* statement nor it is deterministic. On the one hand, it is presented as a general principle, which particular cases are the well-known extremal principles of classical mechanics (as the Heron of Alexandria's principle of minimum path, and the principle of minimum time of Fermat, soon followed by Maupertuis's principle of least action), and the guiding principle of Broglie. On the other hand, as it was said, this principle is exemplified in *statistical* terms. Nothing determines that an eurhythmic behavior always takes place in all circumstances. It just says that there is a natural *tendency* that conducts the

[39] Cf. J. R. Croca, "The Principle of Eurhythmy. A Key to the Unity of Physics", in *First Colloquium for the Philosophy of Science: Unity of Science, Non traditional Approaches*, Lisbon, October, 25-28, 2006.

acron to follow a *stochastic* path that in average leads it to the regions where the intensity of the wave field is greater.

This principle thus asserts the existence of an inherent tendency for physical systems to keep their self-preservation and persistence. Thus defined, this principle of eurhythmy seems to be the physical expression of the metaphysical principle of *conatus* formulated by Spinoza. As the philosopher wrote in his *Ethics*, "*Each thing, as far as it can by its own power, strives to persevere in its being*"[40].

This is an immanent principle of self-organization of Nature simultaneously *naturans* and *naturata*. In the chapter V of the *Short Treatise*, Spinoza referred to this *conatus* – this strife, this effort – in these terms: "the striving which we find in the Nature as a whole, as well in the individual things, to maintain and preserve their own existence. For it is manifest that no thing could, through its own nature, seek its own annihilation, but on the contrary, that every thing has in itself a striving to preserve its condition and to improve itself"[41].

Despite the obvious similarity between the notions of *conatus* and *eurhythmy* – they are both immanent principles of self-persistence –, there is at least one crucial difference: the first one constitutes a deterministic principle, while the second one represents a statistic pattern of behavior. Nevertheless they both represent principles of *self*-organization to the extent that they refer to a spontaneous behavior, in the sense that it is not programmed, nor teleologically oriented. On the other hand, the very notion of self-organization requires, or implies, the notion of a process leading to self-persistence: self-organization means that something is readapting or rearranging itself in order to persist.

In this sense, can we say there is a kind of *finality*, be it deterministic or not? The language of 'striving' can be taken to suggest something like that. It seems problematic to understand the principle of *conatus* within a philosophy that strove to be as radically as possible mechanistic and anti-teleological. What kind of purpose is therefore in question? How should be understood the relationship between teleological or purposive explanations and mechanistic explanations? Can the final causality be reconciled with a mechanistic philosophy?

[40] "*Unaquaeque res, quantùm in se est, in suo esse perseverare conatur*." (Spinoza, *Ethica*, III, prop. 6, in *Opera* im *Auftrag der Heidelberg Akademie der Wissenschaften, her ausgegeben von Carl Gebhardt*. Heidelberg, Carl Winter, vol. II, 1972 (1924), p. 146).

[41] Spinoza, *Korte Verhandeling van God de Mensch en deszelfs Welsatnd*, I, cap. V, in *Opera*, ed. cit., vol. I, 1972 (1925), p. 40.

As it is well-known, the problem of finality and of teleological reasoning has always been to look for, or to place, the causes *after* – and *not* before – their affects.

It was partly to solve this dilemma that in Biology the idea of a program recorded in the genes by natural selection was developed – it should serve to liberate Biology from fatalism and vitalism. The basic idea was to replace teleology by *teleonomy* (an expression created by Piitendrigh, and later resumed by authors like E. Mayr and J. Monod): intentional processes must be replaced by mechanic and mathematizable processes. Modern biology has been devoted to separating intentionality from finality, using the metaphor of the computer program and theories of biological adaptation in the context of neo-Darwinian evolution, and models of self-organization. The purposefulness and goal-directedness exhibited by some systems must therefore be the expression of the development of some special kind of causal-mechanistic processes.

The branch of Physics called Mechanics always involved principles of evolution, in which the final state plays a decisive role in the description of temporal processes. However, the final mechanical causality pretends to be different, and in fact it can be found in the well-known *extremal principles*, as the principle of least action or the principle of maximum entropy. The only purposefulness rationally acceptable, in Biology as in Physics, would then be a mechanistic one – that is mathematizable, calculable and unintentional. The goal or the purpose – as Atlan noted – is not even generally defined as such since it is expressed in the form of principles of optimization, that is, in terms of minimum and maximum, stating that this or that magnitude, expressed by a mathematical function, must attain a maximum or a minimum at the end of a process. For example, the free energy of some substances that can perform chemical reactions will be minimal at the end of the reaction, or the entropy of an isolated spontaneous process will tend towards a maximum. These principles allow estimation by calculating the evolution of certain procedures and do not imply, in any way, a consciousness or an intentionality that drives them. Moreover, these principles of optimization in Physics are always limited to very well defined domains of validity. For instance, the principle of entropy growth applies only to closed and isolated systems. As Atlan says, there is here a kind of 'optimal mechanism'[42].

Therefore, if there is purposefulness and goal-directedness, and if these phenomena must be conceived from a mechanical point of view, then any teleonomic behavior performed by a system must be explained on the basis of its *structural organization* – that is, in terms of the network of

[42] Cf. H. Atlan, *Tout, non, peut-être: Éducation et verité*, Paris, Seuil, 1991, chap. 2.

interactions between its component agents, and their causal-mechanical dynamics. In this sense, self-organizing mechanisms in biological, chemical and physical systems will *causally* explain the occurrence of teleonomic processes. The basic idea is that teleonomic processes must be conceived as effects of previous formal (organizational) causal-mechanical processes.

Partly caused by the impact of cybernetic studies on servo-mechanisms and auto-regulatory machines, and thanks to developments in physical and mathematical techniques, the classical notion of causality has been reformulated, involving the acknowledgement of feedback and feed-forward causal processes.

As Jean Piaget has noted, while Biology was trying to get free from its restricting mechanistic ideas, and when some thinkers, confronted with this deficiency in traditional physical causality, were toying with the idea of a return to Vitalism and old finality, a *complete re-elaboration of the mechanistic approach* opened up new perspectives along lines which corresponded exactly to the notions of circular or feedback systems or of cyclic rather than linear causality[43]. In short, according to Piaget, "we can today retain all that is positive in the idea of finality but at the same time replace the notion of a 'final cause' by an intelligible cyclic causality"[44].

5. SOME PHILOSOPHICAL LESSONS

There are some philosophical lessons to learn from all this. Whatever the challenges that a new mechanistic model of analysis and explanation still has to face, one thing seems certain: we cannot go back in time in order to resurrect the classical mechanism of the ideal and archaic Atomistic-Eleatic worldview of linearity. No serious ontological thought is nowadays possible if it does not incorporate the interactive, organizational and dynamic nature of reality.

However, it is not enough to appeal to a mere recognition of the existence of relationships or interactions. The Greek Atomists themselves

[43] Cf. J. Piaget, *Biologie et Connaissance. Essai sur les relations entre les régulations organiques et les processus cognitifs*, Paris, Gallimard, 1967, pp. 183-189. We find the same kind of analysis in the work of Ludwig von Bertalanffy, e.g., *Problems of Life: an Evaluation of Modern Biological Thought*, London, Watts & Co., New York, J. Wiley & Sons, 1952 [*Das biologische Weltbild: Die Stellung des Lebens in Natur und Wissenschaft*, Bern, Francke AG.,1949].

[44] *Idem*, p. 189. Piaget wrote 'causalité à boucles' (= cyclic causality through causal loops).

admitted the existence of relations among their atoms (in terms of their local movements, collisions, spatial disposition, their order, composition or arrangement), and still that didn't prevent them from developing a fundamentally *linear* conception of reality strongly supported by the Eleatic Metaphysics. The widespread rhetorical use of notions such as relationship, arrangement, interaction, is not enough to omit this central ontological question: How can we defend and promote linear explanatory and reductionist approaches *if* we seriously acknowledge the existence of relationships as genuine interactions, so that the component agents (parts or causes) behaviors are influenced (changed and/or constrained) by those very relations?

It is not possible to admit this *mutability*, this *interactive context-dependency* of the elements' behaviors, and at the same time stubbornly insist on the obsolete paradigm of linearity and on their associated epistemological methods of analysis and explanation. Consider the following statement of Jaegwon Kim:

> "supervenience theses, when applied to the layered model, turn into claims of mereological supervenience, the doctrine that properties of wholes are fixed by the properties and relations that characterize their parts. A general claim of macro-micro supervenience then becomes the *Democritean atomistic doctrine* that the world is the way it is because the microworld is the way it is"[45].

As it is easy to see, Kim is here thinking about 'relations' in a linear or additive way, not as relations in the sense of genuine interactions irreducible to the intrinsic properties of their *relata*. Otherwise, it would make no sense to mention the sponsorship of the ontological doctrine of Democritus...

Therefore, it is not enough to admit the existence of relationships in order to elaborate a truly relational or interactive perspective. As Richard Lewontin and Susan Oyama very strongly emphasized, interactionism is 'the beginning of wisdom', 'a first step in the right direction', but in its traditional version interactionism is not enough.

It is worth reading the following passage from Lewontin's foreword to the recent new edition of Oyama's *The Ontogeny of Information*:

> "The usual interactionist view is that there are separable genetic and environmental causes, but the effects of these causes acting in combination are unique to the particular

[45] J. Kim, *Mind In a Physical World*, Cambridge (Mass.), The MIT Press, 1998, p. 18 (italics mine).

> combination. But this claim of the ontologically independent status of the causes as causes, aside from their interaction in the affects produced, contradicts Oyama's central analysis of the ontogeny of information. There are no 'gene actions' outside environments, and no 'environmental actions' can occur in the absence of genes. The very status of environment as a contributing cause to the nature of an organism depends on the existence of a developing organism. Without organisms there may be a physical world, but there are no *environments*. In like manner no organisms exist in the abstract without environments, although there may be naked DNA molecules lying in the dust. Organisms are the *nexus* of external circumstances and DNA molecules that make these physical circumstances into causes of development in the first place. They become *causes* only at their nexus, and they cannot exist as *causes* except in their simultaneous action. That is the essence of Oyama's claim that information comes into existence only in the process of ontogeny. It is this claim about causes that Oyama, in this new edition, calls 'constructivist interactionism', but that I would characterize as *dialectical* in order to emphasize its radical departure from conventional notions of interaction"[46].

Classical interactionism errs in treating *relata* as separated and independent elements, neatly separated from which other, rather than as interpenetrating and interdependent entities.

As we saw, even the very last level of reality known so far, the quantum domain, constitutes a specific level of organization of physical reality whose start-protagonists can no longer be taken as Atomistic-Eleatic units, but rather as organized systems of local relations, more or less stable, that interact through a network of reciprocal interactions and dependencies within the subquantum medium (or as is usually named the zero point field).

As noted above, this suggests the idea that our basic ontology must be defined in an organizational perspective, where relations and individual entities should be taken as items of equally fundamental ontological status. On this view, individual entities (particular things) are nothing but organized

[46] R. Lewontin, "Foreword" to S. Oyama, *The Ontogeny of Information. Developmental systems and evolution*, Durham (North Carolina), Duke University Press, 2000 (2th ed. revised and expanded), pp. xiv-xv. See also R. Levins, "Dialectics and Systems Theory", *in* R. Lewontin and R. Levins, *Biology Under the Influence*, ed. cit., pp. 101-124; and S. Oyama, "Terms in Tension: What do we do when all the good words are taken?", *in* S. Oyama, Paul E. Griffiths, and R. D. Gray (ed.), *Cycles of Contingency. Developmental Systems and Evolution*, Cambridge (Mass.), MIT Press, 2001, pp. 177-193.

systems of local internal relations with external relations with their environments – that is, with other organized systems of local relations. Things and relations are definable and characterizable only in terms of their reciprocal interdependence. What we conceive as an individual entity is nothing but an organized system of local relations that behaves as a *relatum* when we consider it in his relations with other organized systems of local relations – like organisms and their environments.

As history of philosophy and science shows, the different mechanistic perspectives largely depend on the ontologies that they explicitly assume or presuppose. That is why Stuart Glennan recently noted that the new mechanism, while retaining much of the methodology, explicitly rejects the metaphysics of the classical mechanistic doctrine. Contemporary mechanism is not associated with the mechanistic doctrine as championed by Galileo or by philosophers such as Descartes or Boyle[47]. We need to overcome the old atomistic materialism in order to get free from its mechanistic doctrine.

But this quest is not so recent. Already in 1927, the American emergentist philosopher Roy Wood Sellars wrote:

> "The (…) doctrine which I identified with traditional materialism was the emphasis upon *stuff* rather than upon *organization*. Clearly this emphasis was an expression of the *mechanical* ideal itself in its *atomic* form. If relations are external and integration not recognized as intrinsic and strategic, the stuff of nature simmers down to the bare elements which are tossed hither and thither like flotsam and jetsam. Pattern and fibres of connection are ignored; the whole is but the parts or, to put it more exactly, there is no whole (…). There is heterogeneity, qualitative diversity in the material world. *The truth of materialism was in its naturalism more than in its oversimplified ontology*"[48].

From an ontological point of view, non-linear approaches represent the assumption of an organizational ontology, essentially based on a dialectical and dynamical way of dealing with the reality. But this tension between the organizational and the Atomistic-Eleatic worldviews is not, of course, new. We can trace its path through the history of philosophy, starting from the opposition of Aristotle to the Greek atomistic and eleatic doctrines:

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[47] Glennan, "Mechanisms", *ed. cit.*, pp. 376-377.

[48] R. W. Sellars, *Principles of Emergent Realism: Philosophical Essays* (ed. by W. Preston Warren), St. Louis (Missouri), Warren H. Green, 1970, p. 138 [orig. publ.: "Why Naturalism and Not Materialism", *Philosophical Review* 36 (1927), pp. 216-225] (italics mine).

consider the different kinds of opposition of Spinoza, Leibniz and Diderot to the modern mechanistic worldview developed by Galileo, Descartes and Boyle; the development of the Emergence theory between the middle of the nineteenth century and the first decades of the twentieth century; or the arising of structuralist, organicist or systemic kinds of approach (between the 30s and the 70s of the XXth century) as developed in the domains of Epistemology and of Philosophy of Biology by authors such as Ernst Cassirer, Jean Piaget, or Ludwig von Bertalanffy.

Notably, the close relationship between the non-linear perspective and the Emergence theory is indeed easily understandable, since the well-know classical definition of emergence was given (by Stuart Mill) in terms of 'non-additive' relationships between different causes: an emergent phenomenon represents the case where an ('heteropathic') effect is not the sum of what would have been the effects of each cause acting alone or in other type of relational context. In such kind of cases, the 'principle of composition of causes' fails. That is, the principle of the proportionality of effects to their causes (the superposition principle) fails. Emergent or non-linear phenomena cannot be understood and explained in terms of the traditional analytic methods of science. No wonder that this notion of emergence as a function of the non-linear joint action of different agents (causes or parts) is widely used nowadays by complex systems researchers, such as S. Kauffman, W. Wimsatt, J. Holland, D. Rumelhart, J. McClelland, W. Bechtel, or H. Atlan[49].

As it was said above, in a positive sense, to adopt a non-linear approach to reality is to assume an essentially organizational and dynamical perspective about our natural world – it is in short to assume the point of view of a *dynamic structuralism*. And since the old Atomistic-Eleatic metaphysics, on which the paradigm of linearity of classical materialism has been founded, is no longer tenable, such a new non-linear perspective on reality must be assumed right from the start.

[49] Consider only these two brief characterizations: "[e]mergence is above all a product of coupled, context-dependent interactions. Technically these interactions, and the resulting system, are *nonlinear*. The behavior of the overall system cannot be obtained by summing the behaviors of its constituent parts." (J. Holland, *Emergence. From Chaos to Order*, Oxford, Oxford Univ. Press, 1998, pp. 121-122); "(...) *emergence of a system property relative to the properties of the parts of that system indicates its dependence on their mode of organization.*" (W. Wimsatt, "Aggregate, composed, and evolved systems: Reductionistic heuristics as means to more holistic theories", *Biology & Philosophy* 21 (2006), p. 673). This is why Wimsatt rightly conceives aggregativity as the complete antithesis of emergence.

THESES TOWARDS A NEW NATURAL PHILOSOPHY

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Summary: In this paper I address some philosophical questions regarding the impact quantum mechanics has in the classical conceptions about reality and knowledge. I stress that onto-gnosiological realism still is an option to the issues regarding the relationship between knowledge and reality. Rejecting some radical aspects of Copenhagen interpretation of quantum formalism, I emphasize the advantages of de Broglie's realistic and causal model. To finish with, I discuss the limits of the Cartesian concept of matter and the split between matter and mind.

Keywords: Realism, Quantum Mechanics, Gnosiology, Natural Philosophy.

Philosophers dramatically misunderstand the sense of their task when they embrace a kind of scholasticism, writing entire books about the opinions of other distinguished philosophers instead of talking about the things themselves. Unfortunately, this has become a widespread situation since philosophy was confused with the history of philosophy. This kind of category-mistake was first committed by Hegel. After him, in the late 19th century, with Dilthey and others, it degenerated into a historicism of sceptical and relativistic guises. However, before philosophy came to this regrettable situation, philosophers like Descartes, Leibniz, Kant –or even Helmholtz, to mention only a case of a nineteen-century philosopher of non-Hegelian extraction– were concerned with the formidable task of producing an organized and sound understanding of reality. They were not worried about erudite questions concerning the history of philosophy, nor did they come to their own opinions reading the philosophers of the past. The lack of historical erudition was a common feature of thinkers and natural philosophers like Bruno, Galileo, Descartes, Newton or even Kant. Instead, they were fully aware of the scientific atmosphere of their time. Indeed, it was the

accuracy of their answers to problems concerning our knowledge of reality that gave them the label of classical scientific thinkers and philosophers.

Surely, I am not making a case for ignorance and lack of relevant information. I am only suggesting that the kind of dialogue we should have with other philosophers is one which proves capable of improving our knowledge of reality – that is: a dialogue not for the sake of interpretation, but for the sake of truth.

This productive attitude towards the intellectual legacy of philosophy is particularly appropriate in our times. In fact, we are experiencing the aftermath of an intense intellectual turmoil, which has taken place in the 20th century and has shaken the very foundations of the classical image of reality that three hundreds of continuous scientific developments had given us. The turmoil started in the first three decades of the last century. It was centred on the conception – or a lack thereof – of reality that recent advancements in physics and cosmology were suggesting. This debate about the ideas that new physics and cosmology were dramatically changing and reshaping engaged outstanding researchers like Planck, Bohr, Heisenberg, Einstein, Schrödinger, de Broglie, Bohm, de Sitter, Gamow, Hoyle, and so many others. The hard work they did was not related to physics or cosmology in a narrow sense. It was too deep and far-reaching to be locked-up in a particular science. In fact, it spread to domains like gnosiology and ontology, i.e., it entailed a revision –or at least a reappraisal– of such basic ideas as our general conceptions of reality, the material Universe and knowledge.

As a matter of fact, regarding our scientific knowledge and technical domination of nature, the 20th century was a time of astonishing progress. For the first time since the Greek split between *theoria* and *techne,* the capabilities for a technical transformation were systematically linked with science, sometimes benefiting from its achievements, at other times promoting them. These achievements embraced the smallest as well as the largest structures of nature – they went from the atomic and subatomic structures to stars, galaxies and the formation of the cosmos. Two new theoretical constructions guided this amazing progress: quantum mechanics, on the one hand, and, on the other, the complex and not-so-straightforward conglomerate of observations, hypotheses and hard speculations that extended from Einstein's *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie,* published early in 1917, to the Big Bang theory and the standard cosmological models of today.

Alongside this extraordinary growth, a baffling situation emerged: our classical image of nature was becoming more and more blurred but, despite the enormous work of the fathers of quantum mechanics and modern cosmology, nothing very clear, well defined and uncontroversial was put in its place. This is tantamount to saying that modern physics and cosmology

have put us in the middle of a crisis. Physics, particularly quantum mechanics in the prevailing "Copenhagen Interpretation" by Bohr and his associates, suggested a non-causal and rather indeterminist behaviour of atomic and subatomic entities, proposing a puzzling description of the deepest structures of matter. In fact, quantum jumps (assumed by Bohr since 1913, when he was in Rutherford's laboratory at Manchester), wave-particle duality and the complementarity-principle (Bohr's way out of the conundrum of the double-slit experiment), the collapse of the wave-function (attributing a kind of creative power to measuring acts), and the new interpretation of quantum probabilities (which was Bohr's appropriation of Schrödinger's equation), lead to a non-causal, rather indeterminist and mind-dependent conception of reality. The question concerning Bohr's interpretation of quantum mechanics, especially his considerations about the principle of complementarity, was not meant to bring forward a new understanding of the atomic and subatomic world, but to signal some unbreakable limits in the human efforts to understand nature. With quantum mechanics, we had eventually a mathematical formalism with extraordinary predictive capabilities but no clear interpretation of the *physical meaning* of these very formulae we managed to put together so well. Bohr himself wrote on this subject several times, so that his interpretation sounds as a farewell to a full understanding of nature as such. On the other hand, modern cosmology, based both on Einstein's equations of general relativity (without the cosmological constant) and on the interpretation of the galactic red-shift, discovered by Hubble, as a Doppler-effect, led to the idea of an expanding universe that at earlier times was much hotter and denser than today. Tracing this expansion backwards and supposing it was permanent, cosmologists constructed the hazardous hypothesis of a definite time in the past when the universe would have been infinitely dense. This time, the genuine "beginning" of the Universe, as we are told in a somewhat biblical fashion, has come to be called the "Big Bang", ironically the very same name Hoyle used for mocking it.

In a word, despite the independent paths they have treaded during the 20th century, quantum mechanics in "Kopenhagener Geist" and Bing Bang cosmology have together given an indelible sense of strangeness both to physical matter and to the Universe as a whole. Regarding cosmology, with the guiding hypothesis of an expansion, both the old cosmogonist ways of thinking and the idea of the creation of the material Universe were revitalized. Despite new questions, like those pertaining to the overall geometry of the Universe (if it is flat or otherwise) as determined by the value of the Omega Cosmological Parameter, we are now facing some old-fashioned problems like the alleged beginning and end of the Universe, its duration, its finite or infinite magnitude in space-time, and so on – problems

that remind us of some metaphysical queries of the past and also of the lessons Kant gave us about it in the Antinomies of his *Kritik der reinen Vernunft*. In addition to this, we are facing the perplexing situation of a singularity to which the laws of physics no longer apply, so that the very beginning of the Universe (earlier then the so-called "Planck epoch"), postulated by cosmology, is no longer a matter of physics but of some inexplicable processes that took place nobody knows how and why. This puzzling singularity is, then, the point where *physis* comes to an end and conceals both its nature and origin. On the other hand, the classical conception of physical matter as something fully determined and submitted to necessary causal laws has been strongly challenged by particle physics. Furthermore, with the complementarity-principle and the putative impossibility of attributing determined states to a system before measuring operations, some strong general ontological conceptions about what a being is and about the relationship between reality and consciousness (knowledge) were challenged; we do not know if the principle according to which every being is necessarily fully determined in all its states is actually a good principle to characterize reality or, on the contrary, an oversimplification based on the apparent constancy and completion of objects of our macroscopic perceptual world; we also do not know if we can retain a mind-independent, non-idealist conception of reality. As a result, we are comfortable neither epistemologically nor metaphysically or even scientifically, given the rather confused image we now have of the Universe we live in. As everyone can see, more than the right answers, we are in need of the right questions, because we do not know if contemporary trends in physics and cosmology are on the good paths at all. In face of so many conundrums and seemingly dead ends (particularly in the standard cosmological model), we do not need more physics and more cosmology conducted in the usual ways, but, instead of this, we need a lot of *critical discussions* about the very foundations of physics and cosmology and possibly some really new physics and some really new cosmology in the ways these critical discussions will pave for us. In a word: we need to turn back to the open space of philosophy.

For this is, in fact, a matter of philosophical inquiry, a problem for a natural philosopher, not as somebody who reads (all) but as someone who (incompletely) responds to the huge problems reality addresses to us. A *gigantomachia peri tes ousias,* i.e., a battle of giants concerning what is, as Plato wrote: this is what philosophy is really about.

So we will address the following questions:

In what relationship does knowledge stand to reality? Is reality conceptually independent of knowledge?

What is it to know? How can we describe the chain of cognitive operations that must take place, so that we can go from (the bare existence

of) *x* to the (cognitive) situation *x-is-know*? Do classical conceptions about knowledge still hold?

And what is a being after all, an *ens,* to take up again the Latin jargon of the old metaphysics and ontology? Can we conceive a being which is not fully determined regarding all possible predicates that can be attributed to it? Does causality hold universally if there is something like non-complete determination?

Finally, what is *physis*? Is *physis* fully understandable through the classical Galileo-Cartesian concepts of physics? And if not, what else do we need? In light of particle physics, can we have a cosmogenetic conception for *physis* as a whole?

1. Reality and Knowledge: in Search of a Conceptual Clarification

Reality is prior to knowledge. Yet, the traditional way of defining this priority (*viz*., reality is what it is regardless of being known or not) is not entirely accurate.

The word "prior" above means a relation determined by the semantic content of concepts alone, not a temporal relation or a genetic one between the things referred to by the concepts. So the proposition does not mean that reality comes before knowledge or that knowledge springs from reality. It means that, in order to define the concept of knowledge, we must presuppose the concept of reality and attribute some content to it that is independent of any relationship to knowledge. So we must define precisely what this independent content is.

To begin with, let us say something about the priority-relation that holds between reality and knowledge. It has several meanings that are interrelated. Nevertheless, they do not imply each other. Going from the weakest to the stronger, they are the following:

(i) *A particular sense of aboutness*. Like perceiving, imagining, supposing, and so on, knowing is always about something, i.e., it has a content we can describe. Unlike the other forms, however, if, say, it is true that *p* is known (for instance, if it is known that the Earth is a planet of the Solar System), then *p* is the case, i.e., "*p*" can be asserted as a true proposition. The same does not happen in cases like imagining that *p,* supposing that *p*, thinking, or even perceiving, because from the truth of "I suppose that *p*" it does not follow that *p* must be the case and that "*p*" can be asserted as true. Indeed, "I suppose that *p*" can be true and "*p*" false, or inversely. If I suppose that Scott was the first man to arrive at the South Pole, my

supposition is wrong, but it is nevertheless true that I have such a supposition. On the contrary, if I say that I *know* Scott was the first man to arrive to the South Pole, the falsity of the assertion implies that it is not the case that I have authentic knowledge. So, unlike the other cases (supposing, imagining, thinking, and so on), I cannot say here that "*p*" is false and that "I know that *p*" is true: if *p* is false I do not have any knowledge at all.

So only what is the case qualifies as an actual or possible object of knowledge, and, therefore, knowledge is always about what is. For instance, we cannot know anything about the Olympic gods because they do not exist. However, the cultural facts of ancient Greece are real, and we can know them, namely the stories about fictional characters like Zeus or Poseidon. As a general rule, we must say: if it is true that *p* is known, then *p* is the case (we cannot say yet what the criteria of true knowledge are). The negative judgments (e.g., "Earth is not a star") express no cognizance of unrealities as such, but a cognition that it is the case that something is not true (*viz.,* that "Earth is a star" is a false proposition). The sum of all that is the case is what we call here "reality" in a wide sense, including what is actual, possible, probable, and so on. Reality is what authentic knowledge asserts, even when this true knowledge is not about an actuality but, say, a possibility, a probability, or something else (if it is true that *p* is probable, then it is the case that this probability is a "real" state of affairs). Being real is, then, a necessary condition for being known. On the contrary, if one thing or state of affairs is real, it does not immediately follow that it is or can be known. So being-real is not a sufficient condition for being-known.

(ii) *Irreducibility*. Knowledge is about reality, but reality is irreducible to knowledge. This does not simply mean that there are always more aspects and domains of reality to be known. Such an assertion would be a matter of common sense and an everlasting but trivial truth. This does not mean either that there are absolute limits to knowledge. That would be a very controversial gnosiological thesis. The proposition that states here the irreducibility of reality to knowledge means that what is effectively known is always conceptually understandable and explainable in another theoretical framework. It states, then, that there is no ultimate theory that captures reality –a "theory mirror", so to speak– because there is no "true image of reality" as such, but only reality under a certain description in a determinate conceptual framework. For instance, gravitation phenomena can be conceptually grasped and explained in terms of concepts like force, action at a distance, absolute space, or in terms of space-time curvature, matter-energy density, tensors, and so on. The conceptual universes are pretty different, but the underlying phenomena are very much the same, with a somewhat

changing degree of observational accuracy determined by the conceptual and mathematical sophistication of each theory.

So we must distinguish between the conceptual and (if any) mathematical framework of a theory, on the one hand, and the underlying material nucleus, on the other hand, i.e., the material kernel of what the theory is about. We cannot say: "the underlying material *facts*", because a fact is almost always the function of a theory – it is already a reality under a determinate description. So, for a theory like the ancient Atomism, the falling-down of atoms in void was a fact. Atomists could even "see" it in phenomena like the fall of bodies on Earth. Nevertheless, for Newton's conception of an absolute, homogeneous physical space, there are no such facts as the falling-down of bodies: the facts are, instead of this, the uniform and accelerated motions due to the action of forces, namely the gravitational force, for the case of those fall-phenomena on Earth that experience immediately shows to us. Thus our description of the facts varies –or simply the facts vary– as the theories that imply them change.

So, if we look at reality as irreducible to any form of conceptual thinking, no matter how successful it can be, we must realize that there is no privileged form of access to it. The thesis that perception, as a supposed pre-conceptual, pre-theoretical experience, would be the original form for the presentation of reality –a thesis put forward, in phenomenology, by Merleau-Ponty and, to a certain extent, by Husserl himself– must be denounced as an illusion. This is so, firstly, because perception is already a conceptual grasping of reality, and, secondly, because perception, as a presentation of nature, entails a kind of "folk-physics", so to speak, as it suggests a conception of physical reality in terms of sensible qualities, the four elements, material bodies and forces, psychic causes for movements, and a lot of oppositions like day-night, earth-sky, plenum-void, etc. Perception is not an original or trustworthy, but an anthropological, culturally laden, conceptual and sensible presentation of reality: it has no special rights, neither regarding originality nor regarding truth. Considering reality by itself amounts to taking notice of certain recurrent regularities and patterns of organization as a basis for conceptual formation. For instance, some regular kinetic phenomena, like the behaviour of bodies on Earth's surface, give the empirical basis for different conceptual understandings and theoretical constructions, like Aristotle's theory of natural places or Newton's gravitation law, which are applicable to the same patterns of earthly kinetic regularity.

If, on the other hand, we look to the multiple forms of theoretical thinking, we must acknowledge its relativity, but not embrace relativism. Actually, theories are not equivalent, and the election of one is not a matter of free choice. There are constraints. The refusal of an "ultimate theory" (the irreducibility-thesis) inhibits us from talking about a progressive

approximation to the "true image" of reality. Nevertheless, this is not tantamount to accepting Kuhnian conceptions about the discontinuities of paradigm shifts and their incommensurability, or to talk about the supposed loss of theoretical explanatory power in the substitution of one theory for another, as pointed out by Laudan, while discussing birefringence in face of Huygens' and Newton's optical conceptions. There is progress in the succession of theories: that Newton's gravitational theory is more accurate than Aristotle's theory of natural places is a blatantly truth. Furthermore, partial losses of explicative power are compensated in the long run, as in the case of the return to an explanation of birefringence in terms of the wave-theory of light in the 19th century. In general, theories fit or do not fit a number of phenomena and regularities already known. Theories have or do not have the capability of predicting new phenomena. Theories can grow or not, and are able, or not, to assimilate other theories. These are just some of the constraints determining the election of one theory.

We started by saying that reality is what true propositions assert. We now reach a deeper understanding. Reality is what true propositions of competing theories are about, i.e., the underlying phenomena and regularities. Surely, phenomena are only expressible in a conceptual framework (as "facts of the theory"); nevertheless, phenomena are always able to say "no" to theories and explanations. In a rough sketch, we could say that propositions "A < B" and "B > A" express the same metric state of affairs between A and B. The state of affairs by itself is not expressible without the point of view of one or the other propositions. Still, it makes good sense to say that the state of affairs is none of them, but the –unique– thing the two propositions are about. The relation between reality and theoretical thinking is the same, with the qualification that some theories are better fitted than others both to express and to explain phenomena.

(iii) *Non-substitutability*. Every attempt to ascribe to knowledge an object that is immanent to it will misconstrue and destroy the very concept of knowledge.

In order to show this, let us considerer two classical cases that are instructive to several contemporary conceptions. British empiricism explained knowledge as a set of "operations of the mind" over its own sense-data, by means of which an indirect cognizance of an external reality was obtained, as John Locke put it. These directions in gnosiological thinking have led, with Hume and Berkeley, to a sceptical doubt about "external" reality and even to the denial of such a thing as a material, mind-independent world. As a matter of fact, when Berkeley put forward his thesis of immaterialism he was only extracting a radical consequence from the original presuppositions of Locke's empiricism: if knowledge is constructed as a set

of operations by the mind over sense-data, the very concept of a mind-independent, external reality will be pointless and fruitless. So when Berkeley said famously that a Law of Nature was not a description of some processes taking place in a material universe, but simply a rule for predicting a set of future sensations given a particular set of present sensations, he was just taking the final step in full accordance with empiricist gnosiology – the total substitution of objective reality by predictions about our own future subjective observations as the very object of knowledge. Maybe not so surprisingly, some formulations by Heisenberg about the kind of knowledge we have in quantum mechanics are not very far from this epistemic conception. As a matter of fact, since the very beginning, Heisenberg affirmed that his matrix mechanics was no more than an algorithm for correlating results of experimental observations and making new predictions on that basis. That is to say that his theory was unable to give some glimpses beyond the realm of our empirical observations – it was not a depiction of atomic reality. Surely, for Heisenberg and contrary to Berkeley's standpoint, there is a reality underlying sensible objects (empirical objects). But –he insisted– statements about the physical world are statements about what we can experimentally perceive and verify, and so, science has no good grips beyond this empirical level. These epistemic conceptions were so deeply rooted in Heisenberg's mind that he reaffirmed them in 1962, long after his controversy with Schrödinger's wave mechanics and the necessity he had of justifying the artificiality (from a physical point of view) of his matrix mechanics. Actually, in a text based on his Gifford-lectures, called *Physik und Philosophie,* he stated once more that scientific statements in quantum mechanics were not about an underlying reality, but about our own knowledge, i.e. about predictions over probabilities of future observational states.

The Kantian distinction between appearance and thing-in-itself was an attempt to overcome the sceptical trends stemming from the empiricist stance. The assumption of a mind-independent reality was preserved by defending the possibility of thinking (not of cognizing) a realm of things in themselves, while, on the other hand, the empiricist concept of sense-data was retained and transformed into the concept of an "appearing object" (*erscheinender Gegenstand*) for knowledge, i.e., something that is not a state of mind, a simple sensation (it is already an *object* – *Gegenstand*), but something that, nevertheless, has an intuition and concept-dependent objectivity (it is still an *appearance* to the mind – an *Erscheinung*). However, this Kantian conceptual distinction between a double way of considering things –as they are in themselves and as they appear to us– boils down to the following dilemma: either our cognition of appearances leads to the cognition of things as they are in themselves, so that the distinction

cancels itself out, or our cognition of appearances does not lead to a cognizance of things as they are, so that the cognition we have is a fake presentation of reality. There is no way out of this dilemma. The very opposition between reality as it is as such and as it appears to us must be overthrown, not to mention that we do not fully understand what is really meant by expressions like "as such" and "to us". A lot of conceptual work is in need here.

So empiricism as well as Kantian gnosiology are classical cases of a construction of the object of knowledge as something immanent to knowledge itself. The concept of immanence we use here does not signify sheer interiority. As a matter of fact, while sensations are internal in a psychological sense, Kantian phenomena are typically external objects in space and time (not considering another group of phenomena Kant calls "objects of the inner sense", i.e., states of mind, *Vorstellungen*). Nevertheless, sense-data and phenomena (more precisely: appearing objects) are cases of immanent objects in the sense of something that has no independent existence outside its relationship to knowledge itself. And this is just the point. If the general purpose of knowledge is only the prediction of subjective observational states, the concept of reality does not need a precise definition and can be characterized as the unknown cause of sensations in the knowing subject. This unknown cause can be a material universe as well as God's will or something else, as Berkeley emphasized (Berkeley himself supported the radical hypothesis of God's direct action without the mediation of a –for him superfluous– material universe). But this entails that the concepts of science lose their foundation, because no relationship can be established between them and the ontological content of reality as such. As a matter of fact, we use concepts like space, body, mass, acceleration, field, and so on, to pick out and bind sense-data. But we can no longer trace any connection between the content of these concepts and the unknown content of the "outside" reality. This is a clear admission that all our concepts have no basis at all and, thus, this counts as a self-destruction of the objectivity of knowledge itself. By the same token, the Kantian distinction between a world for us and a realm of things-in-themselves refers epistemic procedures back to the forms of intuition and the concepts of the knowing subject without any justification for the fact that he uses precisely these forms and these concepts and not any other forms or concepts. For instance, the forms of our intuition impose the presentation of a spatiotemporal world. But why do we have these forms and not others? Why must a spatiotemporal world exist (for us)? The Kantian answer (*viz.,* because these are precisely the a priori forms of our faculty of intuition) is not good enough, for we would like to know why we have these forms instead of others and, in the end, if these forms are homogeneous or, at least, a good approximation to the para-

mount reality, i.e., the realm of things as they are when considered in themselves.

So we cannot construct the concept of knowledge avoiding a reference to reality as such. If the object of knowledge is not a reality independent of the knowing operations, but just our observational future states or a (minor) object entirely dependent on our faculties of cognition –an object naively defined as being only "for us"–, we will contravene the normal sense of the concept of knowledge, which amounts to an intentional grasping of something that precedes the very acts that apprehend it, or, to say it somewhat more technically, to something that has not those very acts of cognition as a condition for the possibility of its own existence. As a result, the concept of reality cannot be replaced by another in the definition of knowledge. If we say that knowledge is about predictions of observational states or about objects entirely constructed by our subjective forms of apprehension and thought, we will lose the very objectivity of knowledge, because we can no longer trace any relation between the concepts we use (like space, time, mass, force, field, etc.) and the very reality we have excluded from the outset from the definition of knowledge.

Thus we get a final characterization of the way reality is prior to knowledge: the fundamental concepts of knowledge must give a sound presentation of reality as something whose existence is independent from the very operations by means of which it is known. That is to say: the fundamental concepts we use must give rise to a "realistic" interpretation. In the present state of our knowledge, our fundamental concepts have imposed an interpretation of reality as a physical realm, and this physical realm as a domain of events understandable in terms of concepts such as mass, field, particle, wave, and so on. Yet this is not the final word on these matters. In accordance with what has been said, we reject the idea, so cherished by Schlick and Reichenbach, that physical knowledge is simply a matter of coordination (*Zuordnung*) between mathematical equations and sense-data. Fundamental concepts must lead to a coherent realistic representation, or just to a representation of a realm of physical reality, if we take for granted the assumption we do not want to discuss that the basic form of reality is *physical* reality.

And so are we carried to the question: what is *physis*?

But before that we must say something else about knowledge itself.

2. REAPPRAISING REALISM: WHAT IS REALLY IN QUESTION IN QUANTUM PHYSICS?

Bearing in mind what we said above, let us establish some important conclusions.

First, if we are right, knowledge is bound to strive for the following goals:

1. If, going away from the sterile womb of semantics, we associate to the propositional attitude report "I know that…" the idea of some procedure for obtaining knowledge, be it experimental or otherwise, so that the clause "I know that…" refers the state of knowing to a certain previous cognitive activity culminating in the verification that *p* is the case, then we can state what we will call from now on the "objectivity rule" ("OR" for short):

> **OR:** If I know that *p, p* is the case *simpliciter* and not just under the conditions it has been known, so that *p* can be released from the particular conditions under which it has been known and asserted as true regardless of any verification method.

If the objectivity rule applies to the content of an assertion, e.g., "The average distance between the Sun and the Earth is 149 million kilometres", this signifies that the physical state of affairs referred to by the statement is equally the same for a multiplicity of different verification methods, so that it is independent of any disturbance a chosen measuring device could impinge on it. That is to say that an objectified assertion talks about some objective feature of reality invariant despite a change in verification method, i.e., it talks about what simply is the case, not about the results of an interaction between a physical system and a measuring device or about the proper states of the measuring apparatus.

2. We have talked above about irreducibility as the second sense of priority. Let us extract one important lesson from that.

Instead of the common but somewhat naïve conception that there are facts we can simply collect, we state that it is not possible to collect immediately facts from nature, but that they must be constituted through conceptual and methodological procedures. For instance, the fact of acceleration in Newton's mechanics supposes the concepts of absolute space and time, as well as the mathematical method of fluxions, so that the concepts of applied force and inertial mass can subsequently intervene as explanatory concepts. Thus we must say that knowledge is not simply a matter of putting facts under explanatory concepts. The very facts we

explain point back to constitutive concepts. These are not bound to direct empirical justification. They are kinds of postulates of empirical knowledge. Hence, there are several strata of disagreement between theories. They can disagree at the level of the explanation of facts, i.e., at the level of explanatory concepts (for instance, Doppler-effect versus tired-light as an overall explanation for the galactic red-shift), or they can disagree about the very facts they are facing, that is, at the level of the constitutive concepts and the mathematics associated with them. A clear example is the understanding of light *as* a wave or *as* a beam of particles (no need to say that the mathematical properties of waves and particles are pretty different). Another clear example is Thomson's model and Rutherford-Bohr's model as conflicting depictions of the inner structure of the atom. Whenever we arrive at a primary characterization of the nature and structure of an entity, we use models of intelligibility that are constitutive concepts. Thus we go from phenomena to facts by means of constitutive concepts, and from facts to explicative theories by means of explanatory concepts, so that we can state a kind of hierarchy rule (HR) applying to the several layers of concept formation:

> **HR:** Constitutive concepts give structure to phenomena, converting them into facts; explanatory concepts connect facts to other facts in dependence chains; a change in constitutive concepts can change facts and often leads to the discovery of new realm of phenomena; a change in explanatory concepts leads only to a new explanatory hypothesis.

Explanatory concepts enter explicative theories that either fit or do not fit the facts already recognized; on the other hand, the justification of constitutive concepts lies more deeply in its aptitude to give intelligible structure to phenomena, to disclose new realms of phenomena and to establish new kinds of facts.

3. Finally, let us stress that, in accordance with the third sense of priority, to give a representation of an observer-independent reality is not to depict reality as it is without knowledge –this is a self-contradictory idea– nor to subtract the impact knowledge has on reality –this will bring us back to the point of departure–, but to accept what we will call from now on the "objective reasoning rule" (ORR). This rule allows us to think in terms of an objective reality that follows its own development independently of the circumstance of being proved by a measuring device or not. We can state it as follows:

> **ORR:** The state of any physical system is self-determined by its own laws of development; two systems acquire new states at the moment of their interaction and pursue after that separated world lines; reasoning can anticipate what will be the future state of a system relying solely on its past-story, so that our measuring devices do not create the states of physical systems, but merely prove an observer-independent reality.

This condition used to be articulated as a demand for both causality and determinism. In fact, causal thinking is a way of thinking "as if we were not there", so to speak: we simply figure out what is happening. So causal thinking is intimately connected to the idea of an objective realm governed by its own laws. In addition, the separation of physical systems implies locality and the refusal of connections between events faster than any velocity of signal transmission (although we begin to suspect that time-like intervals of space-time light cones are no limits at all for signal transmission –there are experimental grounds to admit superluminal velocity, i.e., space-like intervals *with* causal connection–, no instantaneous transmission of signals is compatible both with causal connection *and* separation of physical systems). Nevertheless, causal reasoning need not be deterministic in the classical way of defining it. As we will see, there are good reasons to believe that, if we reach a deeper understanding of phenomena, laws of nature will incorporate processes of spontaneous generation of order and self-organization that are incompatible with a Laplacian, deterministic, linear approach. So we will retain causal thinking and locality in ORR, though we shall be ready to give-up determinism.

We will call OR and ORR jointly the thesis of *onto-gnosiological realism.*

Now, *secondly,* let us go straight to the burning question. As Heisenberg expressly recognized in *Physik und Philosophie*, the assertions of quantum mechanics do not respect what we have called "OR", i.e., they cannot be fully objectified and are, thus, incompatible with what he calls "dogmatic realism". Heisenberg certainly recognizes that every researcher aims at what is "objectively true". But quantum mechanics goes so deep and so sharply into the minute structure of nature that the searching apparatus provokes uncontrolled changes in the very system it is intended to scrutinize. So we can make assertions not truly about what objectively is, but only about the results of the interactions between measuring devices and physical system. In other words, it is as if all assertions of quantum mechanics were about an entirely new domain: not the quantum phenomena themselves, but the new global systems formed through the fusion between quantum entities and measuring devices (which have a quantum reality of their own). And these quantum mechanical assertions, now referring to this "in-between

realm", would not respect OR either, because the application of any other verification method should engage another measuring device creating another physical system in which, possibly, the former phenomena could no longer be detectable.

For Heisenberg, this circumstance proves nothing against quantum theory as such (or, better, against the Copenhagen interpretation of quantum formalism). It proves only that natural science is possible without strong realist claims. Rejecting dogmatic realism (*viz.,* All scientific assertions respect OR) and metaphysical realism (*viz.,* Things studied by science exist absolutely), Heisenberg proposes a kind of "practical" realism (*viz.* Some scientific assertions respect OR) which amounts to a denial that natural science requires OR and to a confession that idealism or some kind of instrumentalism is an even better foundation. In fact, in light of Heisenberg's considerations, we could say that these latter epistemic doctrines are preferable because they are, at least, more general, given that only macroscopic phenomena can supposedly give rise to assertions respecting OR, and that idealism or instrumentalist versions of science could encompass assertions respecting OR as well as assertions not respecting it.

Nevertheless, we must make a distinction. We can say that sometimes measuring devices disturb the physical systems in an essential way and that, some other times, the impact of measuring devices on the observed systems can be disregarded. The talk about "disturbance", and about disturbances that can or cannot be ignored in experimental situations, is plainly consistent with OR. Inconsistency with OR arises when quantum mechanics insists that the measuring device "creates" or "realizes" the properties of a system that was not in a determinate physical state before the measuring operations were done. Heisenberg's argument against OR requires talking about "creation" and not talking about simple "disturbance". In fact, the realism implicated in OR is not put in question in case our measuring devices disturb or do not disturb the system to an appreciable extent, but only in case the measuring devices bring about a kind of "passage" from potentiality to actuality. Indeed, we can always say that, if *p* is know, then *p* is the case *simpliciter,* with a slightly (or even a considerable) deviation due to disturbances originating in the very act of knowing (in its physical basis). It would be something different altogether to assert that there is no *p* before the very act of knowing pushes the physical system to a certain outcome, so that we could not talk about a state independent of the verification method associated with it. So, the fact that measuring is itself a physical procedure that always disturbs the physical system measured (in a degree that can or cannot be ignored) says absolutely nothing against OR.

On the other hand, it is common knowledge since Bohr's essay of 1927 "The Quantum Postulate and the Recent Development of Atomic

Theory" that quantum mechanics puts severe limitations on ORR, disallowing causal *and* space-time descriptions that could be simultaneously pursued with an arbitrary degree of precision (technically: they refer to observables of non-commuting operators). According to Bohr, the validity of the superposition principle and of the conservation laws entails the impossibility of a full causal and space-time description. This is so because, as Bohr himself puts it, in attempting to trace the laws of the time-spatial propagation according to the quantum postulate, we are confined to statistical conclusions in light of Schrödinger's wave-equation, and, on the other hand, the claim of causality for the individual processes, characterized by the quantum of action, forces us to renounce to this space-time description.

In fact, for values under the lower limit of $h/4\pi$ for the product of the two measuring uncertainties –the so called Heisenberg's indeterminacy-relations– we can have one or the other, but not both descriptions at the same time. Heisenberg presented for the first time these indeterminacy-relations in 1927. In a *Gedankenexperiment* with a gamma-ray microscope, he reasoned first about the possibility of measuring simultaneously the position and momentum of an electron. Fixing the position of the electron with a total precision would imply an infinite indeterminacy in the electron's momentum, and vice-versa. After that, he extended these indeterminacy-relations to the pair time-energy and found the same lower limit for the product of the two uncertainties.

For a Heisenberg caught in the middle of a struggle with Schrödinger's wave-mechanics, all that these indeterminacy-relations were showing us was no more than the essential discontinuity and unpredictability of the quantum world. He avoided any talk about real particles and (*hélas!*) real waves. However, Bohr had other views on the issue, and a definite non-realistic, non-Schrödingean interpretation regarding the physical meaning of the wave-function. In harsh and relentless discussions with Heisenberg, he stressed that the very kernel of the indeterminacy-relations was the unavoidable wave-particle duality of quantum phenomena and the complementarity-principle he was proposing at the same time. Indeed, as it is generally recognized, with Bohr's ingenious concept of complementarity we have reached the very heart not only of indeterminacy-relations, but also of the Copenhagen interpretation of quantum mechanics as a whole.

Assessing the overall philosophical consequences of this principle, we could say that it tells us the following somewhat disappointing story:

1. Quantum entities *show* us nothing by themselves.

2. If proved by an apparatus, quantum entities can *manifest* wave-like behavior or particle-like behavior.

3. The behaviors of quantum entities are always behaviors *relative to* and *in the context* of a definite apparatus.

4. These particle-like and wave-like behaviors are not manifested simultaneously and by the interaction with the same apparatus.

5. For that reason, the concepts of particle and wave do not contradict each other, given that they are not applied at the same time and in the same experimental context to the same quantum entity.

6. Instead, they exclude each other, but are both jointly necessary to the description of the diversity of quantum manifestations.

7. So, firstly, we have no grips on the quantum entities, but only the possibility of applying to them concepts we have borrowed previously from the macroscopic world of classical physics.

8. Secondly –as it seems Bohr himself has put it– for us there is no such thing as a quantum world, but only a quantum physical description.

9. And, thirdly, quantum entities are a well founded, although limited, conjecture from the macroscopic (classical) world into the microscopic (non-classical) world which returns back to some processes and phenomena in the macroscopic (classical) world (e.g., the classical concept of a particle gives rise to the quantum concept of an electron movement, which serves to interpret a trace of condensed water vapor in the ionization chamber).

We see that complementarity puts severe limitations to ORR (and OR), concerning not only the possibility of conjoining causal description and space-time coordination, but also regarding the very possibility of taking quantum phenomena as a realm of real entities, i.e., of taking them as appearances of an *actual world* of things and events. As a matter of fact, the quantum phenomenon, as described in Bohr's and Heisenberg's interpretations, lacks the conditions to be considered an *entity* in the full sense of the word: it has no independence from the measuring device, no integrated description of its own multiple manifestations, no concepts borrowed in its very nature, and no such fundamental things as continuity across space-time, determined actual states and separation. And so we face, with the Copenhagen interpretation, the following devastating situation for our hopes to understand nature: fundamental concepts are not realistically interpretable

– we must only say that we are constructing a science as if there were a quantum world consisting of waves, on the one hand, and particles, on the other hand, but knowing at the same time that we have no appropriate concepts to understand the way what we are describing as particles combines with what we are describing as waves in the same quantum entity. That is to say: the very concepts of physics we are using do not allow us to construct a sound and coherent representation of *physis.*

All this pertains to the story and the problem of quantum mechanics and is well know to all. Let us only stress that the ensuing development in the Copenhagen paths since 1926-7 only accrues the perplexities. We have:

1. *A non-classic interpretation of probability.* Unlike Boltzmann's probabilities, quantum probabilities do not reflect our ignorance about the individual real processes of a complex system, but rather express the likelihood the interaction with a measuring device would then create a determined outcome as the actual state of a physical system. Max Born proposed in a paper published in 1926 the probabilistic interpretation of Schrödinger's wave-function. According to him, the square of the amplitude of the wave-function in same region of configuration space is related to the probability of finding the quantum particle in that region. So the wave-function does not depict a quantum entity, but only our knowledge about the probability of certain outcomes. The descriptions made by quantum probabilities relate to many experiences. Projecting them onto an individual system entails that the individual system will be a superposition of all its possible actual states. In fact, quantum mechanics stresses that whenever a quantum system can be in a plurality of states, the superposition of states is itself a state in which the system is until some measures are made on it.

2. *An appeal to consciousness in the passage from potentiality to actuality.* The mathematical framework of quantum mechanics does not describe the passage from potentiality to actuality. The collapse of the wave-function is simply supposed to occur, so that the system can pass from the superposition of all its possible states to an actual definite state. Facing the infinite regress implied in the fact that the measuring apparatus is itself a quantum entity, entering in a superposition with the physical system under observation, von Neumann introduced his famous thesis that the wave-function collapses when and if the system interacts with consciousness. In fact, even the sense organs and the brain are quantum systems entering in a new composite wave-function. Only consciousness is supposed to be a

reality beyond the physical realm that puts an end to the infinite regress.

3. *Entangled states and non-locality.* Two particles that have interacted are described by a single two-particle state vector. When a measure is made on one of them, the state vector collapses into a definite state, forcing the other particle to realize instantaneously a specific correlated state, no matter how far it will be from the former (it can be on "the other side" of the Universe). Bohr began to give up the talk about "disturbance" introduced by measuring operations in his rejoinder to the EPR argument in October 1935. Since then, he defended an indeterminacy of the system before its relationship to an apparatus that is tantamount to a real action at a distance in the case of entangled particles. This was a direct vindication of non-locality. The particles lost their individuality in space-time. After him, Bell's inequality theorem and experiments promoted by Aspect and Wheeler gave experimental, but very controversial support to the reality of entanglement.

Must we give up OR and ORR together, that is, must we give up onto-gnosiological realism and the general way we used to do science? Must we *alter* the very concept of scientific knowledge? Is this a reasonable price to pay: entanglement, that is: non-locality; *ad hoc* consciousness, that is: the determination of the physical world relying on a non-physical entity; superposition, that is: the bare actuality of potentiality; and complementarity, that is: renunciation even to a not full-fledged realistic description of a quantum world?

To finish with this point, let us only say, *thirdly,* that this is not such a desperate situation. There have been winds of change blowing since 1926-7. Facing the puzzling aspects of quantum wave-like and particle-like manifestations, the French physicist Louis de Broglie proposed a description of the inner structure of quantum entities that turned these disparate manifestations into phenomena of a well defined underlying quantum reality. Against Born's interpretation, de Broglie proposed in 1926 to view quantum entities as singularities (point-like particles) moving in a real field, while, at the same time, there was a second field which had the statistical and probabilistic significance of Schrödinger's wave-function, as interpreted by Born and Bohr. After this first proposal, de Broglie simplified his theory about the inner structure of a quantum entity. It was, now, a point-particle moving in a continuous, real wave-field. The particle follows the wave-field, and its position is more likely to be in those regions where the amplitude of the wave-field is larger. This is precisely "la loi du guidage", as de Broglie

himself put it: the particle in not only directed by the wave-field, the particle is nothing more than a singularity of the wave itself. Thus, contrasting with Bohr's rather mysterious talk about complementarity, we have no duality between waves and particles and no permanent necessity of speaking the language of waves or the language of particles, never knowing how something can reveal both particle-like behavior and wave-like behavior. On the contrary, we have now an insight into the deep structure of a quantum entity that is able to turn such disparate manifestations into well defined and predictable phenomena of one and the same entity. The interpretation of the double-slit experiment is now trivial: the singularity passes through one and only one slit, while the real wave passes through both slits and produces constructive and destructive interferences determining the path of the singularity. No need to say that, contrary to Copenhagen interpretation, every particle has in its field a definite position and a definite momentum. Probabilities revert now to their classical significance: they express only our ignorance about the intricacies of the quantum world. This theory by de Broglie was developed by Bohm a few decades later. He introduced a non-classical quantum potential, U, to explain the motion of particles. When this quantum potential decreases to zero, the equations of motion of the de Broglie-Bohm interpretation revert to the classical equations of Newtonian mechanics in the so-called Hamilton-Jacobi formalism. Recently, Croca and his associates proposed a reappraisal of the de Broglie's theory, as it will be profusely explained in this book. Taking de Broglie's realistic approach, Croca distinguished between the real-wave, now called theta-wave, and the point-like singularity within the wave, called acron, that is, the higher energetic region of the whole structure.

As we can see, the clash between Born's and Bohr's interpretation, on the one hand, and de Broglie-Bohm-Croca's interpretation, on the other hand, is not a simple disagreement at the level of explicative concepts. On the contrary, referring to our HR above, we must say that there is a real conflict at the level of constitutive concepts. We must, then, expect that a new realm of phenomena and new facts will be uncovered by the challenging theory. As a matter of fact, the initial approach by de Broglie predicts superluminal velocities for the singularity; recently, in 1993, Peter Holland emphasized in his book *The Quantum Theory of Motion* that de Broglie-Bohm theory had something different to say about the time taken by particles to tunnel through a potential barrier. Recent work by Croca about tunneling shows some different results too, and, more importantly, some new phenomena at the sub-quantum level (concerning theta-waves) are coming near to being discovered and converted into new facts of the quantum world.

In a word, de Broglie-Bohm-Croca's realistic interpretation of the wave and of the singularity (Crocas's acron) as a point-like region *in* the real

wave bypasses the conundrums and conceptual perplexities of the Copenhagen interpretation: there is no quantum jumps, no collapse of the wave-function, no mysterious contribution of consciousness to measuring operations, and probabilities return to their classical meaning. In addition, it seems that the realistic interpretation is also able to point in the direction of new realms of phenomena (at the sub-quantum level) and to constitute entirely new facts. Only the subsequent experimental and conceptual work can show what the final decision will be.

3. Underpinning Onto-Gnosiological Realism: Knowledge and its Entities

Meanwhile, we need to come to a decision about the following double way of considering the general issue of quantum mechanics: is quantum mechanics really presenting us new and utterly insoluble paradoxes about knowledge and about nature (as the Copenhagen interpretation suggests), or is quantum mechanics just destroying some naïve conceptions we had, as to give rise to a new coherent conception about knowledge and its object (as a debroglian realistic interpretation would maintain)? The answer is somewhere in between these two extremes. Quantum mechanics showed that classical ideas about nature and knowledge were inapplicable to it – this is undeniable. However, we have reasons to question whether the paradoxical look of quantum mechanics is an ultimate and insuperable situation (and if so, in what regards), or whether these paradoxes can be overturned in a new post-classical conception that is still to come. In addition, we must remark that, on the one hand, quantum mechanics corrected some oversimplifications about knowledge and its object, and these very corrections have diminished, in turn, the weight of quantum paradoxes; on the other hand, some paradoxes of quantum mechanics are still based on the entrenched naïve ideas, and it is foreseeable that the criticism of those ingenuities will contribute to dissolve them.

Up until now, a huge literature emphasized the paradoxes: entanglement, measuring acts collapsing the wave-functions, wave-particle duality, a general lack of strong objectivity and the rise of a nondeterministic, somewhat irrational description of nature. Let us now stress, the other way around, both the naïve conceptions we used to be acquainted with and which quantum mechanics have begun to beat down, and those naïve conceptions quantum mechanics still accepts without a doubt.

Given their special character, we will call them “fictions”. The word is used here not as meaning something non-real or patently false, but in the following precise sense: a fiction is some assumption not justified in itself

that allows us to think *as if* some things really were true, so that we embed phenomena in a particular conceptual framework and give them a correlated meaning. Fictions are in most cases simplifications and highly abstract hypothesis that dispense us to inquire into the true nature of reality. They are not a theory and they are not constitutive concepts either, they are, rather, a useful device to deal with complex realities in the absence of a true knowledge or a sound theory about its nature. For instance, the liberty and responsibility of people brought to justice is an overall fiction that allows the adjudication of guilt or innocence in a trial: the jury decides as if the defendant were really free and really responsible of all the criminal acts he is accused of. However, the full liberty and responsibility of individuals in society are assumptions never proved by anybody. Political theory also furnishes us with a lot of fictions. The idea that the commonwealth began by means of a mutual pact between individuals, so that this social contract and not brute force counts as the very beginning of the state, is an evident fiction developed by some political thinkers like Thomas Hobbes or John Locke. This fiction allowed them to consider political society as if all individuals were free and equal from the start, and to justify political obligation before the sovereign as if it had sprung from an original act of consent (the original pact).

We cannot take a simple pragmatic approach to fictions. The question is not simply whether they are useful, but also whether they are true or, at least, plausible assumptions. We cannot have an understanding of reality fully based on fictions. They must retreat in face of an effective knowledge of what there is. The fiction about the liberty of individuals is a good approximation to truth, if we disregard the metaphysical question of free-will and define liberty in terms of voluntary acts. Besides, in order to prevent social disorder it is useful to have a penal system, and penal systems are based on criminal law, which depends on the idea of imputability, that is, of the liberty and responsibility of individuals. The fiction of the original contract is, however, entirely implausible nowadays, and this implausibility hinders all utility it may have had in the past: nobody believes it now and nobody acts in accordance with it, that is to say, as if it were true.

Classical gnosiological thinking was also prolific in fictions. Let us examine those that are most important for our issue.

1. *The immaterial eye fiction,* that is, knowledge mostly conceived as a simple action of taking notice of what is, i. e., as a (theoretical) regard that does not affect or alter the things looked upon. Vision was paradigmatic for this fiction about knowledge. And theory, conceptual thinking in general, was often conceived as an intellectual vision too – as an “intuitus mentis”, as Descartes named it. The ancient idioms

theorein and *contemplare* had the same idea behind them in the ancient noetic parlance. According to this fiction, all happens as if things were what they are and, then, a supervening regard only came to register their existence and properties. Naïve realism uses this fiction surreptitiously in order to talk about a reality that is what it is independently of being known or not. For neutralizing the impact of knowledge on reality, this fiction suggests that knowledge is an event belonging to a mind that forms no part of the physical-material universe and does not produce any effects on it. Linked with this fiction there is, thus, a sketchy dualism between matter and mind. Not so surprisingly, von Neumann's conception about the role of consciousness in measurement only radicalizes this fiction of the immaterial eye: instead of merely taking notice of what things already are we now have the idea of a non-material, non-physical regard that makes things happen and be what they are. However, this fiction about an action of taking notice that has no roots and no effects in the reality concerned is completely untenable. Quantum mechanics, without Copenhagen thesis about the creative power of measuring acts, suggested the idea of an inevitable (even if depreciable) disturbance of physical systems by the very act of knowing, which is an obvious state of affairs when the act of knowing is not a pure theoretical inspection, but an experimental operation involving technical apparatus (but even macroscopic vision is, after all, a physical event that disturbs infinitesimally the surrounding world). Thus, contrary to this fiction, we must considerer, from the very start, the act of knowing as a physical action which produces modifications in the things that are its objects, so that knowledge never reaches a nude reality, undisturbed by the very act of knowing. We can say as a motto: every act of knowing a physical universe is itself a physical event in the universe.

2. *The certain-inside versus the uncertain-outside fiction,* that is, the idea that knowledge is an event taking place in the internal arena of consciousness, in a "theatrum mentis" (inner stage), so to speak, so that reality is conceived as a realm of "exteriority" unattainable by a direct grasping. This is one of the most widespread fictions in gnosiological thinking. Descartes inaugurated a doubt about the existence of an external world based on the assumption that the mind has direct access only to its own ideas. If the mind is closed in upon itself, reality becomes a matter of "exteriority", while the mind moves back to an extra-worldly "internal" space. This gives a semblance of plausibility to the extremely odd idea that there is a split between a material,

external universe, and an immaterial, internal mind. From now on, under the impressive suggestion of this fiction, the events in the internal stage of the mind are considered as certain and beyond all conceivable doubt, while things in the external universe –to which the internal events allegedly refer– are uncertain and doubtful. Malebranche, Berkeley, David Hume – all of them were cases of this inner-outer, certain-uncertain fiction. Kant tried to amend this in his famous "Refutation of Idealism" in the *Kritik der reinen Vernunft.* His point was that the inner sense is possible only through the mediation of the outer sense (the intuition of time depending on the intuition of something permanent in space), so that the certainty of the outer sense is equal to the certainty of the inner sense. Nevertheless, Kant's argument is pointing only halfway beyond this fiction, because he continues to accept the very distinction and a definition of physical reality in terms of exteriority relative to the mind. One of the most conspicuous consequences of this fiction was the Humean thesis that, when the mind is affected by some external objects, the only things that are given to it are its own internal modifications, called "impressions", so that the mind does not have a solid ground to state, for instance, "The thing perceived is hot", but only to say "There is now a sensation of hotness in me". This is tantamount to saying that any measuring apparatus, a thermometer, say, can only report its own internal states and not causally refer these states to an objective reality. As a matter of fact, if, according to this fiction, this reference takes the form of a mapping of the internal states onto an external universe, the skeptic will always raise doubts as to whether there is truly an external reality outside" the mind to which these internal states refer. When quantum mechanics considers the measuring apparatus as the only thing whose states physics can describe, it suffers somewhat from the same Humean delusion. Against it, we must state that any sensible organism interprets its states as a result of the interconnection between itself and the surrounding world. These states do not represent something external – they do not "depict" an external reality. We must drop this way of considering things. Those states are *states of reality,* not of an "inside" (the mind) referred to an "outside" (the "external" world). This reality is not bare reality (a deeper consideration shows that this concept of "bare reality" is meaningless), but reality submitted to the set of operations that we call "interaction with the measuring apparatus" or "interaction with a living organism." So we must say: an apparatus (or a sense organ – vision, say) is not a device to scan a reality external to it, but a procedure to induce a certain kind of events in reality. For instance, the

complex system eye-brain-consciousness is a device to bring about a chromatic world. "Outside" this system there is no chromatic world at all, i.e., in order to have a chromatic world we must insert in reality the set of operations produced by the system eye-optical nerves-brain-consciousness (by itself, the reality from which we are starting is not "nude reality", but already the result of other sets of interconnected and stratified operations). Thus Bohr's hunch was half-right after all. In a way, the measuring apparatus pushes the physical system to a certain outcome that was not there before. The apparatus (or our sensibility) is not a way to figure out an outside, "external" reality – it is part of the same reality, and a more complex level of reality, bringing about phenomena that constitute new realms in their own right. Bohr's general conception about the nature of an apparatus (or the sense organ of a living organism) is then quite right when stating, as in the essay of 1935 "Quantum Mechanics and Physical Reality," that the procedure of measurement has an essential influence on the conditions on which the definition of the physical quantities rests. The fault of Bohr's conception lies in his refusal to account for the dependence of physical properties upon the experimental apparatus in terms of causal processes producing growing levels of complexity.

3. *The fiction of the label-like properties of objects,* that is, the idea that objects by themselves have individualizing, well-defined properties before any interaction takes place between them. Notwithstanding the seemingly strong evidence supporting this idea, it is a naïve way of thinking about reality. A property is not like a label that a thing has in order to be distinguished from any other thing. Things do not have properties – things acquire properties as long as they interact with each other. Every property expresses an interaction. For instance, speaking about the polarization of light or the spin of an electron, which are intrinsic properties, is a way of expressing some patterns of interactions of light or electrons with some other physical entities (like a filter or a magnetic field). Quantum mechanics contributed powerfully to a break down of this old naiveté of ontological and gnosiological thinking. As a matter of fact, the best examples to illustrate this are given by quantum entities, because they cannot be described as macroscopic objects that are the apparently enduring bearers of permanent properties. Nevertheless, in other realms, the same general conception applies. Talking about the sincerity or the generosity of a person means that some behavior usually took place when this person was submitted to a specific social situation (say, being always veracious in the expression of his own

feelings, giving assistance to others, and so on). It makes no sense to talk about the properties of being sincere or generous of a person that does not have social interactions with other people. The fiction of a label-like property of objects gave rise to an ontology that conceded a primary rank to the concept of "thing". In accordance with this fiction, ultimately we could even speak of a universe composed of just one thing which had, say, properties *a, b* and *c* instead of properties *d, f* or *g,* even if this way of putting things amounts to an obvious non-sense. In this respect, metaphysics has for a longtime discussed the "principium individuationis," i.e., the conditions that allow a thing to be one and, at the same time, distinct from any other. A multitude of answers was given to this secular problem, from Aristotle and Aquinas to Leibniz and Kant. The Leibnizian way of dealing with the problem was a very interesting one. It consisted in putting the principle of individuation not in a space-time location but instead into form, that is, by way of determination, and in considering that the individuality of a thing is given by an infinite series of predicates, describing all its real properties (the word "real" meaning, in the scholastic jargon Leibniz was using here, a property that is inherent to the thing itself and that is not a relation to another thing, i.e., something expressed by a monadic and not by an n-adic predicate). However, so far, so… bad, we could say, because this is the same as thinking candidly that we can answer the ontological question about what there is by simply stating that there are a plurality of substances (i.e., bearers or subjects of a collection of real predicates), and that substances differ from others in having some label-like properties peculiar to them or a set of properties that is not replicated by any other thing. Notwithstanding this naiveté, when we wonder why the infinite set of properties of one substance cannot be replicated by another thing, we come to an interesting answer: Leibniz suggests that every property of one thing is necessarily correlated with a correspondent property of any other thing, so that, if two sets of properties were not correlated but wholly identical, they will constitute the very same substance, expressing the universe from the same point of view. That is to say: it is true that, for Leibniz, substances are series of real predicates and are closed in upon themselves, according to the label-like fiction; nevertheless, each real predicate of one substance is necessarily in correlation with at least one property of any other existing substance. This is tantamount to saying that properties express universal interconnections of one substance with all substances in the universe. Going beyond Leibniz's metaphysics, we could say that properties are just the expression of causal interconnections, something like the

> result of an interaction between all physical realities. In a word: a property is not a state a thing simply has from the start, but a state a being acquires as a result of an interaction. And if we can say with Leibniz that a substance is an infinite set of properties, this will mean now that a substance is an infinite process of interaction with other things. Thus we must take determination or property not as a label, but always as an interaction result. The fiction of things as bearers of properties is overturned: a "thing" is now just a never-ending process of causal interactions in which it gets determination. Spinoza said once *omnis determinatio est negatio*. Adjusting a bit Spinoza's dictum, we could put it as motto: *omnis determinatio est interactio.*

In putting these things together, we must say: (i) physical knowledge is itself a physical disturbing process involving measuring apparatus (the same applies to the sense organs of our perceptual acquaintance with the surrounding world); (ii) a measuring apparatus is a physical way for producing phenomena of a special kind; and (iii) properties of physical systems are the results of interaction between these very systems and other physical systems (namely, but not forcibly, an apparatus). This is our non-classical version of onto-gnosiological realism: not a word about a reality that is what it is regardless of being known or not; also not a word about a conception of knowledge as a non-disturbing process; and, finally, a place for the observer, and for a physics closely related to the observer's point of view in a universe that goes beyond the observational content (linked to the measuring apparatuses or just to the sense organs) by means of a causal connection between this content and the underlying reality. Keeping this in mind, we will not only see the limits of classical gnosiological theories, but we will also have some glimpses into what is solid and what is doubtful in the paradoxes quantum mechanics presents to us.

As a matter of fact, if we examine the major attempt to give an epistemological foundation to classical Newtonian mechanics, that is, if we turn our eyes to the *Kritik der reinen Vernunft,* by Immanuel Kant, we will see that the operations involved in knowledge are described as follows:

I. Univocal location of any event in space and time. i.e., space-time coordination;

II. Causal connection of any event with any other event simultaneous to it (reciprocal action) or situated in the future (causality in a narrow sense), i.e., settlement of an universal causal net;

To those operations of coordination and causation the following conditions apply:

A. Definition of an object in terms of continuity of events across space-time;

B. The principle of conservation (of "matter", or better yet, of "energy") as a necessary condition for the passage of an object from one position to other (contiguous) position in space-time and from one state to another;

C. The principle of determination of any state of an object by (i) its previous states (if it is a closed system), plus (ii) the causal interaction with all other objects (if not a closed system);

D. The principle of complete determination of an object regarding all its possible properties.

Kant (and Newton too) thought that I. was a simple matter of Euclidian geometry, plus the reference to a universal time order. It is well known that the special theory of relativity brought major complications to this very simple idea. An univocal location of events and determination of distance between pairs of events turned out not to be measuring the distances in space and time relative to a given frame of reference, but determining a new magnitude, invariant to all possible observers, named "space-time interval." However, the great correction relativity introduced relates to II. – the quest for a maximal velocity of signal transmission destroyed the (supposedly) Newtonian idea of an instantaneous action at a distance –as far as gravity is concerned– and the Kantian assumption that there is a causal web connecting simultaneously all objects in empirical space.

Nevertheless, it was quantum mechanics that introduced a real rupture into this classical scheme. And so, we come, at last, to the paradoxes. To begin with, the possibility of performing conjointly coordination operations and causal connections to an arbitrary degree of precision was severely restricted by the complementarity principle; as we saw above, either we have position (doing coordination operations) or we have momentum (developing causal reasoning). Moreover, the quantum entity – as the Copenhagen interpretation stresses– is by itself *not* determined in its position and momentum under a certain lower limit, which turns out to be the Heisenberg's indeterminacy relations. Epistemological uncertainty (Heisenberg's *Ungewissheit*) is based on ontological (physical) indeterminacy (Heisenberg's word was precisely *Unbestimmtheit*). This directly rebuffs D. In addition,

quantum mechanics also shakes A, C, and D anew with its quantum jumps, the restriction to probabilities, and the superposition of linear states before the interaction with an apparatus produces a determinate state through the collapse of the wave-function. The only principle still unshaken is B, if we interpret it as an energy conservation law (Bohr himself pondered several times if he had to drop it). In turn, the non-locality of entangled states is a fresh departure from the constraints raised by relativity and a return to a Newtonian conception of something like an action at a distance connecting all things in the universe.

What does a neo-debroglian approach mixed with our onto-gnosiological realism have to say about all this? That was the question. To begin with, let us say that quantum entities are certainly not like classical, macroscopic objects. This circumstance imposes some qualifications onto A and C: first, it is a matter of discussion if we do not have something like acron jumps *inside* the finite theta-wave (if we consider that the real theta-wave, despite its extension, is all contained in each time-point, say t_n, the subsequent position of the acron in t_{n+1} can be any point on the theta-wave in t_{n+1} even if not contiguous to the point it had in t_n – tunneling could be an effect directly related to this, with the acron "jumping" to the region of the theta-wave that stands behind the barrier); second, the displacement of the acron along the theta-wave is nondeterministic – it integrates stochastic process and gives rise only to a probabilistic description. However, despite this non-classical view, we can reach a rather different general explanation for the reason why we cannot simultaneously hold coordination and causal determinations to an arbitrary level of precision. In fact, considering what we said above, we must ponder freezing all talk about complementarity and try to substitute for it the concept of pairs of mutually-dependent properties. We can define this concept as follows: properties *A* and *B* are a pair of mutually-dependent properties if any alteration in one of them disturbs the other, so that we cannot go from the determination of a precise value of *A* to a precise value of *B* and return from *B* to the same value of *A,* and vice-versa, because the alteration of one quantity produces an alteration in the other quantity in a probabilistic, nondeterministic way. Now we could say: given that a quantum entity has at each moment a set of interactions with other quantum entities from which it gets some precise properties defining its state, and given that some interactions interfere with other mutually-dependent properties, disturbing the values they have (position and momentum, for instance), a quantum entity must be considered as a *balance* always in adjustment between the properties it has (from previous interactions), the new properties it gets (from actual interactions), and the disturbing processes that impinge chains of alteration-loops in the mutually-dependent properties. That is: a quantum entity is actually determined in all properties it has, it has

definite position and definite velocity, even if it is in a never-ending process of alteration-loops between its mutually-dependent properties (this is the same as stating that we cannot have simultaneously a precise measuring-value for them all). Thus a quantum entity, if not closed upon itself but submitted to some measuring process, is never stabilized in a well-defined global state.

To conclude, we must state that our approach must reject the classical principle of complete determination. Kant said that it is not a condition for the possibility of experience, but only a rational rule for conceiving what a thing is, tracing for it a place in the sum of all possibilities. As he puts it in his *Kritik* (B599-600), "Every *thing*, as regards its possibility, is … subject to the principle of *complete* determination, according to which if *all the possible* predicates of *things* be taken together with their contradictory opposites, then one of each pair of contradictory opposites must belong to it." That is to say: to conceive the possibility of a thing is to put it (better: to conceive the possibility of such a procedure) before all pairs of possible predicates and to decide for each one of the pair, so that a thing is conceived as *real* when it is (or is taken as) completely determined by all conceivable predicates. Two things are different if and only if they have different global series of predicates. We approach Leibniz. And, if we enter the realm of physical knowledge with this principle in hand, we come to the idea that any physical object must be in causal interconnection with all objects in the universe. As a matter of fact, comparing one thing to all possible predicates turns now into a process of putting a thing in connection with all other existing things. So we must embrace the idea that there is a universal causal web linking each thing with all other things, so that the full knowledge of one simple physical being would imply the knowledge of the physical universe as a whole. We would come near Laplace's demon, which dwells in a full deterministic universe, where everything is connected to everything. However, if we do not drop the relativistic idea of a maximal finite velocity for signal transmission and causal connection (no matter if it is *c* or otherwise), given that a property is an interaction, as we said, and that an object has no interaction with all other objects (relativity's *must*), we cannot define a real object as something which is completely determined regarding all is possible predicates, because a physical object *never is* in causal connection with all physical objects and so necessarily lacks the (possible) properties related to those interactions: in this regard, it is patently undetermined, and it is just *as partially undetermined* or as *incompletely determined* that we must affirm its very existence.

4. *PHYSIS* – WHAT IS IT?

Here is the question that turns all of us into no more than apprentices. We will try to get some glimpses beyond the customary paths, even if by taking the risk of being a bit speculative.

For a long time, natural philosophy fit comfortably within the Cartesian split between matter and mind. Descartes thought that the former, pre-Galilean way of conceiving the physical realm suffered from a huge misconception, namely, the blend of properties pertaining to mind and to matter, as if they formed a whole. In his criticism of scholastic and Aristotelian physics and biology, concepts like "sensitive soul", "vegetative soul" or "substantial form" were denounced as a kind of anthropomorphic projection into the material realm of concepts belonging only to mind. Specifically, the scholastic physics, following Aristotelian principles, made a massive use of the concept of form, which, from Descartes' point of view, could only be understandable through the erroneous import of properties that pertain exclusively to mind and, for that very reason, eventually veiled the properties that truly belong to matter.

For the sake of a critical assessment, it would be important to distinguish Descartes' explicit argument from what is Descartes' ultimate point. The argument starts in metaphysics. It begins with Descartes' double thesis that his mind is better known (to him) than his body (or any other "outer" body) and –this is crucial– that he himself knows undoubtedly that he has a mind while it is doubtful whether he has a body linked to his own mind. Long after this initial split between certain self-knowledge as "mens" and doubtful self-knowledge as "corpus", Descartes makes an additional appeal in his *Meditations* to the "veracitas dei" in order to settle that what is clearly and distinctly conceived of one thing belongs certainly to it, and that – following from the former thesis– if one thing can be conceived without the other (i.e., without the properties that are clearly and distinctly conceived in the other), then this very thing is separated from the other, or exists apart from it. The conclusion was, at this point, looming very near: considering that mind and body could be conceived apart, mind and body also could exist apart, so that the science of bodies and the science of minds do not conflate.

The argument is flawed in several regards. The appeal to the *veracitas dei* proves nothing, because the certainty about the existence of God depends circularly on the clear and distinct rule; in addition, it is by no means sure that we can even conceive a mind without the understructure of a body and, conversely, a body without the organization superimposed by a "mind", as the argument settles as premises (nothing that belongs to mind belongs to body, and nothing that belongs to body belongs to mind, states Descartes).

This double assumption that we can *understand* what a body is without some properties supposedly belonging to mind (more about this later), and, the other way around, that we can have a *full concept* of the mind (as a "substance") without the properties belonging to body, this pair of assumptions are precisely Descartes' point. He never really demonstrated them; he simply took them as true premises for his overall argument (and as the faith underlying his entire work as physicist, embryologist, biologist, and so on).

But what is mind? And body? What consequences follow concerning physics? Let us see more closely what Descartes' point really was. It was a very bold bet: nothing less than suppressing systematically in the description of nature the reference to patterns of organization –which were entrenched in the concept of "form"– in order to come to them as results of the mechanistic, blind and not finalistic local motion of matter alone. That is to say: while Aristotelian-scholastic physics was going top-down, from the organization-pattern, called "form", to the material base in which it was stamped, going down successively until the ultimate "undetermined matter", the so-called *substratum*, was reached, Descartes' bet was to go bottom-up, starting with autonomous laws of matter in order to arrive at the organized structures not as principles, but as results of these core-laws. Given that the reference to "form" (i.e., to organization patterns) was removed from the front door in the study of nature, physical substances were reduced to matter, and matter to motion. The laws of matter were, now, simple laws of impact and communication of movement. Physicist had to discuss the relation between such scalar and vectorial quantities as force, movement, velocity, acceleration, mass, and so on. Regarding the organized top-structures, like living beings (or even crystals), the old names "substantial form" and "soul" were, thus, no longer applicable; considering that they were explainable in terms of the local motions and arrangements of their own constitutive parts, their new and true name was instead (natural) "machines".

Surely, the old concept of "form" was poorly illuminating. It explained really nothing. Notwithstanding, it had several advantages in the understanding of nature. Namely, it directed the attention from the very start to patterns of organization, and not simply to bits of bare mass (the "quantitas materiae"), and, in addition to this richer point of view, it endorsed a broader concept of movement, which encompassed no only local displacement, but also the phenomena of growing and alteration, which were characteristics of structured and organized beings. The Cartesian "reform" of physics was, then, based on several very hard although controversial assumptions. First, that bare matter could be considered by itself as a realm of local motion and forces acting by impact, that is, that kinematics and dynamics were the very core of physics; second, that organized natural

structures could be explained mechanically, without the involvement of teleological principles and final causes; third, that every time we were talking about something like a "plan", an "organizing principle" or an "intention", we would be talking about mind, i.e., about a psychic being, and definitively not about matter.

So it was with Descartes. What we gained by this *tour de force* is well known to all. No need to describe it again. Modern natural philosophy took physics as the fundamental science, and physics was defined as the science of local motion until Faraday and Maxwell's works in the 19th century about electromagnetism introduced a new branch. In line with this leading old idea, in the late 18th century Kant tried to present nothing less than an a priori deduction of the object of physics. In his *First Metaphysical Principles of the Science of Nature,* he presented physics as the science of matter, and the science of matter as the science of motion, to conclude, supposedly still in an a priori fashion, that the several parts of the science of matter in motion were Phoronomy (Kinematics), Dynamics, Mechanics, and Phenomenology (in a very particular sense).

But Kant himself realized at the same time the limits of mechanistic explanatory schemes. Let us return briefly to Descartes' theoretical decisions. They were certainly a tremendous step forward. But they were also a dramatic impoverishment. First of all, there are natural processes that seem to be driven by something like a plan, or a pattern, that determines in advance the disposition and the arrangement of the parts. Living organisms are such a case. But also global arrangements like the ones we can see in crystals or in snow and ice patterns suggest to the natural philosopher that there is something like a "natural technique" or a "natural plan", that is, something like an unconscious natural intention (or propensity) in several physical processes. However, the very center of this insight into the deep structure of matter lies in organization as such, not in finalistic or intentional processes. Final causes or the talk about a "natural technique" or a "natural intention" are a rather bewildering response to the problem of organization, i.e., to organization as a phenomenon of nature in need of a *natural* explanation. We can see now what happened: Descartes' reform suppressed from the science of nature any talk about final causes or intentions acting in natural phenomena, and, in so doing, the fact of organization, to which the concepts of final causes and forms used to apply, vanished from the regard of the natural philosopher. Refusing a bad answer to the problem of natural organization patterns (substantial forms, final causes, and so on), Descartes imposed to drop the very fact of organization or, better, the concepts that constitute organization as a fact. This is precisely the first impoverishment we wanted to emphasize: organization was formerly picked-up in nature through the concepts of forms, inner intentions, ends and plans; all those

concepts were now interpreted through the model of the purposive action of a conscious mind and, for that very reason, they were excluded from the science of matter; as a result, organization was regarded only as a secondary upper-level phenomenon of something more fundamental, namely, as a consequence of the pure and simple local motion of matter. Now we can see the second dramatic impoverishment Descartes' reform brought with it. Organization is a high-level phenomenon that is not explainable by a linear reasoning going from the parts to the whole and considering the whole as the simple sum of the parts. Organization is a pattern that emerges on an underlying multiplicity of elements, but in such a way that it *seems* that it was the pattern that directed in advance the disposition and arrangement of these elements. This was the "intentional" or "teleological" delusion to which the concepts of substantial form and final causes gave expression. Even Kant recognizes that organized structures, such as living beings, exhibit a complexity that lead to think *as if* the whole was not the result of the parts, but, contrariwise, as if it was the whole that commanded in advance the disposition of the parts. Kant considers, thus, that organized beings put an unbreakable limit to mechanistic explanation. Nevertheless, his proposal that we must reflect as if there was a "natural wisdom" and a "natural technique" (another fiction: the teleological fiction, here somewhat lessened by Kant's famous distinction between determining and reflective judgments) suffers from the old fault of considering that teleological concepts are the best (or even the sole) answer to the phenomenon of organization. As Kant saw it, organization puts before us an epistemological problem: we do not see how organization can be thinkable as a natural phenomenon (without teleological concepts). In refusing teleology –even if it simply refers to the unconscious patterns enclosed in the concept of a natural form– by assimilating all forms of design to the purposive action of conscious minds and in refusing, obviously, to talk about properties of minds in the science of nature, Descartes' reform closed at the same time all other paths to the organization phenomenon, so that the only intellectual scheme able to explain the top-properties of an organized natural structure was to try to recover them as an effect of the blind arrangement of the parts, as if they happened by pure chance.

Now the question is: how can we constitute the phenomenon of organization as a *natural fact*? We must give-up, obviously, the bold and straightforward teleological concepts. We can no longer think as if the whole preceded and directed in advance the arrangement of the parts. Instead, in order to take organization as a strict natural phenomenon, we must consider organization as a *self*-organization process.

For doing this, the following constitutive concepts are worth being considered:

Knotting – i.e., a tendency to join on the basis of some common or matching feature, like, for instance, frequency, energy level, opposite charge, or pairs of opposite spins, etc.: the entities resulting from knots can be "homeomeric" (the parts add to produce only one entity of the same nature, like the superposition of waves in phase), or "heteromeric" (the parts join to produce a twofold or n-fold entity, like opposite electron spins in an orbital).

Reinforcement – i.e., jointed physical entities strengthen the knots by mutually catalyzing the elements necessary to each other, so that they form whole that conserves them and constitute a new entity that interacts with the surrounding medium in a new global way; the systems formed by reinforcement can be stabilized systems or unstable systems, that need to get in the environment ever new conditions in order to endure.

Economy or least dispense – i.e., the feature that knotting and reinforcement are favoured because they reduce the losses that each entity in isolation would have through the interactions with the surrounding world, as if nature obeyed a principle of least action or as if nature chose according to a principle of the good path; instead, nature simply conserves the elements that get the conditions for preserving themselves by knotting and reinforcing, while the others disappear or enter other wholes in a process of physical natural selection, so to speak. Crocas's principle of eurhythmy goes this way: it speaks about good paths indeed, while avoiding the enchantment of teleological concepts.

An organized system is not a necessary event of nature. It has stochastic processes at its base and, for that very reason, is not predictable in a full deterministic way. A science of self-organization must deal with non-linear processes and, thus, it has to overthrow the old Cartesian linear ways of thinking. Yet, only organized structures can last and remain in nature, so that we can see the springing of *physis* as the coming about of organized systems in never ending growing degrees of complexity. We reach then a pretty different conception about matter. Descartes and modern thinking in general had a conception of matter as something which was lying passively under universal and unbreakable laws. Modern thinking used many times the fiction of God's legislative will in order to understand matter as such a passive substratum. Very suggestively, Descartes said that God gave His laws to nature as a king to his kingdom. Now we have some glimpses at a deeper dimension of matter. They show us matter as a process of self-organization, i.e., not as something that is just under laws, but as something that creates the lawful patterns (or the "forms") of its own existence. We can

finally see *physis* or nature as an ever-growing process of organization; that is to say: a never-ending process in which emerging patterns give the basis for the springing of new, more complex patterns, and so on indefinitely (are there laws of breakdown processes beyond a certain upper-limit of complexity?)

How deep does this self-organizing process go? Generally, we perceive it clearly in living organisms. But this is already halfway through the story. In order to characterize life, we must talk about cycles, i.e., loops of internal processes, replication, i.e., production of new similar organisms, agency, i.e., some active imprint in the surroundings, and cognizance, i.e., a kind of discernment or sensitivity to the environment. All this supposes the more basic processes of knotting and reinforcement, which belong to the lower levels of organization processes. Thus, our guess is that organization starts at the deepest level, and that all matter is a continual organization process, so that there is nothing as naked matter or some substance that is passively under laws (of motion or else) which are superimposed unto it. As we see it, when the simpler quantum entity springs in the theta-wave of the sub-quantum medium, the photonic acron, this *is* the emergence of organization at the most fundamental level. The photonic acron, while having zero rest mass and no charge, is indeed an already complex, organized entity: it has a physics that others will describe in this book.

Let us finish by aligning several insights about what the cosmos seems to be from this new point of view.

1. There is no initial singularity, but an everlasting sub-quantum medium.

2. There is no creation of the universe in an initial expansion, but a permanent process of creation of matter as a perturbation of the sub-quantum medium – this perturbation (whose physics is still to come) is the emergence of the acronic, quantum entity.

3. There is nothing before space and time, preceding the putative creation of the universe, but there is a permanent sub-quantum medium to which the concepts of time and space no longer apply.

4. In order to describe the complexity of the universe, we do not need to start with a violent expansion and to adjust *ad hoc* the initial parameters in order to recover the structure of the existing universe (e.g., the formation of galaxies), reasoning as if the development of matter was a blind, mechanistic and all-deterministic process completely defined in advance by the sole initial conditions; instead, we must follow the successive levels of

complexity, starting from the fundamental formation of quantum entities in the sub-quantum medium and ascending to the upper levels, understanding the propensity of matter to create ever new patterns of organization. We must give matter a chance, so to speak: if we allow, matter will find its way…

5. Entire universes can spring from the sub-quantum everlasting medium, having their complete story, their emergence and downfall; in this process of genesis and destruction, it is likely that many worlds have come and are still to come, and not only a unique universe, like the Big Bang story parochially suggested us.

For if we have the modesty to gaze beyond our nose and above our shoulders into the immensity of being, it is likely that we come at last to the conviction that there is an infinity of ever new worlds, as Giordano Bruno once said (and died for it).

THE CONCEPT OF TIME

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Summary: In New Physics, the so-called hyperphysis, Time is a primordial basic concept used to help in the understanding of physical reality. On the other hand, Space plays a minor role secondary to Time, remaining an important notion nonetheless.

Keywords: Time, space, Hyperphysis, subquantum medium, principle of eurhythmy, frequency, complex interaction, alteration, becoming.

1. INTRODUCTION

In this note, it is intended to show that Time is an utterly basic concept albeit being an important tool which helps to integrate the diverse interaction that one experiments in a relatively comprehensible yet global picture.

In classical physics (meaning relativistic physics and orthodox quantum mechanics) the concepts of energy and momentum play a most fundamental role whereas in Hyperphysis[1] the basic concepts are the concept of frequency, both temporal and spatial. Moreover, this new physics aims at describing the system's complex and nonlinear interaction at its upper and diverse scales of organization. The formalization of this New Physics is done mainly through the organizational principle of eurhythmy[2].

2. THE CONCEPT OF TIME

One is familiar to the categories of both time and space as something that is of assistance so to understand the multiplicity of interactions one is

confronted with. The same is to say to aim for the understanding of this global interaction process we call reality.

Naturally, outside organized[3] regions (the ones that we are able to conceive) it is not possible to talk neither of time nor of space. The most we allowed to do is to state that an indefinite medium exists… the subquantum medium, or the Apeiron if one likes best. In this picture the most important concept is the one of subquantum medium, where the concepts of space and time are only mere derivate concepts.

The problem, nonetheless, is to decide whether if one of these concepts, either the one of time or the one of space, has primacy over the other. If one does, it should be the concept of Time.

Could it be that when we speak about space we are implicitly speaking about Time?

In the context of Hyperphysis, Time is to be understood as a measurement… as an evaluation of change and of the alteration that is the modification of the complex interacting systems. We know that "change" has the utmost relevance in New Physics, being deeply connected with the interaction.

On the other hand, we also are aware that interaction means alteration, modification … change, that is. So, if there are no modifications there cannot be any interactions, for interaction implies modification. Without modifications there are no interactions and vice-versa. Overall, interaction is essentially a modification.

Considering these conditions, our world (the only one we can truly understand being a simple section of the subquantum medium) is nothing more than what endures the change, or the alteration. Our world is in permanent becoming.

At this point we must stress that this change must be validated by Time. Hence, it follows necessarily that, in the context of the New Physics and in this sense, the primary concept is the one of Time. Without Time we cannot begin to understand change, so no physics would be possible at all.

3. THE CONCEPT OF SPACE

All in all, the primacy of Time has been established.

Now, the problem is to understand the role played by space.

Knowing that in Hyperphysis the most basic concept is the one of interaction, of change, of the alteration of the becoming (in other words, the one of Time), it follows that the notion of space must somehow be derived from this primary concept.

Given this, everything leads us to believe that the notion of space has to do only with the necessity of identification, or with the conceptual separation of the emergent subquantum beings at the diverse scale of description.

In these conditions, and according to the scale and process of observation, different beings may occupy the same region in space. So, the conclusion to draw is that space is nothing more than a mere tool, more or less useful, however devoided of any privileged ontological status.

4. CONCLUSION

These new developments in the meaning of time and space, in the light of New Physics, the Hyperphysis, open a whole new realm of possibilities for understanding Nature.

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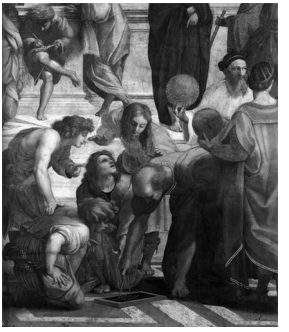

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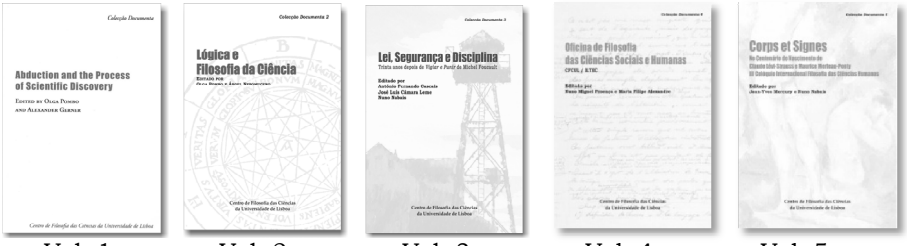
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